Optimal Design of Step Stress Partially Accelerated Life Test under Progressive Type-II Censored Data with Random Removal for Gompertz Distribution

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Abstract This paper deals with random removal of progressively type II censored data. The removal of the data is assumed to follow a binomial or a uniform distribution, and the life time testing is assumed to follow a Gompertz distribution. Parameters of these distributions are estimated using the Maximum Likelihood estimation procedure. Fisher information matrix is used to estimate the asymptotic mean squared error and to construct confidence intervals of model parameters. The optimal partially accelerated lifetime testing (PALT) is estimated by minimizing the Generalized Asymptotic Variance (GAV). Simulation study is performed to Clarification the statistical properties of the parameters. A simulation results reveal that for the fixed values of the parameters, the error and optimal time decrease with increasing sample size n; estimates of binomial are more stable with a relatively small error with increasing sample size; and the test design is robust and works well for binomial removal.

Keywords: *partially accelerated life test, PALT, SS-PALT, progressively type II censored data, Gompertz distribution optimal design, D-optimality*

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1. Introduction

Traditional life testing may show no failure or few failures of some highly reliable units because of some severe conditions (stresses) that may occur in the form of pressure, voltage, temperature, vibration, load, cycling rate, etc. In such situations life testing has to be performed at higher than usual conditions in order to obtain failures as fast as possible. The data collected under stresses or severe conditions should not be used to estimate the life distribution at normal use assumptions. There are three types of stress, a) step stress; b) progressive stress, and c) constant stress; Ismail, [1] when testing is conducted under stress, it is called either "Accelerated Life Testing (ALT)" or "Partially Accelerated Life Testing (PALT)". In 'Alt" testing, units are put under stress to reach more failure in short time, and the mathematical stress model is known; while in "PALT", data can be extrapolated to normal use conditions, and the statistical model parameters could be estimated and step-stress could be applied. Nelson [2] proposed the step stress PALT (SS-PALT) where units are tested at normal use conditions within a specified time interval. Censored schemes arise when not all lifetime testing units are observed. When a unit does not fail within that time interval, it is put under stress, until failure

or until the termination of the time interval (censored scheme). Rao [3] and Balakrishnan and Aggarwala [4] have mentioned that step stress saves money, effort and time. There are two censoring scheme types. Type I arises when the experiment continues up to a pre-specified time T and Type-II censoring scheme requires the experiment to continue until a pre-specified number of failures m ≤ n occur.

These two types do not allow for the removal of any unit until life testing is terminated. Nevertheless, this allowance may be desirable to compromise between at least one observation is sought and a reduced time interval for the experimentation. At this situation, progressive censoring is needed.

In the progressive Type II censoring scheme, the experimenter selects n (iid) units for life testing; and observes the occurrence of the first failure r_1, at time $t_{(1)}$, and this r_1 is removed from testing. When the second failure occurs r_2 at time $t_{(2)}$, one of the surviving units are randomly selected and removed from the test, the experiment terminates when the mth failure occurs at time $t_{(m)}$ and the remaining surviving units $r_m = n - m - r_1 - r_2 - \cdots - r_{m-1}$ are all removed from the Test Progressive type-II censoring scheme with fixed number of removals $r_1, r_2, \ldots, r_m$ is considered by Cohen [5] and Cohen and Norgaard [6]. In some reliability

experiments and when the number of removals is not fixed, a progressive censoring with random removals model could be considered [7,8,9]. The random removal is assumed to follow the binomial distribution, and used to estimate the parameters of the reliability model.

Ismail [1] discusses step-stress partially-accelerated model. The model differs than the progressive type II censoring scheme in that the lifetime of the test follows the exponential distribution rather than the binomial.

This paper considers the step-stress partially-accelerated model, where the removals are assumed to follow the binomial and the life time-testing follows the Gompertz distribution. The optimal stress change time is also being determined by minimizing the generalized asymptotic variance of the MLE parameters. In Section 2, assumptions of the Gompretz distribution are given; Section 3 presents the assumptions of the partially accelerated model. Estimation of model parameters is given in Section 3; simulation study results are given in Section 4.

2. The Gompretz Distribution

The probability density function of the Gompretz distribution takes the form:

$$f(x) = \theta \alpha e^{\theta x} \exp\left\{-\alpha\left[e^{\theta x} - 1\right]\right\}, \ 0 < x, \alpha, \theta > 0 \quad (1)$$

And the cumulative distribution function is:

$$F(x) = 1 - \exp\left\{-\alpha\left[e^{\theta x} - 1\right]\right\}, \ 0 < x, \alpha, \theta > 0. \quad (2)$$

The following assumptions are used:
1. n identical and independent units are selected for life testing.
2. The lifetime of each unit has Gompertz distribution.

The following steps are to be followed:
a) Each of the n units is first run under normal use condition. If it does not fail or remove from the test by a pre-specified timeτ. it is put under accelerated condition (stress).
b) At the ith failure a random number of the surviving units, $R_i, i = 1, 2, \ldots m - 1$, are randomly selected and removed from the test.
c) Finally, at the mth failure the remaining surviving units $R_m = n - m - \sum_{i=1}^{m-1} R_i$ are all removed from the test and the test is terminated.

The lifetime, say X, of a unit under SS-PALT can be rewritten as

$$X = \begin{cases} T & \text{if } T \le \tau \\ \tau + \dfrac{T - \tau}{\beta} & \text{if } T \ge \tau \end{cases}$$

Where, the pdf of X is given by

$$f(x) = \begin{cases} f_1(x) = \theta \alpha e^{\theta x} \exp\left\{-\alpha\left[e^{\theta x} - 1\right]\right\}, 0 < x \le \tau \\ f_2(x) = \begin{cases} \beta \theta \alpha e^{\theta[\tau + \beta(x-\tau)]} \\ \times \exp\left\{-\alpha\left[e^{\theta[\tau+\beta(x-\tau)]} - 1\right]\right\} \end{cases}, x > \tau \end{cases} \quad (3)$$

In addition, the survival functions under normal and accelerate use conditions respectively is given by

$$S_1(x) = \exp\left\{-\alpha\left[e^{\theta x} - 1\right]\right\}, 0 < x < \tau. \quad (4)$$

And

$$S_2(x) = \exp\left\{-\alpha\left[e^{\theta[\tau + \beta(x-\tau)]} - 1\right]\right\}, x > \tau. \quad (5)$$

3. Estimation of Parameters

Let $(x_i), i = 1, 2, \ldots, m$, denote the observation obtained form a progressively type-II censored sample with random removals in a step-stress PALT. Here $x_{(1)} \le x_{(2)} \ldots \le x_{(m)}$.

Given a pre-determined number of removals $R = (R_1 = r_1, \ldots, R_{m-1} = r_{m-1})$, the conditional likelihood function of the observations $x = \{(x_i, r\ ), i = 1, 2, \ldots, m\}$ takes the following form

$$L_1(x_i, \alpha, \theta, \beta, u_{1i}, u_{2i} | R = r)$$
$$= \prod_{i=1}^{m}\left(\left\{f_1(x_i)(S_1(x_i))^{r_i}\right\}^{u_{1i}} \left\{f_1(x_i)(S_1(x_i))^{r_i}\right\}^{u_{2i}}\right). \quad (6)$$

Equations (1) and (3) are inserted in (6) and simplify, we get

$$L_1(x_i, \alpha, \theta, \beta, u_{1i}, u_{2i} | R = r)$$
$$= \prod_{i=1}^{m}\left[\begin{array}{c} \theta \alpha e^{\theta x_i}\exp\left\{-\alpha\left[e^{\theta x_i} - 1\right]\right\} \\ \times \exp\left\{-\alpha r_i\left[e^{\theta x_i} - 1\right]\right\} \end{array}\right]^{u_{1i}} \quad (7)$$

$$\times \left[\begin{array}{c} \beta \theta \alpha e^{\theta[\tau + \beta(x_i - \tau)]} \\ \times \exp\left\{-\alpha\left[e^{\theta[\tau + \beta(x_i - \tau)]} - 1\right]\right\} \\ \times \exp\left\{-\alpha r_i\left[e^{\theta[\tau + \beta(x_i - \tau)]} - 1\right]\right\} \end{array}\right]^{u_{2i}}.$$

3.1. Parameter Estimation with Binomial Removals

Given that the number of units removed $\boldsymbol{R_1}$ from the test at each failure time follows a binomial distribution $\boldsymbol{bin(n - m, p)}$, and $\boldsymbol{R_i \sim bin\left(n - m - \sum_{j=1}^{m} r_j, p\right)}$ for $\boldsymbol{i = 1, 2, \ldots 3., r_m = n - m - r_1 - r_2 - \cdots - r_{m-1}}$. Thus, the number of units removed at each failure time follows a binomial distribution such that

$$P(R_1 = r_1) = \binom{n-m}{r_1} p^{r_1}(1-p)^{n-m-r_1},$$

And for $\boldsymbol{i = 2, 3, \ldots, m - 1}$

$$P(R_i = r_i | R_{i-1} = r_{i-1} \ldots, R_1 = r_1)$$
$$= \binom{n - m - \sum_{j=1}^{i-1} r_j}{r_i} p^{r_i}(1-p)^{n-m-\sum_{j=1}^{i} r_j}. \quad (8)$$

The likelihood of the sample of size n is given as follows

$$L(x_i, \alpha, \theta, \beta, p) = L_1(x_i, \alpha, \theta, \beta, p | R = r) P(R = r), \quad (9)$$

Where

$$P(R = r) = P(R_1 = r_1, R_2 = r_2, ..., R_{m-1} = r_{m-1})$$

$$= P(R_{m-1} = r_{m-1} | R_{m-2} = r_{m-2}.., R_1 = r_1)$$

$$\times P(R_{m-2} = r_{m-2} | R_{m-3} = r_{m-3}.., R_1 = r_1)$$

$$\times ... P(R_2 = r_2 | R_1 = r_1) P(R_1 = r_1).$$

That is

$$P(R = r) = \frac{(n-m)!}{\left(n-m-\sum_{i=1}^{m-1} r_i\right)! \prod_{i=1}^{m-1} r_i}$$

$$\times p^{\sum_{i=1}^{m-1} r_i} (1-p)^{(m-1)(n-m) - \sum_{i=1}^{m-1}(m-i)(r_i)}. \quad (10)$$

The log-likelihood function [Eq. 10] can be written as follows:

$$l(x_i, \alpha, \theta, \beta, p) = \ln L_1(x_i, \alpha, \theta, \beta) + \ln P(R = r). \quad (11)$$

$$l(x_i, \alpha, \theta, \beta, p)$$

$$= m \ln \alpha + m \ln \theta + m_2 \ln \beta$$

$$+ \theta \sum_{i=1}^{m_1} x_i - \alpha \sum_{i=1}^{m_1} (1+r_i) e^{\theta x_i} + \theta \beta \sum_{i=1}^{m_2} x_i \quad (12)$$

$$- \alpha \sum_{i=1}^{m_2} (1+r_i) e^{\theta[\tau + \beta(x_i - \tau)]} + \alpha \sum_{i=1}^{m} (1+r_i) + \ln p \sum_{i=1}^{m-1} r_i$$

$$+ \ln n(1-p) \left[(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)(r_i) \right] + c$$

where $c = \left(\dfrac{(n-m)!}{\left(n-m-\sum_{i=1}^{m-1} r_i\right)! \prod_{i=1}^{m-1} r_i} \right)$ and

$$\sum_{i=1}^{m_1} u_{1i} + \sum_{i=1}^{m_2} u_{2i} = m.$$

To obtain the estimate of the α, θ, β **and** p, the first partial derivatives with respect to α, θ, β and p of Equation (12) are obtained and equated to zero as follows:

$$\frac{\partial l}{\partial \alpha} = \frac{m}{\alpha} - \sum_{i=1}^{m_1} (1+r_i) e^{\theta x_i}$$

$$- \sum_{i=1}^{m_2} (1+r_i) e^{\theta[\tau + \beta(x_i - \tau)]} + \sum_{i=1}^{m} (1+r_i) = 0. \quad (13)$$

$$\frac{\partial l}{\partial \theta} = \frac{m}{\theta} + \sum_{i=1}^{m_1} x_i - \alpha \sum_{i=1}^{m_1} (1+r_i) x_i e^{\theta x_i} + \beta \sum_{i=1}^{m_2} x_i + m_2 \tau (1-\beta)$$

$$- \alpha \sum_{i=1}^{m_2} (1+r_i) [\tau + \beta(x_i - \tau)] e^{\theta[\tau + \beta(x_i - \tau)]} = 0 \quad (14)$$

$$\frac{\partial l}{\partial \beta} = \frac{m_2}{\beta} - \sum_{i=1}^{m_2} x_i - m_2 \tau \theta$$

$$- \alpha \theta \sum_{i=1}^{m_2} (1+r_i) x_i e^{\theta[\tau + \beta(x_i - \tau)]} = 0 \quad (15)$$

$$\frac{\partial l}{\partial p} = \frac{\sum_{i=1}^{m-1} r_i}{p} - \frac{(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i)(r_i)}{1-p} = 0. \quad (16)$$

From Equation (16), p is estimated as follows

$$\hat{p} = \frac{\sum_{i=1}^{m-1} r_i}{(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i-1)(r_i)}. \quad (17)$$

There is no closed-from solution to this system of equations 13, 14 and 15. using the Newton-Raphson iteration method, for root finding. $\emptyset = (\boldsymbol{\alpha, \theta, \beta})$ is **estimated** Iteratively as follows:

$$\hat{\emptyset}_{j+1} = \hat{\emptyset}_j - G^{-1} g \quad (18)$$

where g is the vector of normal equations,

$$g = \begin{bmatrix} g_1 & g_2 g_3 \end{bmatrix}$$

and

$$g_1 = \frac{m}{\alpha} - \sum_{i=1}^{m_1} (1+r_i) e^{\theta x_i}$$

$$- \sum_{i=1}^{m_2} (1+r_i) e^{\theta[\tau + \beta(x_i - \tau)]} + \sum_{i=1}^{m} (1+r_i)$$

$$g_2 = \frac{m}{\theta} + \sum_{i=1}^{m_1} x_i - \alpha \sum_{i=1}^{m_1} (1+r_i) x_i e^{\theta x_i}$$

$$+ \beta \sum_{i=1}^{m_2} x_i + m_2 \tau (1-\beta)$$

$$- \alpha \sum_{i=1}^{m_2} (1+r_i) [\tau + \beta(x_i - \tau)] e^{\theta[\tau + \beta(x_i - \tau)]}$$

$$g_3 = \frac{m_2}{\beta} - \sum_{i=1}^{m_2} x_i - m_2 \tau \theta$$

$$- \alpha \theta \sum_{i=1}^{m_2} (1+r_i) x_i e^{\theta[\tau + \beta(x_i - \tau)]}$$

and G [Equation 17] is the matrix of second derivatives

$$G = \begin{bmatrix} \dfrac{\partial g_1}{\partial \alpha} & \dfrac{\partial g_1}{\partial \theta} & \dfrac{\partial g_1}{\partial \beta} \\ \dfrac{\partial g_2}{\partial \alpha} & \dfrac{\partial g_2}{\partial \theta} & \dfrac{\partial g_2}{\partial \beta} \\ \dfrac{\partial g_3}{\partial \beta} & \dfrac{\partial g_3}{\partial \theta} & \dfrac{\partial g_3}{\partial \beta} \end{bmatrix} \quad (19)$$

Where

$$\frac{\partial g_1}{\partial \alpha} = \frac{-m}{\alpha^2} \quad (20)$$

$$\frac{\partial g_1}{\partial \theta} = \frac{\partial g_2}{\partial \alpha} = -\sum_{i=1}^{m_1}(1+r_i)x_i e^{\theta x_i}$$
$$-\sum_{i=1}^{m_2}(1+r_i)\left[\tau + \beta(x_i - \tau)\right]e^{\theta[\tau+\beta(x_i-\tau)]} \qquad (21)$$

$$\frac{\partial g_1}{\partial \beta} = \frac{\partial g_3}{\partial \alpha} = -\theta\sum_{i=1}^{m_2}(1+r_i)x_i e^{\theta[\tau+\beta(x_i-\tau)]} \qquad (22)$$

$$\frac{\partial g_2}{\partial \theta} = \frac{-m}{\theta^2} - \alpha\sum_{i=1}^{m_1}(1+r_i)x_i^2 e^{\theta x_i} \qquad (23)$$

$$\frac{\partial g_2}{\partial \beta} = \frac{\partial g_3}{\partial \theta} = \sum_{i=1}^{m_2}x_i - m_2\tau$$
$$-\alpha\theta\sum_{i=1}^{m_2}(1+r_i)\left[\tau + \beta(x_i - \tau)\right]x_i e^{\theta[\tau+\beta(x_i-\tau)]} \qquad (24)$$

$$\frac{\partial g_3}{\partial \beta} = \frac{-m}{\beta^2} - \alpha\theta^2\sum_{i=1}^{m_2}(1+r_i)x_i^2 e^{\theta[\tau+\beta(x_i-\tau)]}. \qquad (25)$$

Convergence of the Newton-Raphson algorithm for the estimates of α, θ and β depends on the tolerance limit change with each successive iteration, to $\hat{\alpha}, \hat{\theta}$ and $\hat{\beta}$.

Numerically inverting the above G matrix above, we easily obtain Fisher Information matrix, i.e. $\mathbf{F=G^{-1}}$

The approximate $\mathbf{100(1-\gamma)}$ % two sided confidence intervals for α, θ and β can be, constructed as follows:

$$\hat{\alpha} \pm Z_{\gamma/2}\sigma_{\hat{\alpha}}, \hat{\theta} \pm Z_{\gamma/2}\sigma_{\hat{\theta}} \text{ and } \hat{\beta} \pm Z_{\gamma/2}\sigma_{\hat{\beta}}.$$

3.2. Parameter Estimation with the Uniform Removals

The model assumes that removed units are independent and that the probability of each removed unit is the same, such that:

$$P(R_1 = r_1) = \frac{1}{n-m+1}. \qquad (26)$$

And for $i = 2, 3, .., m - 1$

$$P(R_i = r_i | R_{i-1} = r_{i-1} .., R_1 = r_1)$$
$$= \frac{1}{n - m - \sum_{j=1}^{i-1}r_j + 1}.$$

The joint probability distribution of

$$R = (R_1 = r_1, .., R_{m-1} = r_{m-1})$$

is given by

$$P(R = r) = \frac{1}{n - m - \sum_{i=1}^{m-1}r_i + 1} \qquad (27)$$

where $0 \le r_i \le n - m - \sum_{j=1}^{i-1}r_j$, $i = 0, 1, ..., m - 1$.

The maximum likelihood estimators can be derived directly by maximizing the equations (12) and then solving for equations (13), (14) and (15).

3.3. D-optimality

Fisher's information matrix is used to determine t *he optimal value of τ* in the SS-PALT type II progressive censoring scheme proposed criterion is based on the determinant of Fisher's information matrix. The generalized asymptotic variance (GAV) is the reciprocal of the determinant of Fisher's information matrix F [10], thus

$$GAF(\hat{\alpha}, \hat{\theta}, \hat{\beta}) = \frac{1}{|F|}. \qquad (28)$$

The D-optimality criterion is the optimal value of τ that maximizes the determinant of the Fisher's information matrix F and minimizes the GAV.

Table 1. Simulation study results with Binomial Removals for θ=2, α=2.1, β=2.3, τ=3

n	m	Estimates			95% Confidence Interval Coverage			τ	$|F^{-1}|$	$Bias_{\hat{\alpha}}$	$Bias_{\hat{\theta}}$	$Bias_{\hat{\beta}}$
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\beta}$	$CP_{\hat{\alpha}}$	$CP_{\hat{\theta}}$	$CP_{\hat{\beta}}$					
20	9	2.173	2.035	2.114	0.3865	0.4519	0.3089	2.7310	0.968	0.0062	0.0085	0.0077
	19	2.360	2.065	2.154	0.5699	0.6421	0.3449	2.7862	0.985	0.0055	0.0078	0.0086
50	9	2.177	1.995	2.064	0.3935	0.4519	0.2623	2.8459	1.082	0.0074	0.0075	0.0066
	19	2.134	2.026	2.103	0.3471	0.4124	0.2975	2.8160	1.111	0.0052	0.0073	0.0074
	29	2.115	2.042	2.124	0.3235	0.3991	0.3134	2.8055	1.180	0.0041	0.0071	0.0078
	39	2.088	2.089	2.185	0.2912	0.3771	0.3687	2.8169	1.194	0.0034	0.0067	0.0092
	49	2.053	2.033	2.113	0.2681	0.3300	0.3088	2.8362	1.273	0.0027	0.0064	0.0077
80	9	2.185	2.039	2.121	0.3979	0.4633	0.3150	2.7959	1.361	0.0025	0.0060	0.0079
	19	2.143	2.053	2.138	0.3533	0.4255	0.3291	2.8362	1.375	0.0024	0.0055	0.0082
	29	2.128	2.058	2.145	0.3349	0.4140	0.3327	2.8450	1.465	0.0022	0.0052	0.0083
	39	2.088	2.072	2.164	0.2929	0.3754	0.3494	2.8055	1.563	0.0020	0.0042	0.0087
	49	1.962	2.077	2.169	0.1632	0.2525	0.3512	2.8353	1.651	0.0018	0.0039	0.0088
	59	1.940	2.082	2.176	0.1395	0.2323	0.3565	2.8634	1.686	0.0018	0.0036	0.0089
	69	1.936	2.093	2.190	0.1317	0.2314	0.3671	2.8450	1.833	0.0017	0.0034	0.0092
	79	1.930	2.101	2.200	0.1230	0.2296	0.3742	2.8257	1.928	0.0015	0.0032	0.0094
100	9	2.186	2.022	2.099	0.4014	0.4633	0.2948	2.8274	2.036	0.0013	0.0028	0.0074
	19	2.172	2.032	2.111	0.3857	0.4510	0.3054	2.8459	2.118	0.0012	0.0025	0.0076
	29	2.154	2.039	2.121	0.3638	0.4360	0.3116	2.8046	2.204	0.0011	0.0021	0.0078
	39	2.134	2.044	2.128	0.3402	0.4193	0.3152	2.8353	2.297	0.0010	0.0018	0.0079
	49	2.114	2.053	2.139	0.3183	0.4008	0.3248	2.8599	2.446	0.0009	0.0015	0.0081
	59	2.097	2.059	2.147	0.2982	0.3876	0.3293	2.8950	2.553	0.0008	0.0010	0.0082
	69	2.084	2.067	2.157	0.2833	0.3762	0.3372	2.8862	2.600	0.0007	0.0008	0.0084
	69	2.066	2.071	2.163	0.2624	0.3621	0.3399	2.8652	2.732	0.0007	0.0007	0.0085
	79	2.056	2.080	2.173	0.2484	0.3550	0.3470	2.8546	2.820	0.0006	0.0007	0.0087
	89	2.028	2.087	2.183	0.2169	0.3303	0.3532	2.8739	2.992	0.0005	0.0005	0.0088
	99	2.018	2.102	2.202	0.2004	0.3276	0.3656	2.8774	3.051	0.0004	0.0004	0.0091

Table 2. Simulation study results with uniform Removals for $\theta = 2$, $\alpha = 2.1$, $\beta = 2.3$, $\tau = 3$

n	m	Estimates			95% Confidence Interval Coverage			τ	$|F^{-1}|$	$Bias_{\hat{\alpha}}$	$Bias_{\hat{\theta}}$	$Bias_{\hat{\beta}}$
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\beta}$	$CP_{\hat{\alpha}}$	$CP_{\hat{\theta}}$	$CP_{\hat{\beta}}$					
20	9	2.478	2.320	2.411	0.4408	0.5152	0.3522	3.1140	1.104	0.0071	0.0097	0.0088
	19	2.691	2.355	2.456	0.6498	0.7322	0.3933	3.1770	1.123	0.0063	0.0089	0.0098
50	9	2.482	2.275	2.354	0.4487	0.5153	0.2991	3.2450	1.234	0.0084	0.0085	0.0075
	19	2.433	2.310	2.398	0.3958	0.4702	0.3392	3.2110	1.267	0.0059	0.0083	0.0085
	29	2.412	2.328	2.422	0.3689	0.4551	0.3573	3.1990	1.345	0.0047	0.0081	0.0089
	39	2.381	2.382	2.491	0.3320	0.4300	0.4204	3.2120	1.361	0.0039	0.0076	0.0105
	49	2.341	2.318	2.409	0.3057	0.3763	0.3522	3.2340	1.451	0.0031	0.0073	0.0088
80	9	2.491	2.325	2.418	0.4538	0.5282	0.3592	3.1880	1.552	0.0029	0.0068	0.0090
	19	2.444	2.341	2.438	0.4028	0.4852	0.3753	3.2340	1.568	0.0027	0.0063	0.0094
	29	2.427	2.347	2.446	0.3819	0.4721	0.3794	3.2440	1.671	0.0025	0.0059	0.0095
	39	2.381	2.363	2.467	0.3340	0.4280	0.3984	3.1990	1.782	0.0023	0.0048	0.0100
	49	2.237	2.368	2.473	0.1860	0.2880	0.4005	3.2330	1.882	0.0021	0.0045	0.0100
	59	2.212	2.374	2.481	0.1591	0.2649	0.4065	3.2650	1.922	0.0020	0.0041	0.0102
	69	2.207	2.387	2.497	0.1502	0.2638	0.4186	3.2440	2.090	0.0019	0.0039	0.0105
	79	2.201	2.396	2.509	0.1402	0.2618	0.4267	3.2220	2.198	0.0017	0.0036	0.0107
100	9	2.493	2.306	2.393	0.4577	0.5283	0.3362	3.2240	2.321	0.0015	0.0032	0.0084
	19	2.477	2.317	2.407	0.4398	0.5142	0.3482	3.2450	2.415	0.0014	0.0028	0.0087
	29	2.456	2.325	2.418	0.4148	0.4972	0.3553	3.1980	2.513	0.0013	0.0024	0.0089
	39	2.433	2.331	2.426	0.3879	0.4781	0.3594	3.2330	2.619	0.0011	0.0021	0.0090
	49	2.410	2.341	2.439	0.3630	0.4570	0.3704	3.2610	2.789	0.0010	0.0017	0.0093
	59	2.391	2.348	2.448	0.3400	0.4420	0.3755	3.3010	2.911	0.0009	0.0011	0.0094
	69	2.376	2.357	2.459	0.3231	0.4289	0.3845	3.2910	2.965	0.0008	0.0009	0.0096
	69	2.356	2.362	2.466	0.2992	0.4128	0.3876	3.2670	3.115	0.0008	0.0009	0.0097
	79	2.344	2.372	2.478	0.2832	0.4048	0.3957	3.2550	3.215	0.0006	0.0008	0.0099
	89	2.312	2.380	2.489	0.2473	0.3767	0.4028	3.2770	3.412	0.0005	0.0006	0.0101
	99	2.301	2.397	2.511	0.2285	0.3735	0.4169	3.2810	3.479	0.0004	0.0005	0.0104

4. Simulation Study

A simulation study is performed to study the properties of the estimators using ML method; the study involves the computation of Mean squared errors (MSEs), the construction of confidence intervals for different sample sizes; and the determination of, the optimal stress change time. The following steps were followed:

a)Value of n and m to be specified.

b)Value of the parameters were set as: $\theta = 2$, $\alpha = 2.1$, $\beta = 2.3$, $\tau = 3$.

c) A random sample with size n and censoring size m were generated, with random removals, $r_i, i = 1, 2, \ldots, m - 1$ from the random variable X given by (3).

d) Generate a group value $R_i \sim bino\left(n - m - \sum_{j=1}^{m} r_j, p\right)$ and also $R_i \sim unif\left(0, n - m - \sum_{j=1}^{i-1} r_j\right)$ where $0 \leq r_i \leq n - m - \sum_{j=1}^{i-1} r_j$, $i = 0, 1, \ldots, m - 1$ and $r_m = n - m - r_1 - r_2 - \cdots - r_{m-1}$.

e) ML estimates were computed, for n= 20, 50, 80 and 100,

f) The mean squared error (MSE), the 95% confidence interval of parameters and the bias associated with the MLE of the parameters, optimal value of τ, the Optimal GAV of the MLEs of the model parameters are obtained numerically for each sample size.

5. Conclusions

The (SS-PALT) under progressive type-II censored data with binomial and uniform random removal assuming Gompertz distribution was studied. The Newton-Raphson method is applied to obtain the optimal stress-change time which minimizes the Generalized Asymptotic Variance (GAV).

The maximum Likelihood estimation procedure was used the estimation of model parameters, mean squared error and the optimum plan for the binomial and uniform removals for different sample size were computed and shown. Simulation results show that the error and the optimal time decrease as sample size increase, when parameters are fixed. that both the average value of τ and the average value of GAV for type-II progressive censoring are getting close to those of complete sample with the bigger m and close faster for bigger n. Hence from the numerical result we can conclude that estimates of binomial are more stable with relatively small error with increasing sample size. Therefore, the test design obtained here is robust design and work well for binomial removal.

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