

Generalised Rational α_s -Meir-Keeler Contraction Mapping in S-metric Spaces

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Abstract In this paper, we introduce the concept of generalised rational α_s -Meir-Keeler contraction mapping on S-metric spaces. The existence of fixed points is also discussed.

Keywords: α -admissible, α_s -admissible, α_s -Meir-Keeler contraction mapping, S-metric space

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1. Introduction and Preliminaries

Banach contraction principle is one of the most interested topics for many researchers because of its applications in various fields, simplicity and easiness. They attempted to generalise Banach contraction principle in different dimensions. Samet et. al. [1] made an attempt by introducing the idea of α -admissible mappings and further introducing the concept of α - ψ -contractive type mappings in metric spaces. The results of Samet et. al. [1], show that Banach fixed point theorem and a large number of results in the literature are consequences of their results. On the other hand, as one the result of generalisation of metric space, Sedghi et. al. [2] introduced the definition of S-metric space. There are various works on generalisation of Banach contraction principle and generalisation of metric space in the literature. Some of these works can be found in the research papers through [3-29] and references mentioned in these papers.

Definition 1.1. [2] In a non-empty set X let $S: X \times X \times X \rightarrow [0, \infty)$ be a mapping satisfying

- (1) $S(a, b, c) \geq 0$,
- (2) $S(a, b, c) = 0$ if and only if $a = b = c$,
- (3) $S(a, b, c) \leq S(a, a, e) + S(b, b, e) + S(c, c, e)$

for all a, b, c, e in X then S is known as S- metric and pair (X, S) is known as S-metric space.

Definition 1.2. [2] We have for an S-metric space

$$S(a, a, b) = S(b, b, a).$$

Definition 1.3. [2] Let (X, S) be an S-metric space.

i) A sequence $\{a_n\}$ in X converges to a if and only if $S(a_n, a_n, a) \rightarrow 0$ as $n \rightarrow \infty$. That is there exists $n_0 \in \mathbb{N}$ for each $\varepsilon > 0$ such that $S(a_n, a_n, a) < \varepsilon$ for all $n \geq n_0$, and denoted as $\lim_{n \rightarrow \infty} a_n = a$.

ii) A sequence $\{a_n\}$ in X is said to be a Cauchy sequence if there exists $n_0 \in \mathbb{N}$ for each $\varepsilon > 0$, such that $S(a_n, a_n, a_m) < \varepsilon$ for each $n, m \geq n_0$.

iii) If every Cauchy sequence is convergent then S-metric space (X, S) is said to be complete.

Meir-Keeler [25] introduced a generalisation of Banach contraction principle. According to them, a self mapping U in a metric space (X, d) is said to be Meir-Keeler contraction if for an $\varepsilon > 0$ there exists $\delta > 0$ such that $\varepsilon \leq d(a, b) < \varepsilon + \delta(\varepsilon)$ implies $d(Ua, Ub) < \varepsilon$ for all a, b in X . They also state and prove that in a metric space, if a self mapping U is a Meir-Keeler contraction, then U has a fixed point in X .

In this paper we introduced the concept of α_s -Meir-Keeler contraction on S-metric space and proved a fixed point theorem.

Now, we recall the definition of α -admissible mappings and its generalisations in metric space, G-metric space, S-metric space and S_b -metric space.

Definition 1.4. [1] In a metric space (X, d) , let U be a self mapping and let $\alpha: X \times X \rightarrow [0, +\infty)$ be a function. U is said to be an α -admissible if $a, b \in X, \alpha(a, b) \geq 1$ becomes $\alpha(Ua, Ub) \geq 1$.

Example 1. Let $X = [0, +\infty)$, and $U: X \rightarrow X$ be a self mapping defined by $Ua = 5a$ for all $a, b \in X$. Let $\alpha: X \times X \rightarrow [0, +\infty)$ be defined by

$$\alpha(a, b) = \begin{cases} e^{b/a}, & \text{if } a \geq b, a \neq 0 \\ 0, & \text{if } a < b. \end{cases}$$

Then, U is an α -admissible mapping.

Let X be a metric space for further consideration if otherwise stated.

Definition 1.5. [3] Suppose $U, V: X \rightarrow X$ and $\alpha: X \times X \rightarrow [0, +\infty)$ then (U, V) pair is said to be α -admissible if $\alpha(a, b) \geq 1$ for $a, b \in X$, then $\alpha(Ua, Vb) \geq 1$ and $\alpha(Va, Ub) \geq 1$.

Definition 1.6. [4] Suppose $U: X \rightarrow X$ and $\alpha: X \times X \rightarrow (-\infty, +\infty)$ then mapping U is said to be a triangular α -admissible if

- (i) $\alpha(a, b) \geq 1$, implies $\alpha(Ua, Ub) \geq 1, a, b \in X$,
- (ii) $\alpha(a, c) \geq 1, \alpha(c, b) \geq 1$, implies $\alpha(a, b) \geq 1, a, b, c \in X$.

Definition 1.7. [3] Suppose $U, V: X \rightarrow X$ and $\alpha: X \times X \rightarrow [0, +\infty)$ then (U, V) pair is said to be a triangular α -admissible mapping if

(i) $\alpha(a, b) \geq 1$, implies $\alpha(Ua, Vb) \geq 1$ and $\alpha(Va, Ub) \geq 1$, $a, b \in X$,

(ii) $\alpha(a, c) \geq 1, \alpha(c, b) \geq 1$, implies $\alpha(a, b) \geq 1, a, b, c \in X$.

Definition 1.8. [5] Suppose $U: X \rightarrow X$ and let $\alpha, \eta: X \times X \rightarrow [0, +\infty)$ be two functions then U is said to be α -admissible mapping with respect to η if $\alpha(a, b) \geq \eta(a, b)$ implies $\alpha(Ua, Ub) \geq \eta(Ua, Ub)$ for all $a, b \in X$.

If we consider $\eta(a, b) = 1$, in definition 1.8 then we get definition 1.4. Also, U is said to be a η -sub admissible mapping if $\alpha(a, b) = 1$.

Lemma 1.1. [6] Let (X, d) be a metric space and $U: X \rightarrow X$ be a triangular α -admissible mapping. Suppose $a_0 \in X$ such that $\alpha(a_0, Ua_0) \geq 1$. Let us define a sequence $\{a_n\}$ as $a_{n+1} = Ua_n$. Then $\alpha(a_n, a_m) \geq 1$ for all $m, n \in \mathbb{N} \cup \{0\}$ with $n < m$.

Lemma 1.2. [7] Let (X, d) be a metric space and $U, V: X \rightarrow X$ be a triangular α -admissible mapping. Suppose $a_0 \in X$ such that $\alpha(a_0, Ua_0) \geq 1$. Let us define sequences $a_{2i+1} = Ua_{2i}$ and $a_{2i+2} = Va_{2i+1}$, where $i = 0, 1, 2, \dots$. Then $\alpha(a_n, a_m) \geq 1$ for all $m, n \in \mathbb{N} \cup \{0\}$ with $n < m$.

The notion of α -admissible mappings was generalised by Alghamdi and Karapinar [8] to make it suitable for G -metric space and termed it as β -admissible. The definition itself is as follows.

Definition 1.9. [8] Let (X, G) be a G -metric space, $U: X \rightarrow X$ and $\beta: X \times X \times X \rightarrow [0, +\infty)$, then U is said to be β -admissible if for all $a, b, c \in X$, $\beta(a, b, c) \geq 1$ implies $\beta(Ua, Ub, Uc) \geq 1$.

They had given suitable example for β -admissible mappings. Further, they also generalised the α - ψ contractive mappings by introducing generalised G - β - ψ contractive mappings of type I and II.

Further, Hussain et. al. [9] generalised the concept of α -admissible mappings in G -metric space. They introduced the concept of rectangular G - α -admissible and extended this concept for two mappings.

By introducing G - η -sub admissible mapping and α -dominating map Ansari et. al. [10] also studied α -admissible mappings in G -metric space. Another type called η -sub dominating map is also introduced by them. They also introduced α -regular in the context of G -metric space, partially weakly G - α -admissible and partially weakly G - η -sub admissible mappings, etc.

Zhou et. al. [11] also extended the concept of α -admissible mappings in S -metric space under the name γ -admissible. It is defined as follows:

Definition 1.10. [11] Let $U: X \rightarrow X$ and $\gamma: X^3 \rightarrow [0, +\infty)$ then U is said to be γ -admissible if for all $a, b, c \in X$, $\gamma(a, b, c) \geq 1$ implies $\gamma(Ua, Ub, Uc) \geq 1$.

The notion of generalised S - β - ψ contractive type mappings was introduced by Bulbul et. al. [12] on the similar way as that of generalised G - β - γ contractive type mappings, but in S -metric space. The notion of α -admissible mappings in S_b -metric space was also introduced by Nabil et. al. [13].

Definition 1.11. Let (X, S) be an S -metric space, $U: X \rightarrow X$ and $\alpha_s: X \times X \times X \rightarrow [0, +\infty)$, then U is called α_s -admissible if $a, b, c \in X$, $\alpha_s(a, b, c) \geq 1$ implies $\alpha_s(Ua, Ub, Uc) \geq 1$.

Example 2. Let $X = [0, +\infty)$ and define $U: X \rightarrow X$ and $\alpha_s: X \times X \times X \rightarrow [0, +\infty)$ by $Ua = 4a$, for all $a, b, c \in X$ and

$$\alpha_s(a, b, c) = \begin{cases} \frac{c}{e^{ab}}, & \text{if } a \geq b \geq c; a, b \neq 0 \\ 0, & \text{if } a < b < c. \end{cases}$$

Then U is α_s -admissible.

Definition 1.12. Let (X, S) be an S -metric space, $U, V: X \rightarrow X$ and $\alpha_s: X \times X \times X \rightarrow [0, +\infty)$. We say that the pair (U, V) is α_s -admissible if $a, b, c \in X$ such that $\alpha_s(a, b, c) \geq 1$, then we have $\alpha_s(Ua, Ub, Vc) \geq 1$ and $\alpha_s(Va, Vb, Uc) \geq 1$.

Definition 1.13. Let (X, S) be an S -metric space, $U: X \rightarrow X$ and $\alpha_s: X \times X \times X \rightarrow [0, +\infty)$. We say that U is triangular α_s -admissible mapping if

(i) $\alpha_s(a, b, c) \geq 1$ implies $\alpha_s(Ua, Ub, Uc) \geq 1, a, b, c \in X$.

(ii) $\alpha_s(a, c, e) \geq 1, \alpha_s(b, b, e) \geq 1$ and $\alpha_s(c, c, e) \geq 1$ implies $\alpha_s(a, b, c) \geq 1, a, b, c, e \in X$.

Definition 1.14. Let (X, S) be an S -metric space, $U: X \rightarrow X$ and let $\alpha_s, \eta_s: X \times X \times X \rightarrow [0, +\infty)$ be two functions. We say that U is α_s -admissible mapping with respect to η_s if $a, b, c \in X$, $\alpha_s(a, b, c) \geq \eta_s(a, b, c)$ implies $\alpha_s(Ua, Ub, Uc) \geq \eta_s(Ua, Ub, Uc)$.

Note that if we take $\eta_s(a, b, c) = 1$, then this definition reduces to definition 1.11. Also, if we take $\alpha_s(a, b, c) = 1$, then we say that U is an η_s -subadmissible mapping.

Now we state the following two lemmas in the line of Lemma 1.1 and Lemma 1.2.

Lemma 1.3. Let (X, S) be an S -metric space, $U: X \rightarrow X$ be a triangular α_s -admissible mapping. Assume that there exists $a_0 \in X$ such that $\alpha_s(a_0, a_0, Ua_0) \geq 1$. Define a sequence $\{a_n\}$ by $a_{n+1} = Ua_n$. Then we have $\alpha_s(a_n, a_n, a_m) \geq 1$, for all $m, n \in \mathbb{N} \cup \{0\}$.

Lemma 1.4. Let (X, S) be an S -metric space, $U, V: X \rightarrow X$ be a triangular α_s -admissible mapping. Assume that there exists $a_0 \in U$ such that $\alpha_s(a_0, a_0, Ua_0) \geq 1$. Define sequences $a_{2i+1} = Ua_{2i}$ and $a_{2i+2} = Va_{2i+1}$, where $i = 0, 1, 2, \dots$. Then we have $\alpha_s(a_n, a_n, a_m) \geq 1$ for all $m, n \in \mathbb{N} \cup \{0\}$ with $n < m$.

2. Main Result

We present the following results.

Definition 2.1. In an S -metric space (X, S) let $U: X \rightarrow X$ be a triangular α_s -admissible mapping. Suppose that for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\varepsilon \leq \nabla_1(a, b, c) < \varepsilon + \delta$$

implies

$$\alpha_s(a, b, c)S(Ua, Ub, Uc) < \varepsilon \tag{1}$$

where

$$\nabla_1(a, b, c) = \max \left\{ \begin{array}{l} S(a, b, c), S(Ua, Ub, Uc), \\ \frac{S(a, a, Ua) \cdot S(b, b, Ub)}{1 + S(a, b, c) + S(Ua, Ub, Uc)}, \\ \frac{S(b, b, Ub) \cdot S(c, c, Uc)}{1 + S(a, b, c) + S(Ua, Ub, Uc)}, \\ \frac{S(c, c, Uc) \cdot S(a, a, Ua)}{1 + S(a, b, c) + S(Ua, Ub, Uc)} \end{array} \right\} \tag{2}$$

for all $a, b, c \in X$. Then U is called a generalised rational α_s -Meir Keeler contraction of type-I.

Definition 2.2. In an S-metric space (X, S) let $U : X \rightarrow X$ be a triangular α_s -admissible mapping. Suppose that for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\varepsilon \leq \nabla_2(a, a, b) < \varepsilon + \delta$$

implies

$$\alpha_s(a, a, b)S(Ua, Ua, Ub) < \varepsilon \tag{3}$$

where

$$\nabla_2(a, a, b) = \max \left\{ \begin{array}{l} S(a, a, b), \\ S(Ua, Ua, Ub), \\ \frac{S(a, a, Ua) \cdot S(a, a, Ua)}{1 + S(a, a, b) + S(Ua, Ua, Ub)}, \\ \frac{S(a, a, Ua) \cdot S(b, b, Ub)}{1 + S(a, a, b) + S(Ua, Ua, Ub)} \end{array} \right\} \tag{4}$$

for all $a, b \in X$. Then U is called a generalised rational α_s -Meir Keeler contraction of type-II.

Theorem 2.1. In a complete S-metric space (X, S) , $U : X \rightarrow X$ be a mapping satisfying

(i) U is generalised rational α_s -Meir-Keeler contraction of type-I.

(ii) U is triangular α_s -admissible.

(iii) there exists $u_0 \in X$ such that $\alpha_s(u_0, u_0, Uu_0) \geq 1$.

(iv) U is continuous.

Then U has a fixed point in X .

Proof. Let $u_1 \in X$ be such that $u_1 = Uu_0$. Then, we construct a sequence u_n of points in X such that,

$$u_{2i+1} = Au_{2i} \tag{5}$$

where $i = 0, 1, 2, 3, \dots$

By assumption $\alpha_s(u_0, u_0, u_1) \geq 1$ and U is α_s -admissible, by Lemma 1.3, we have

$$\alpha_s(u_n, u_n, u_{n+1}) \geq 1 \text{ for all } n \in N \cup \{0\}. \tag{6}$$

Then,

$$\begin{aligned} \varepsilon &\leq \nabla_1(u_{2i}, u_{2i}, u_{2i+1}) < \varepsilon + \delta \\ &\rightarrow \alpha_s(u_{2i}, u_{2i}, u_{2i+1})S(Uu_{2i}, Uu_{2i}, Uu_{2i+1}) < \varepsilon \end{aligned} \tag{7}$$

for all $i \in N \cup \{0\}$. Now,

$$\begin{aligned} &\nabla_1(u_{2i}, u_{2i}, u_{2i+1}) \\ &= \max \left\{ \begin{array}{l} \frac{S(u_{2i}, u_{2i}, u_{2i+1}), S(Uu_{2i}, Uu_{2i}, Uu_{2i+1}), \\ S(u_{2i}, u_{2i}, Uu_{2i}) \cdot S(u_{2i}, u_{2i}, Uu_{2i})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(Uu_{2i}, Uu_{2i}, Uu_{2i+1})}, \\ \frac{S(u_{2i}, u_{2i}, Uu_{2i}) \cdot S(u_{2i+1}, u_{2i+1}, Uu_{2i+1})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(Uu_{2i}, Uu_{2i}, Uu_{2i+1})}, \\ \frac{S(u_{2i}, u_{2i}, Uu_{2i}) \cdot S(u_{2i+1}, u_{2i+1}, Uu_{2i+1})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(Uu_{2i}, Uu_{2i}, Uu_{2i+1})} \end{array} \right\} \end{aligned}$$

$$= \max \left\{ \begin{array}{l} \frac{S(u_{2i}, u_{2i}, u_{2i+1}), S(u_{2i+1}, u_{2i+1}, u_{2i+2}), \\ S(u_{2i}, u_{2i}, u_{2i+1}) \cdot S(u_{2i}, u_{2i}, u_{2i+1})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(u_{2i+1}, u_{2i+1}, u_{2i+2})}, \\ \frac{S(u_{2i}, u_{2i}, u_{2i+1}) \cdot S(u_{2i+1}, u_{2i+1}, u_{2i+2})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(u_{2i+1}, u_{2i+1}, u_{2i+2})}, \\ \frac{S(u_{2i}, u_{2i}, u_{2i+1}) \cdot S(u_{2i+1}, u_{2i+1}, u_{2i+2})}{1 + S(u_{2i}, u_{2i}, u_{2i+1}) + S(u_{2i+1}, u_{2i+1}, u_{2i+2})} \end{array} \right\}$$

If

$$\begin{aligned} &\max \{S(u_{2i}, u_{2i}, u_{2i+1}), S(u_{2i+1}, u_{2i+1}, u_{2i+2})\} \\ &= S(u_{2i+1}, u_{2i+1}, u_{2i+2}) \end{aligned}$$

then

$$\begin{aligned} \varepsilon &\leq S(u_{2i+1}, u_{2i+1}, u_{2i+2}) \leq \varepsilon + \delta \\ &\Rightarrow \alpha_s(u_{2i}, u_{2i}, u_{2i+1}) \cdot S(Uu_{2i}, Uu_{2i}, Uu_{2i+1}) < \varepsilon. \end{aligned}$$

Therefore, we deduce that

$$\begin{aligned} &S(u_{2i+1}, u_{2i+1}, u_{2i+2}) \\ &\leq \alpha_s(u_{2i}, u_{2i}, u_{2i+1}) \cdot S(Uu_{2i}, Uu_{2i}, Uu_{2i+1}) \\ &< \varepsilon \\ &\leq S(Uu_{2i+1}, Uu_{2i+1}, Uu_{2i+2}). \end{aligned}$$

which is a contradiction. Hence

$$S(u_{2i+1}, u_{2i+1}, u_{2i+2}) < S(u_{2i}, u_{2i}, u_{2i+1}) \tag{8}$$

This implies that

$$S(u_{n+1}, u_{n+1}, u_{n+2}) < S(u_n, u_n, u_{n+1}) \tag{9}$$

for all $n \in N \cup \{0\}$.

So, sequence $\{S(u_n, u_n, u_{n+1})\}$ is nonnegative and nonincreasing. Now, we have to show that $S(u_n, u_n, u_{n+1}) \rightarrow 0$. It is clear that $\{S(u_n, u_n, u_{n+1})\}$ is a decreasing sequence. Therefore, $\lim_{n \rightarrow \infty} S(u_n, u_n, u_{n+1}) = r$ for some positive number r .

Let $r > 0$. Then we must have

$$0 < r \leq S(u_n, u_n, u_{n+1}) \tag{10}$$

for all $n \in N \cup \{0\}$.

Since the condition (7) holds for every $\varepsilon > 0$, we may choose $\varepsilon = r$ then there exists $\delta(\varepsilon) > 0$ satisfying (3). In other words,

$$\begin{aligned} &r < S(u_n, u_n, u_{n+1}) < r + \delta \\ &\Rightarrow \alpha_s(u_n, u_n, u_{n+1}) \cdot S(Uu_n, Uu_n, Uu_{n+1}) \cdot r. \end{aligned}$$

However, this implies

$$\begin{aligned} &r \leq \nabla_1(u_n, u_n, u_{n+1}) = S(u_n, u_n, u_{n+1}) < r + \delta \\ &\Rightarrow S(u_{n+1}, u_{n+1}, u_{n+2}) \\ &\leq \alpha_s(u_n, u_n, u_{n+1}) S(Uu_n, Uu_n, Uu_{n+1}) < r. \end{aligned} \tag{11}$$

A contradiction and hence $r = 0$, that is

$$\lim_{n \rightarrow \infty} S(u_n, u_n, u_{n+1}) = 0 \quad (12)$$

Next, we have to show that sequence $\{u_n\}$ is Cauchy. If possible let $\{u_n\}$ is not a Cauchy sequence. Then there exists sequences $\{u_{m_k}\}$ and $\{u_{n_k}\}$ with $\varepsilon > 0$ such that,

$$S(u_{m_k}, u_{m_k}, u_{n_k}) \geq \varepsilon \quad (13)$$

and

$$S(u_{m_{k-1}}, u_{m_{k-1}}, u_{n_k}) < \varepsilon \quad (14)$$

where $m_k > n_k > k$.

Using the triangle inequality and (14),

$$\begin{aligned} \varepsilon &\leq S(u_{m_k}, u_{m_k}, u_{n_k}) \\ &\leq 2S(u_{m_k}, u_{m_k}, u_{m_{k-1}}) + S(u_{m_{k-1}}, u_{m_{k-1}}, u_{n_k}) \\ &< 2S(u_{m_k}, u_{m_k}, u_{m_{k-1}}) + \varepsilon. \end{aligned}$$

Applying $k \rightarrow \infty$ we obtain

$$\lim_{k \rightarrow \infty} S(u_{m_k}, u_{m_k}, u_{n_k}) = \varepsilon \quad (15)$$

Also, from the triangular inequality, we have

$$\begin{aligned} &\left| S(u_{n_{k+1}}, u_{n_{k+1}}, u_{m_k}) - S(u_{m_k}, u_{m_k}, u_{n_k}) \right| \\ &\leq 2S(u_{n_k}, u_{n_k}, u_{n_{k+1}}) \end{aligned}$$

and

$$\begin{aligned} &\left| S(u_{m_{k+1}}, u_{m_{k+1}}, u_{n_{k+1}}) - S(u_{n_{k+1}}, u_{n_{k+1}}, u_{m_k}) \right| \\ &\leq 2S(u_{m_{k+1}}, u_{m_{k+1}}, u_{m_k}) \end{aligned}$$

Applying $k \rightarrow \infty$ we obtain

$$\lim_{k \rightarrow \infty} S(u_{n_{k+1}}, u_{n_{k+1}}, u_{m_k}) = \varepsilon$$

and

$$\lim_{k \rightarrow \infty} S(u_{m_{k+1}}, u_{m_{k+1}}, u_{n_{k+1}}) = \varepsilon. \quad (16)$$

By Lemma 1.3, $\alpha(u_{n_k}, u_{n_k}, u_{m_{k+1}}) \geq 1$, we have

$$\begin{aligned} S(u_{n_{k+1}}, u_{n_{k+1}}, u_{m_{k+2}}) &= S(Uu_{n_k}, Uu_{n_k}, Uu_{m_{k+1}}) \\ &\leq \alpha_s(u_{n_k}, u_{n_k}, u_{m_{k+1}}) \cdot S(Uu_{n_k}, Uu_{n_k}, Uu_{m_{k+1}}) \leq \varepsilon \end{aligned}$$

a contradiction. This shows that $\{u_n\}$ is a Cauchy sequence. By completeness of X , there exists a $a \in X$ such that $u_n \rightarrow a$ and hence $u_{2i+1} \rightarrow a$. By continuity of U we get $Uu_{2i+1} \rightarrow Ua$. Thus $a = Ua$ showing that 'a' is a fixed point of U .

3. Conclusion

The concept of generalised rational α_s -Meir-Keeler contraction mapping on S-metric spaces is introduced by giving two contractive definitions. The existence of fixed

points for the new contractive type mappings is discussed. Further study for α_s -Meir-Keeler contraction mapping in S-metric, S_β -metric G-metric and G_β -metric spaces can be carried out.

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Conflict of Interest

There is no conflict of interest.

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