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Life Expectancy of a System with Successive Stages of Deterioration

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Abstract In this paper we present Life Expectancy and Availability of a patient suffering from Kidney disease, where the deteriorating rates follow the Weibull distribution. In our paper, we present modelling and estimate the life expectancy/mean time to absorbtion and availability of the system having four successive stages of deteriorating system. Explicit expressions for the availability, mean sojourn time and life expectancy/mean time to absorbtion are obtained consequently. Graphs of availability and mean time to absorbtion are also plotted to dispose the nature with respect to time for distinct values of deteriorating rates.

Keywords: *life expectancy, mean time to absorbtion, availability*

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1. Introduction

An early stage detection of a disease may be more responsive to treatment. Kidney disease affects our body's ability to clean blood and help to control our blood pressure. There are two kidneys in our body which are located on both side of the spine at the lowest level of the rib cage, commonly called Flank region. It is known that a vital role of the kidneys is to remove toxic waste products and eliminate extra fluids from our body. The elimination of waste products and extra fluid is done by urine. The critical regulation and hemodynamic balance of the body's minerals and acid content is mainly done by the kidneys. Kidneys also produce harmones which affect the functioning of rest of the organs such as Erythropoetin, a harmone that is generated by kidneys stimulates red blood cell production in our body.

Diagnosis of kidney disease at an early stage is the key to keep acute kidney disease from gradually recurring kidney failure. The kidney failure is very fatal to our body. Kidney disease can be due to many reasons. Diabetes and high blood pressure also affect kidneys. Few kidney diseases are inherited from one generation to another. Many kidney diseases can also be caused by drugs and toxins. The use of (OTC) medicines in excessive amounts for a prolonged time causes kidney damage. If your kidneys have a working capacity of less than 15 percent of normal, then call it kidney failure. Kidney failure means that your kidneys are not in a position to do the above said jobs for a long time, as a result, additional health problems also may arise. As your kidney problem is serious, it can be a challenge to maintain your body equilibrium. Treatment with kidney replacement or dialysis will help to increase your life expectancy and it will help to feel better.

Treatment by hemodialysis method can be done at the dialysis unit or at home. It has been evidenced that success rate of kidney transplantation method is very high. The kidney used for transplantation may be come from someone, ie, that person, who donate kidney may be a relative, friend or even a strange person.

For the purpose to model the course of diseases, multistate markov models are commonly used in continuous time. A multistate model is demonstrated in Figure 1. These types of models represent a series of consecutively more critical states of degradation. In case of critical deterioration states, the patient may also be completely healthy or even in the next stages of the diseases. The stages $S_i(t)$ are observed by a number of individuals i at time t which is arbitrary,that may alter between individuals. In our paper we described the model by a transition intensity matrix Q. Using kolmogorov differential equation, explicit expressions for life expectancy and availability of the systems are derived. These types of models with various number of transient states have been used in many application like cancer, chronic diseases, screening of breast cancer etc. Kay etal. [1] Presented the survival analysis of cancer markers and disease states by using a markov model. Andersen [2] discussed a model related to mortality in diabetes. Longini etal [3] studied the step by step statistical analysis of HIV infection by a markov model. Klotz etal. [4] discussed a heart transplant model.Using Markov chain process some models related to breast cancer screening were discussed by Duffy etal [5].

Table 1. Description and States of the system

State No.	Description	State name
1	Both kidneys are working	S0
2	Only one kidney is working	S1
3	Both kidneys deteriorate and does not work properly	S2
4	Patient undergoing dialysis	S3

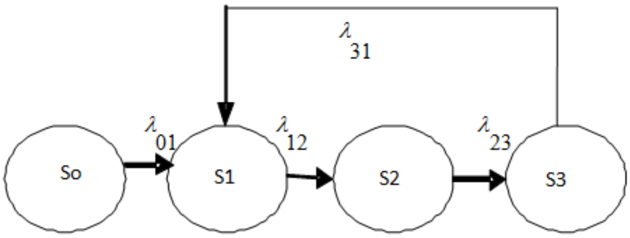

Figure 1.

Depending on the above interpretative model presented in Figure 1, the transition rate matrix of the model is as follows:

$$Q=\begin{bmatrix}-\lambda_{01} & 0 & 0 & 0\\ \lambda_{01} & -\lambda_{12} & 0 & \lambda_{31}\\ 0 & \lambda_{12} & -\lambda_{23} & 0\\ 0 & 0 & \lambda_{23} & -\lambda_{31}\end{bmatrix} \qquad (*)$$

Q is a 4×4 matrix, its elements λ_{ij} are the instantaneous rates of transition from one state to another.

2. Reliability Assessment

$P_i(t)$ is the probability that the system is in state S_i. Let P (t) be the probability row vector at time t, the initial condition for this problem are:

$$P(0)=\left[P_0(0), P_1(0), P_2(0), P_3(0)\right]=[1,0,0,0].$$

We obtain the following differential equation

$$P'(t)=P(t).Q.$$

It is difficult to evaluate the transient solutions hence following [7,8,9], so for the calculation of the life expectancy/mean time to absorbtion of the system, we take transpose of matrix Q and rows and columns for the absorbing state are deleted then new matrix become, say A. From equation (*) we calculated the expected time to reach an absorbing state.

The expected time to reach an absorbing state is obtained from

$$\text{Mean time to absorption}=P(0)\left(-A^{-1}\right)\begin{bmatrix}1\\1\end{bmatrix}$$

where $A=\begin{bmatrix}\lambda_{01} & \lambda_{01}\\ 0 & -\lambda_{12}\end{bmatrix}.$

$$\text{Life Expectancy}=\frac{\lambda_{12}}{\lambda_{01}\lambda_{12}}.$$

3. Availability Analysis

For availability analysis the initial condition is same as that in case of reliability:

P (0) = [1,0,0,0], the differential equations form can be expressed as shown below:

$$\begin{bmatrix}P_0'\\P_1'\\P_2'\\P_3'\end{bmatrix}=\begin{bmatrix}-\lambda_{01} & 0 & 0 & 0\\ \lambda_{01} & -\lambda_{12} & 0 & \lambda_{31}\\ 0 & \lambda_{12} & -\lambda_{23} & 0\\ 0 & 0 & \lambda_{23} & -\lambda_{31}\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\end{bmatrix}.$$

In the steady state, the derivatives of the state probabilities become zero as presented in equation.

$$QP(\infty)=0.$$

Then the steady state Availability can be calculated as revealed in equation

$$A(\infty)=P_0(\infty)+P_1(\infty)$$
$$=\frac{\lambda_{01}\lambda_{23}\lambda_{31}}{\lambda_{01}\lambda_{12}\lambda_{23}+\lambda_{01}\lambda_{23}\lambda_{31}+\lambda_{01}\lambda_{12}\lambda_{31}}.$$

4. Mean Sojourn Time

The mean sojourn time in state of a CTMC is calculated in terms of transition rates. Hence, we calculate that

$$e_0=\int_0^\infty e^{-\lambda_{01}t}dt=\frac{1}{\lambda_{01}},$$

$$e_1=\int_0^\infty e^{-\lambda_{12}t}dt=\frac{1}{\lambda_{12}}.$$

5. Discussion

Since the deterioration rates follow the Weibull distribution f (t) = $\lambda_k\delta t^{\delta-1}$, k =1, 2, 3, δ is the shape parameter. Take $\lambda_{31}=\alpha$ as restoration rate

Then the transition matrix takes the form

$$Q=\begin{bmatrix}-\lambda_1\delta t^{\delta-1} & 0 & 0 & 0\\ \lambda_1\delta t^{\delta-1} & -\lambda_2\delta t^{\delta-1} & 0 & \alpha\\ 0 & \lambda_2\delta t^{\delta-1} & -\lambda_3\delta t^{\delta-1} & 0\\ 0 & 0 & \lambda_3\delta t^{\delta-1} & -\alpha\end{bmatrix}.$$

To express the affect of restoration and deteriorating rate, some numerical examples are presented related to mean time to absorption and steady state availability of the system which are based on some parametric values. For the purpose of computation, following set of parameter values are used:

$$\alpha=0.3, \lambda_1=0.4, \lambda_2=0.8, \lambda_3=0.9, \delta=0.9.$$

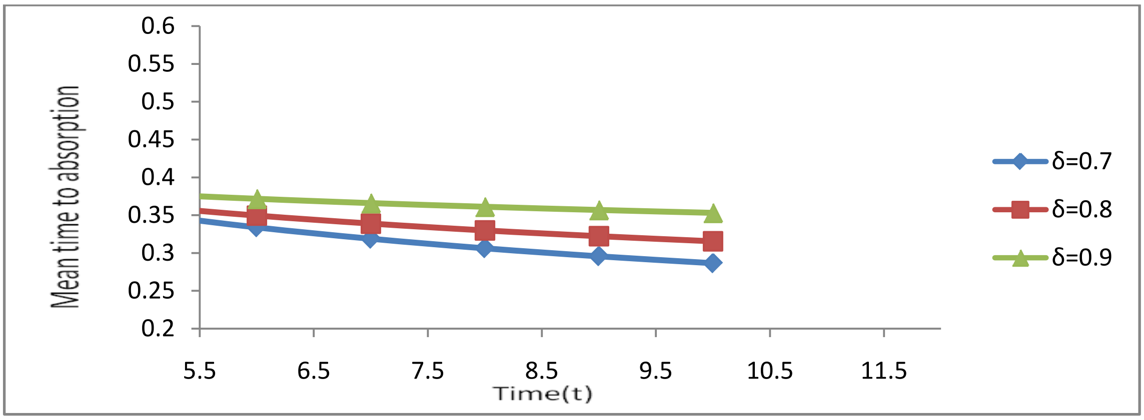

Figure 2. life expectancy/mean time to absorbtion against different values of δ(0.7, 0.8, 0.9)

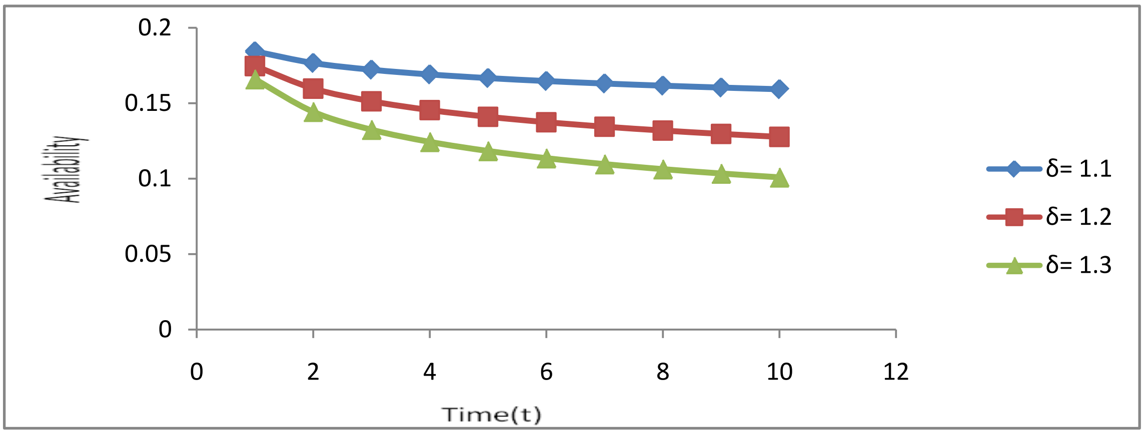

Figure 3. Availability against time for different values of δ (1.1, 1.2, 1.3)

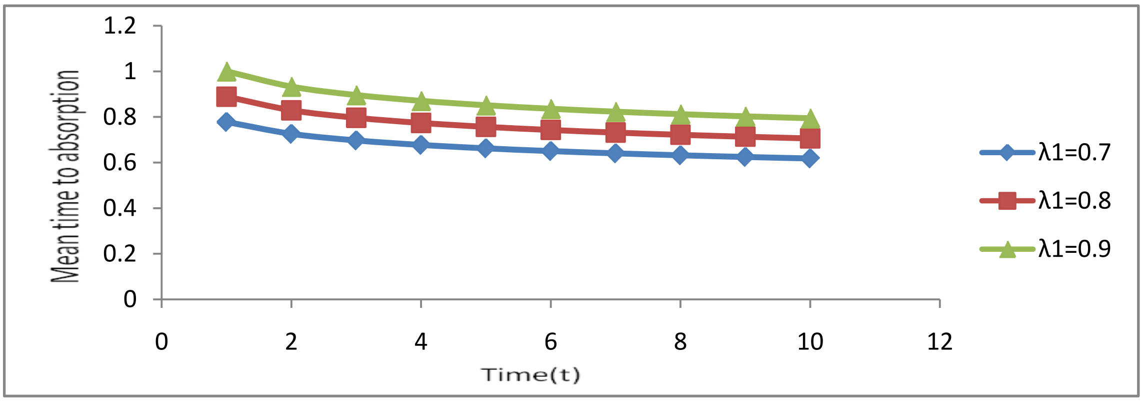

Figure 4. Mean time to absorbtion against for different values of λ_1 (0.7, 0.8, 0.9)

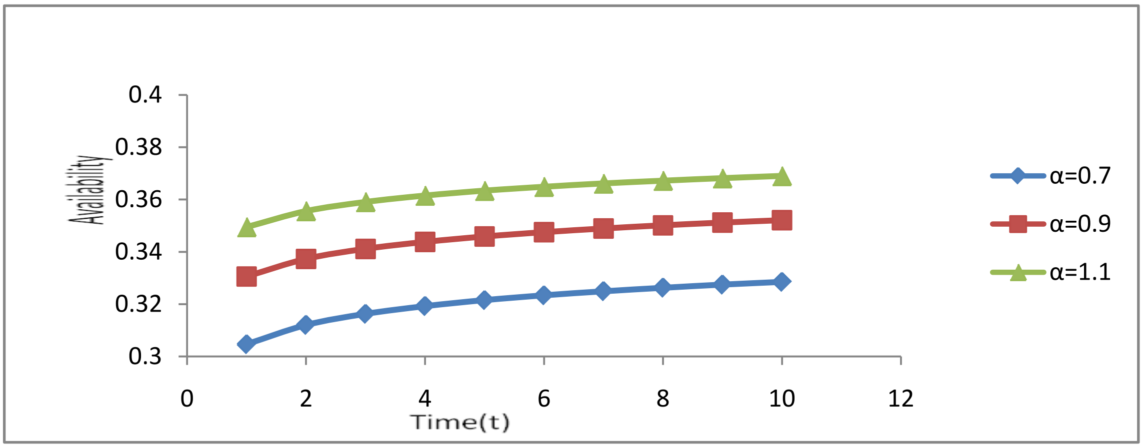

Figure 5. Availability against for different values of α(0.7, 0.8, 0.9)

6. Conclusion

Numerical results of mean time to absorbtion and availability with respect to time are depicted in Figure 2 and Figure 3 for different values of deteriorating rate δ. In these figure the mean time to absorbtion and availability decreases as deteriorating rate δ increases. On the other hand in Figure 4, as deteriorating rate λ_1 increases from 0.7 to 0.9 then mean time to absorbtion decreases and in Figure 5, as restoration rate α increases then availability increases.

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