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Solitons and Periodic Solutions of the Fisher Equation with Nonlinear Ordinary Differential Equation as Auxiliary Equation

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Abstract In this article the new extension of the generalized and improved (G'/G) -expansion method has been used to generate many new and abundant solitons and periodic solutions, where the nonlinear ordinary differential equation has been used as an auxiliary equation, involving many new and real parameters. We choose the Fisher Equation in order to explain the advantages and effectives of this method. The illustrated results belongs to hyperbolic functions, trigonometric functions and rational functional forms which show that the implemented method is highly effective for investigating nonlinear evolution equations in mathematical physics and engineering science.

Keywords: *New extension of the generalized and improved* (G'/G) *-expansion method, Nonlinear evolution equation (NLEE), Fisher Equation, Wave solutions, Nonlinear partial differential equation (NLPDE)*

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1. Introduction

In physical sciences all essential equations are nonlinear and these are often complicated to interpret. So the exact solutions of nonlinear evolution equations (NLEEs) have turned out to be a chief concern for researchers. NLEE is one of the most powerful and important modelled equations among all equations in nonlinear sciences and it plays a vital role in the field of scientific work of engineering sciences such as chemical kinematics, fluid mechanics, chemistry, biology, nonlinear optics, optical fibers, plasma physics, solid state physics, biophysics, geochemistry, quantum mechanics, chemical physics, condensed matter physics, high-energy physics and so on. As they reveal a lot of physical information which help to understand the operation of the physical model better, that is why the explicit solutions of NLEEs play important role in the study of physical phenomena and remains a crucial field for researchers in the ongoing investigation.

For the past few decades, a vast research has been going on to construct explicit solutions of NLEEs, which are used as models in order to describe many important and problematics physical phenomena in various fields of science. So to figure out the exact solutions of NLEEs substantial work are being made by mathematicians and scientists and have developed effective and convincing methods such as the Hirota's bilinear transformation method [1], the tanh-function method [2,3], the exp-function method [4,5], the F-expansion method [6], the Jacobi elliptic function method [7], the homogeneous balance method [8], the homotopy perturbation method [9], the tanh-coth method [10], the direct algebraic method [11], the Backlund transformation method [12], and others [13,14,15,16].

Later in 2008, Wang *et al* [17] introduced a new method called the (G'/G)-expansion method for finding the solutions of traveling waves of NLEEs. This (G'/G) -expansion method shows that it is one of the most powerful and effective method to solve NLEEs since it gives a clear and short to the point results in terms of hyperbolic functions, trigonometric functions and rational functions which is why scientists have carried out a lot of researches to construct traveling wave solutions via this method [18-21].

Further research of (G'/G) - expansion method has been carried out by many researchers to show the possible productivity of the application. For example- Zhang *et al* [22] expanded the original (G'/G) - expansion method and named as the improved (G'/G)- expansion method. Using this method many researches have been carried out in order to find travelling wave solutions for NLPDEs [23-30]. Then Akber *et al* [31] introduced the generalized and improved (G'/G) - expansion method, where the second order LODE were used as auxiliary equation to construct travelling wave solution, this method were also

used in the study of higher dimensional NLPDEs [32]. In the meanwhile, Naher ad Abdullah [33] demonstrated a new method that is the new approach of the (G'/G) - expansion method and new approach of the generalized (G'/G) - expansion method where nonlinear ODE were used as auxiliary equation and the resulted travelling wave solutions of this method were quite different. Many researchers still carrying out experiments using the new extension of (G'/G) - expansion method to generate more new travelling wave solutions of NLEEs.

2 Methodology of New Extension of the Generalized and Improved (G'/G) - Expansion Method

Recently a new application have been introduced called the new extension of the generalized and improved (G'/G) - expansion method for NLEEs. So to demonstrate this method, first a NLPDE is taken with real independent variables x and t i.e.

$$P(u,u_t,u_x,u_{tt},u_{xt},u_{xx}...)=0 \tag{2.1}$$

where P is the polynomial and here $u=u(x,t)$ is an unknown function. In the polynomial P contains different partial derivatives of the function u itself wherein involves the highest order derivatives and the highest nonlinear terms. Now the prime process of this method is being discussed in steps below.

Step 1: Suppose that,

$$u(x,t)=u(\xi),\ \xi=x\pm Wt \tag{2.2}$$

where the constant term W is known as the speed of wave, is substituted in Eq. (2.1), which allows a PDE to convert an ODE with respect to ξ.

$$Q(u,u',u'',u'''...)=0 \tag{2.3}$$

Step 2: Eq. (2.3) is being integrated term by term and if needed it can be integrated more than once and the integral constants may be set to zero to make easy to solve. Now the integrated travelling wave solution of Eq. (2.3) can be represented as.

$$u(\xi)=\sum_{j=-N}^{N}a_j\left(d+H\right)^j+\sum_{j=1}^{N}\frac{b_j}{\left(d+H\right)^j} \tag{2.4}$$

where a_N, a_{-N} or b_N can be zero but all cannot be zero at the same time, $a_j\,(j=0,\pm1,\pm2,...\pm N)$, $b_j\,(j=1,2,3,...N)$, d is the arbitrary constant to be determined later and $H(\xi)$ is

$$H(\xi)=\left(\frac{G'}{G}\right) \tag{2.5}$$

where $G=G(\xi)$ satisfies the nonlinear ordinary differential equation (ODE) i.e.

$$\lambda GG''-\mu GG'-\delta(G')^2-\beta(G)^2=0 \tag{2.6}$$

where $G''=\frac{d^2G}{d\xi^2}, G'=\frac{dG}{d\xi}$ and $\lambda,\mu,\delta\,\&\,\beta$ are the real parameters

Step 3: The positive integer N appearing in the integrated solution of Eq. (2.3) is then determined by considering the homogeneous balance between the highest order derivative and the highest nonlinear term. The value of N is substituted in Eq. (2.4) which gives a complete ODE. Then the completed ODE of Eq. (2.4), Eq. (2.5) along with Eq. (2.6) is substituted in the integrated solution of Eq. (2.3) and collecting all the powers of the term $\left(d+H\right)$ in descending order to the left hand side, thus transforms into another polynomial of $\left(d+H\right)$, here $(d+H)^N\ (N=0,\pm1,\pm2,...)$ and $(d+H)^{-N}\ (N=1,2,3...)$.

Step 4: The coefficient of the $(d+H)$ polynomial is then equated to zero, hence generates a set of algebraic equation. By solving the algebraic equation gives the value for $a_j\,(j=0,\pm1,\pm2,...\pm N)$, $b_j\,(j=1,2,3,...N)$, d and W obtained from Eq. (2.5). Now by solving Eq. (2.6) we obtain a general solution, which is then substituted with the values of constants into Eq. (2.4) we can achieve more general type and more new travelling wave solutions of NLPDE of Eq. (2.1).

Step 5: Using the general solution of Eq. (2.6), the following solutions for Eq. (2.5) are obtained:

Family 1: When $\mu\neq0,\ \Psi=\lambda-\delta$ and

$$\Omega=\mu^2+4\beta(\lambda-\delta)>0,$$

$$H(\xi)=\left(\frac{G'}{G}\right)$$

$$=\frac{\mu}{2\Psi}+\frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}. \tag{2.7}$$

Family 2: When $\mu\neq0,\ \Psi=\lambda-\delta$ and

$$\Omega=\mu^2+4\beta(\lambda-\delta)<0,$$

$$H(\xi)=\left(\frac{G'}{G}\right)$$

$$=\frac{\mu}{2\Psi}+\frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} \tag{2.8}$$

Family 3: When $\mu\neq0,\ \Psi=\lambda-\delta$ and

$$\Omega=\mu^2+4\beta(\lambda-\delta)=0,$$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\mu}{2\Psi}+\frac{C_2}{C_1+C_2\xi} \tag{2.9}$$

Family 4: When $\mu=0,\ \Psi=\lambda-\delta$ and $\Delta=\Psi\beta>0,$

$$H(\xi)=\left(\frac{G'}{G}\right)$$
$$=\frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}. \quad (2.10)$$

Family 5: When $\mu=0,\ \Psi=\lambda-\delta$ and $\Delta=\Psi\beta<0$,

$$H(\xi)=\left(\frac{G'}{G}\right)$$
$$=\frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}. \quad (2.11)$$

3. Application of the Method

Let us consider the Fisher equation to investigate and construct new wave solutions by executing new extension of the generalized and improved $(G'/G)-$ expansion method [34].

The Fisher equation:

$$u_t-u_{xx}-u(1-u)=0. \quad (3.1)$$

By using the wave transformation of Eq. (2.2), into Eq. (3.1), the above equation transforms into the following NLODE:

$$u''+Wu'+u-u^2=0. \quad (3.2)$$

Now by taking the homogeneous balance between the nonlinear term u^2 and the highest order derivative term u'' in Eq. (3.2), we obtain the value for N i.e. $N=2$. Therefore the solution of Eq. (3.2) can be written in the form:

$$u(\xi)=a_0+a_1(d+H)+a_2(d+H)^2+(a_{-1}+b_1)(d+H)^{-1}+(a_{-2}+b_2)(d+H)^{-2} \quad (3.3)$$

where $a_{-2},a_{-1},a_0,a_1,a_2,b_1,b_2$ and d are constants to be determined.

Substituting Eq. (3.3) along with Eq. (2.5) and (2.6) into Eq. (3.2) and by simplifying it transforms into polynomials in $(d+H)^N\ (N=0,\pm1,\pm2,...)$ and $(d+H)^{-N}\ (N=1,2,3...)$. By collecting the resulted polynomials, yields a set of simultaneous algebraic equations for $a_{-2},a_{-1},a_0,a_1,a_2,b_1,b_2,d,c$ and W. After solving the systems of algebraic equations with the aid of Maple, we have obtained the following sets result for travelling waves.

3.1. Results of Travelling Waves

Set 1

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,$$
$$\beta=-\frac{1}{96\Psi}\left(96d^2\Psi^2-\lambda^2\right),W=\pm\frac{5}{\sqrt{6}},$$
$$d=d,a_{-2}=-\frac{1}{1536\Psi^2}\left(1536b_2\Psi^2-\lambda^2\right), \quad (3.1.1)$$
$$a_{-1}=-\frac{1}{16\Psi}\left(16b_1\Psi\pm\frac{\lambda}{\sqrt{6}}\right),a_0=\frac{3}{8},a_1=\mp\frac{6\Psi}{\lambda\sqrt{6}},$$
$$a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 2

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{96\Psi}\left(96d^2\Psi^2+\lambda^2\right),$$
$$W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-\frac{1}{1536\Psi^2}\left(1536b_2\Psi^2-\lambda^2\right), \quad (3.1.2)$$
$$a_{-1}=\frac{1}{16\Psi}\left(-16b_1\Psi\pm\frac{\lambda}{\sqrt{6}}\right),\ a_0=\frac{5}{8},$$
$$a_1=\mp\frac{6\Psi}{\lambda\sqrt{6}},\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 3

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\pm\frac{1}{\sqrt{6}}\right),\beta=0,$$
$$W=\pm\frac{5}{\sqrt{6}},d=d,a_{-2}=-b_2,a_{-1}=-b_1, \quad (3.1.3)$$
$$a_0=0,a_1=0,\ a_2=\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2;$$

Set 4

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda i}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\pm\frac{i}{\sqrt{6}}\right),\beta=0,$$
$$W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1, \quad (3.1.4)$$
$$a_0=1,\ a_1=0,\ a_2=-\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2;$$

Set 5

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\mp\frac{1}{\sqrt{6}}\right),\beta=0,$$
$$W=\pm\frac{5}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,a_{-1}=-b_1, \quad (3.1.5)$$
$$a_0=1,a_1=-\frac{2}{d},\ a_2=\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2;$$

Set 6

$$\lambda=\lambda,\ \mu=\mp\frac{\lambda i}{\sqrt{6}},\ \delta=\frac{\lambda}{d}\left(d\mp\frac{i}{\sqrt{6}}\right),\beta=0,$$
$$W=\pm\frac{5i}{\sqrt{6}},\ d=d,a_{-2}=-b_2,\ a_{-1}=-b_1, \quad (3.1.6)$$
$$a_0=0,\ a_1=\frac{2}{d},\ a_2=-\frac{1}{d^2},\ b_1=b_1,\ b_2=b_2;$$

Set 7

$$\lambda=\lambda,\mu=\mp\frac{\lambda}{\sqrt{6}},\delta=\delta,\beta=0,W=\pm\frac{5}{\sqrt{6}},d=d,$$
$$a_{-2}=-b_2,a_{-1}=-b_1,a_0=\frac{6d^2\Psi^2}{\lambda^2},a_1=-\frac{12d\Psi^2}{\lambda^2},\quad(3.1.7)$$
$$a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 8

$$\lambda=\lambda,\mu=\mp\frac{\lambda i}{\sqrt{6}},\delta=\delta,\beta=0,W=\pm\frac{5i}{\sqrt{6}},d=d,$$
$$a_{-2}=-b_2,a_{-1}=-b_1,a_0=\frac{6d^2\Psi^2+\lambda^2}{\lambda^2},\quad(3.1.8)$$
$$a_1=-\frac{12d\Psi^2}{\lambda^2},\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 9

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda}{\sqrt{6}},\ \delta=\delta,\ \beta=0,$$
$$W=\pm\frac{5}{\sqrt{6}},d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,$$
$$a_0=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}}\mp\frac{12d\lambda\delta}{\sqrt{6}}+6d^2\Psi^2\right),\quad(3.1.9)$$
$$a_1=\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}}\mp\frac{12\lambda^2}{\sqrt{6}}-d\Psi^2\right),$$
$$a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 10

$$\lambda=\lambda,\ \mu=\pm\frac{\lambda i}{\sqrt{6}},\ \delta=\delta,\ \beta=0,W=\pm\frac{5i}{\sqrt{6}},$$
$$d=d,a_{-2}=-b_2,\ a_{-1}=-b_1,$$
$$a_0=\frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2 i}{\sqrt{6}}\mp\frac{2\lambda\delta i}{\sqrt{6}}+d\Psi^2\right),\quad(3.1.10)$$
$$a_1=\frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta i}{\sqrt{6}}\mp\frac{12\lambda^2 i}{\sqrt{6}}-d\Psi^2\right),$$
$$a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2;$$

where, $\Psi=\lambda-\delta$.

Set 11

$$\lambda=\pm\mu\sqrt{6},\mu=\mu,\ \delta=\pm\mu\sqrt{6},\beta=\beta,W=\pm\frac{5\sqrt{6}}{6},$$
$$d=d,a_{-2}=\frac{1}{\mu^2}\left(\mu^2d^2+\beta^2-2\mu\beta d-\mu^2b_2\right),\quad(3.1.11)$$
$$a_{-1}=-b_1,a_0=0,\ a_1=0,a_2=0,b_1=b_1,b_2=b_2;$$

Set 12

$$\lambda=\pm\mu i\sqrt{6},\ \mu=\mu,\ \delta=\pm\mu i\sqrt{6},$$
$$\beta=\beta,\ W=\mp\frac{5i\sqrt{6}}{6},\ d=d,\quad(3.1.12)$$
$$a_{-2}=\frac{1}{\mu^2}\left(2\mu\beta d-\mu^2d^2-\beta^2-\mu^2b_2\right),$$
$$a_{-1}=-b_1,a_0=1,a_1=0,a_2=0,b_1=b_1,b_2=b_2;$$

Set 13

$$\lambda=\pm\mu\sqrt{6},\ \mu=\mu,\ \delta=\delta,\ \beta=0,$$
$$W=\frac{\mp5\mu d\sqrt{6}-\mu+\delta d}{\left(\pm\mu\sqrt{6}\mp d\delta\sqrt{6}+6\mu d\right)},\ d=d,$$
$$a_{-2}=\frac{1}{\mu^2}\left(\pm2\mu^2d^3\sqrt{6}\mp2\delta\mu d^4\sqrt{6},\right.$$
$$\left.+\mu^2d^2+\delta^2d^4-2\delta d^3\mu+6\mu^2d^4-\mu^2b_2\right),$$
$$a_{-1}=\frac{1}{\mu^2}\begin{pmatrix}\pm4\delta d^3\mu\sqrt{6}\mp4\mu^2d^2\sqrt{6}-2d\mu^2\\+4\delta\mu d^2-2\delta^2d^3-12\mu^2d^3-\mu^2b_1\end{pmatrix},$$
$$a_0=\frac{\begin{pmatrix}18d\mu^3-36\mu^2d^2\delta+18\mu d^3\delta^2\\+36\mu^3d^3\pm18d^2\mu^3\sqrt{6}\pm\mu^3\sqrt{6}\\\mp18\delta d^3\mu^2\sqrt{6}\mp3d\delta\mu^2\sqrt{6}\\\pm3\mu\delta^2d^2\sqrt{6}\mp\delta^3d^3\sqrt{6}\end{pmatrix}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d},\quad(3.1.13)$$
$$a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2;$$

Set 14

$$\lambda=\pm\mu i\sqrt{6},\mu=\mu,\delta=\delta,\beta=0,$$
$$W=\frac{\mp5\mu di\sqrt{6}+\mu-\delta d}{\left(\pm\mu i\sqrt{6}\mp d\delta i\sqrt{6}-6\mu d\right)},d=d,$$
$$a_{-2}=\frac{1}{\mu^2}\begin{pmatrix}\pm2\mu d^4\delta i\sqrt{6}\mp2\mu^2d^3i\sqrt{6}-\mu^2d^2\\-\delta^2d^4+2\delta d^3\mu+6\mu^2d^4-\mu^2b_2\end{pmatrix},$$
$$a_{-1}=\frac{1}{\mu^2}\begin{pmatrix}2d\mu^2-4\delta\mu d^2\mp4\delta d^3\mu i\sqrt{6}\\\pm4\mu^2d^2i\sqrt{6}+2\delta^2d^3\\-12\mu^2d^3-\mu^2b_1\end{pmatrix},\quad(3.1.14)$$
$$a_0=\frac{d\begin{pmatrix}12\mu^3-36\mu^2d\delta\pm18d\mu^3i\sqrt{6}\\\mp18\delta d^2\mu^2i\sqrt{6}+18\mu d^2\delta^2\pm2\delta\mu^2i\sqrt{6}\\-36\mu^3d^2\mp3\mu\delta^2di\sqrt{6}\pm\delta^3d^2i\sqrt{6}\end{pmatrix}}{\pm\mu^3\sqrt{6}\mp d\delta\sqrt{6}+6\mu d},$$
$$a_1=0,\ a_2=0,b_1=b_1,b_2=b_2;$$

Set 15

$$\lambda=\lambda,\mu=\mu,\delta=-\frac{1}{24\beta}\left(\lambda^2-6\mu^2-24\lambda\beta\right),$$
$$\beta=\beta,W=\pm\frac{5}{\sqrt{6}},d=d,a_{-2}=-b_2,a_{-1}=-b_1,$$

$$a_0=\frac{1}{96\lambda^2\beta^2}\begin{pmatrix}\lambda^4d^2\pm\frac{24\beta d\lambda^3}{\sqrt{6}}-12\lambda^2\mu^2d^2\\+24\lambda^2\beta^2+24\lambda^2\mu\beta d\mp\frac{144\mu^2\beta d}{\sqrt{6}}\\\pm\frac{288\lambda\mu\beta^2}{\sqrt{6}}+144\beta^2\mu^2\\+36\mu^4d^2-144\mu^3d\beta\end{pmatrix},$$

$$a_1=-\frac{1}{48\lambda^2\beta^2}\begin{pmatrix}\pm\frac{12\lambda^3\beta}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2}{\sqrt{6}}\\+12\mu\beta\lambda^2-72\beta\mu^3+d\lambda^4\\-12d\lambda^2\mu^2+36\mu^4d\end{pmatrix}, \qquad (3.1.15)$$

$$a_2=\frac{1}{96\lambda^2\beta^2}\left(\lambda^2-6\mu^2\right)^2,\ b_1=b_1,\ b_2=b_2;$$

Set 16

$$\lambda=\lambda,\mu=\mu,\delta=\frac{1}{24\beta}\left(\lambda^2+6\mu^2+24\lambda\beta\right),$$

$$\beta=\beta,W=\pm\frac{5i}{\sqrt{6}},\ d=d,\ a_{-2}=-b_2,\ a_{-1}=-b_1,$$

$$a_0=\frac{1}{96\lambda^2\beta^2}\begin{pmatrix}\lambda^4d^2\mp\frac{24\beta d\lambda^3i}{\sqrt{6}}+12\lambda^2\mu^2d^2\\+72\lambda^2\beta^2-24\lambda^2\mu\beta d\mp\frac{144\lambda\mu^2\beta di}{\sqrt{6}}\\\pm\frac{288\lambda\mu\beta^2i}{\sqrt{6}}+144\beta^2\mu^2\\+36\mu^4d^2-144\mu^3d\beta\end{pmatrix},$$

$$a_1=-\frac{1}{48\lambda^2\beta^2}\begin{pmatrix}\mp\frac{12\lambda^3\beta i}{\sqrt{6}}\mp\frac{72\lambda\beta\mu^2i}{\sqrt{6}}-12\mu\beta\lambda^2\\-72\beta\mu^3+d\lambda^4+12d\lambda^2\mu^2+36\mu^4d\end{pmatrix},$$

$$a_2=\frac{1}{96\lambda^2\beta^2}\left(\lambda^2+6\mu^2\right)^2,\ b_1=b_1,\ b_2=b_2;(3.1.16)$$

3.2. Solutions

Substituting Eq (3.1.1) in Eq (3.3), along with Eq (2.7) and simplifying, yields the following travelling wave solution, (if $C_1\neq0$ and $C_2=0$)

$$u1_1(x,t)=\frac{3}{8}\mp\frac{3\sqrt{\Omega}}{\lambda\sqrt{6}}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)+\frac{3\Omega}{2\lambda^2}\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\mp\frac{\lambda}{8\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}+\frac{\lambda^2}{384\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)},$$

Substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.8) and simplifying, our obtained solution becomes, (if $C_1\neq0$ and $C_2=0$)

$$u1_2(x,t)=\frac{3}{8}\pm\frac{3\sqrt{\Omega}i}{\lambda\sqrt{6}}\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)-\frac{3\Omega}{2\lambda^2}\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\pm\frac{\lambda}{8\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}+\frac{\lambda^2}{384\Omega i^2\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)},$$

Substituting Eq (3.1.1) in Eq (3.3), along with Eq (2.9) and simplifying, our exact solution becomes

$$u1_3(x,t)=\frac{3}{8}\mp\frac{6\Psi C_2}{\lambda\sqrt{6}\left(C_1+C_2\xi\right)}+\frac{6\Psi^2C_2^2}{\lambda^2\left(C_1+C_2\xi\right)^2}\mp\frac{\lambda\left(C_1+C_2\xi\right)}{16\Psi\sqrt{6}C_2}+\frac{\lambda^2\left(C_1+C_2\xi\right)^2}{1536\Psi^2C_2^2},$$

Similarly, substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.10) and simplifying, our obtained exact solution becomes, (if $C_1\neq0$ and $C_2=0$)

$$u1_4(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\sqrt{6}\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)}+\frac{\lambda^2}{1536\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2},$$

Substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.11) and simplifying, we obtain the following solution, (if $C_1\neq0$ and $C_2=0$)

$$u1_5(x,t)=\frac{3}{8}\mp\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\lambda\sqrt{6}}+\frac{6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\lambda^2}\mp\frac{\lambda}{16\sqrt{6}\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}$$

$$+\frac{\lambda^2}{1536\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2},$$

where $\xi = x - Wt$.

Similarly, substituting Eq. (3.1.3) in Eq. (3.3), along with Eqs. (2.7) - (2.11) and simplifying, our travelling wave solutions become:

$$u3_1(x,t)=\frac{\left\{2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2},$$

$$u3_2(x,t)=\frac{\left\{2d\Psi\sqrt{6}\pm\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{24d^2\Psi^2},$$

$$u3_3(x,t)=\frac{\left\{\begin{matrix}2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\\ \pm\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\end{matrix}\right\}^2}{24d^2\Psi^2\left(C_1+C_2\xi\right)^2},$$

$$u3_4(x,t)=\frac{\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2},$$

$$u3_5(x,t)=\frac{\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{d^2\Psi^2},$$

where $\xi = x - Wt$.

Similarly, substituting Eq (3.1.7) in Eq (3.3), along with Eq (2.7) - (2.11) and simplifying, our obtained travelling wave solutions become:

$$u7_1(x,t)=\frac{1}{\lambda^2}\left\{\begin{matrix}6d^2\Psi^2-\dfrac{6d\Psi\left\{\begin{matrix}2d\Psi\sqrt{6}\mp\lambda\\ +\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\end{matrix}\right\}}{\sqrt{6}}\\ +\dfrac{\left\{2d\Psi\sqrt{6}\mp\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\end{matrix}\right\},$$

$$u7_2(x,t)=\frac{1}{\lambda^2}\left\{\begin{matrix}6d^2\Psi^2-\dfrac{6d\Psi\left\{\begin{matrix}2d\Psi\sqrt{6}\mp\lambda\\ -\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\end{matrix}\right\}}{\sqrt{6}}\\ +\dfrac{\left\{2d\Psi\sqrt{6}\mp\lambda-\sqrt{6\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2}{4}\end{matrix}\right\},$$

$$u7_3(x,t)=\frac{1}{\lambda^2}\left\{\begin{matrix}6d^2\Psi^2-\dfrac{6d\Psi\left\{\begin{matrix}2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\\ \mp\lambda\left(C_1+C_2\xi\right)\\ +2\Psi\sqrt{6}C_2\end{matrix}\right\}}{\sqrt{6}\left(C_1+C_2\xi\right)}\\ +\dfrac{\left\{\begin{matrix}2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\\ \mp\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\end{matrix}\right\}^2}{4\left(C_1+C_2\xi\right)^2}\end{matrix}\right\},$$

$$u7_4(x,t)=\frac{1}{\lambda^2}\left\{\begin{matrix}6d^2\Psi^2-12d\Psi\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)\\ +6\left(d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right)^2\end{matrix}\right\},$$

$$u7_5(x,t)$$

$$D=\frac{1}{\lambda^2}\left\{\begin{matrix}6d^2\Psi^2-12d\Psi\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)\\ +6\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\end{matrix}\right\},$$

where $\xi = x - Wt$.

Similarly, substituting Eq. (3.1.9) in Eq. (3.3), along with Eqs. (2.7), (2.9) and (2.11) and simplifying, our obtained solutions become:

$$u9_1(x,t)=a_0+\frac{a_1\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)}{2\Psi\sqrt{6}}+\frac{a_2\left(2d\Psi\sqrt{6}\pm\lambda+\sqrt{6\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2}{24\Psi^2},$$

$$u9_2(x,t)=a_0+\frac{a_1\left\{\begin{matrix}2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\\ \pm\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\end{matrix}\right\}}{2\Psi\sqrt{6}\left(C_1+C_2\xi\right)}+\frac{a_2\left\{\begin{matrix}2d\Psi\sqrt{6}\left(C_1+C_2\xi\right)\\ \pm\lambda\left(C_1+C_2\xi\right)+2\Psi\sqrt{6}C_2\end{matrix}\right\}^2}{24\Psi^2\left(C_1+C_2\xi\right)^2},$$

$$u9_3(x,t)=a_0+\frac{a_1\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)}{\Psi}+\frac{a_2\left(d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2}{\Psi^2},$$

where

$$a_0 = \frac{1}{\lambda^2}\left(\pm\frac{12d\lambda^2}{\sqrt{6}} \mp \frac{12d\lambda\delta}{\sqrt{6}} + 6d^2\Psi^2\right),$$

$$a_1 = \frac{1}{\lambda^2}\left(\pm\frac{\lambda\delta}{\sqrt{6}} \mp \frac{12\lambda^2}{\sqrt{6}} - d\Psi^2\right),$$

$$a_2 = \frac{6\Psi^2}{\lambda^2}$$

and $\xi = x - Wt$.

Similarly, substituting Eq. (3.1.11) in Eq. (3.3), along with Eqs. (2.7) - (2.11) and simplifying, the following travelling wave solutions become:

$$u11_1(x,t) = \frac{4\Psi^2\left(\mu^2d^2 + \beta^2 - 2\mu\beta d\right)}{\mu^2\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2},$$

$$u11_2(x,t) = \frac{4\Psi^2\left(\mu^2d^2 + \beta^2 - 2\mu\beta d\right)}{\mu^2\left\{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2},$$

$$u11_3(x,t) = \frac{4\Psi^2\left(\mu^2d^2 + \beta^2 - 2\mu\beta d\right)(C_1 + C_2\xi)^2}{\mu^2\left\{\begin{array}{l}2d\Psi(C_1 + C_2\xi) \\ +\mu(C_1 + C_2\xi) + 2\Psi C_2\end{array}\right\}^2},$$

$$u11_4(x,t) = \frac{\Psi^2\left(\mu^2d^2 + \beta^2 - 2\mu\beta d\right)}{\mu^2\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2},$$

$$u11_5(x,t) = \frac{\Psi^2\left(\mu^2d^2 + \beta^2 - 2\mu\beta d\right)}{\mu^2\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2},$$

where $\xi = x - Wt$.

Similarly, substituting Eq. (3.1.14) in Eq. (3.3), along with Eqs. (2.7), (2.9) and (2.11) and simplifying, our solutions become:

$$u14_1(x,t) = a_0 + \frac{2\Psi(a_{-1} + b_1)}{\left(2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)} + \frac{4\Psi^2(a_{-2} + b_2)}{\left(2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right)^2},$$

$$u14_2(x,t) = a_0 + \frac{2\Psi(a_{-1} + b_1)(C_1 + C_2\xi)}{\left\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\right\}} + \frac{4\Psi^2(a_{-2} + b_2)(C_1 + C_2\xi)^2}{\left\{2d\Psi(C_1 + C_2\xi) + \mu(C_1 + C_2\xi) + 2\Psi C_2\right\}^2},$$

$$u14_3(x,t) = a_0 + \frac{\Psi(a_{-1} + b_1)}{\left(d\Psi - \sqrt{\Delta}i\tanh\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)} + \frac{\Psi^2(a_{-2} + b_2)}{\left(d\Psi - \sqrt{\Delta}i\tanh\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2},$$

where

$$a_0 = \frac{\left(\begin{array}{l}18d\mu^3 - 36\mu^2d^2\delta + 18\mu d^3\delta^2 \\ +36\mu^3d^3 \pm 18d^2\mu^3\sqrt{6} \pm \mu^3\sqrt{6} \\ \mp 18\delta d^3\mu^2\sqrt{6} \mp 3\mu\delta^2di\sqrt{6} \pm \delta^3d^2i\sqrt{6}\end{array}\right)}{\pm\mu^3\sqrt{6} \mp d\delta\sqrt{6} + 6\mu d},$$

$$a_{-1} = \frac{1}{\mu^2}\left(\begin{array}{l}\pm 4\delta d^3\mu\sqrt{6} \mp 4\mu^2d^2\sqrt{6} - 2d\mu^2 \\ +4\delta\mu d^2 - 2\delta^2d^3 - 12\mu^2d^3 - \mu^2b_1\end{array}\right),$$

$$a_{-2} = \frac{1}{\mu^2}\left(\begin{array}{l}\pm 2\mu d^4\delta i\sqrt{6} \mp 2\mu^2d^3i\sqrt{6} - \mu^2d^2 \\ -\delta^2d^4 + 2\delta d^3\mu + 6\mu^2d^4 - \mu^2b_2\end{array}\right)$$

and $\xi = x - Wt$.

4. Discussions

Various methods have been used to investigate for the solutions of Fisher Equation, such as Kudryashov [35] investigated by using simplest equation method, Wazwaz *et al.* [36] studied by using the Adomain decomposition method, in Ref. [37] Öziş *et al.* implemented by the Exp-function method, the homotopy analysis method executed by Tan *et al.* [38], and Ablowitz *et al.* [39] investigated solutions for a special wave speed. To the best our awareness the Fisher Equation has not been investigated by the new generalized and improved (G'/G) – expansion method. It is important to point out that our some obtained solutions are new, simple, straightforward, and precise compared to the solutions obtained in the open literature.

5. Graphical Representations

With the help of the computational software, Maple, we have illustrated some of the obtained solutions for travelling waves solutions in below.

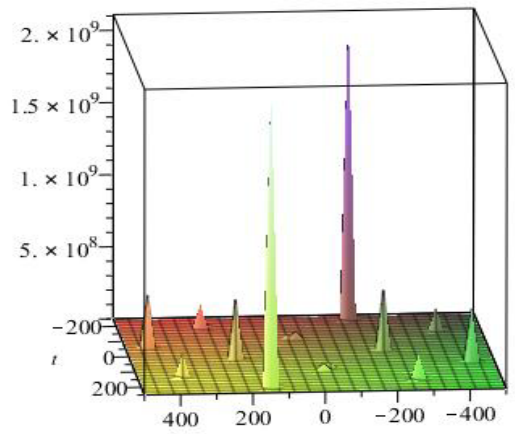

Figure 1. Eq.- $u1_5$, for $\lambda = 5,\ \delta = 15,\ \beta = -\frac{1}{4\Psi}\left(\lambda^2 + 4d^2\Psi^2\right),$ $d = 10,\ W = -\frac{5}{\sqrt{6}}$ and $x = -500..500,\ t = -250..250$

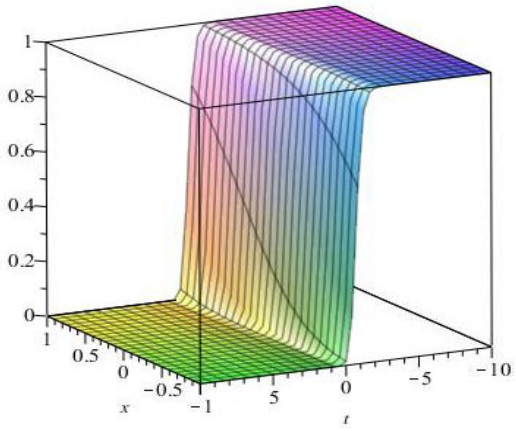

Figure 2. Eq.- $u3_1$, for $\lambda = 3,\ \mu = \frac{\lambda}{\sqrt{6}},\ \beta = 0,$ $\delta = \frac{\lambda}{d}\left(d + \frac{1}{\sqrt{6}}\right),\ d = 2,\ W = \frac{5}{\sqrt{6}}$ and $x = -1..1,$ $t = -10..10$

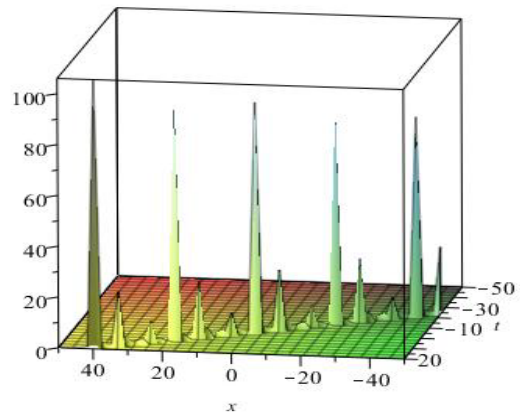

Figure 3. Eq- $u3_3$, for $\lambda = -4,\ c_2 = 0.521\ \mu = \frac{\lambda}{\sqrt{6}},\ \beta = 0,$ $\delta = \frac{\lambda}{d}\left(d + \frac{1}{\sqrt{6}}\right),\ d = 1,\ W = \frac{5}{\sqrt{6}},\ c_1 = 0.7,$ and $x = -50..50,\ t = -50..20$

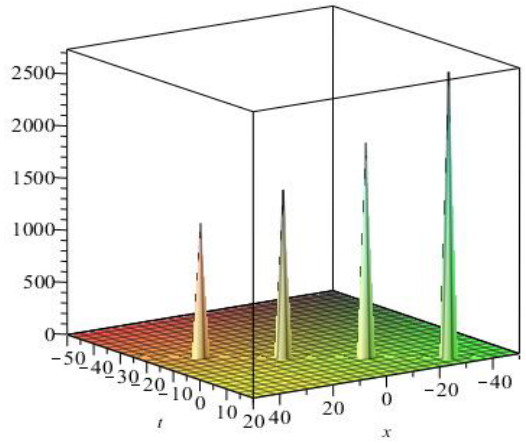

Figure 4. Eq- $u9_2$, for $\lambda = 8.5,\ \delta = \frac{\lambda}{\sqrt{6}},\ \beta = 0,\ d = 0.4,$ $W = -\frac{5}{\sqrt{6}},\ c_1 = 2.1,\ c_2 = 9.5$ and $x = -50..50,\ t = -50..20$

6. Conclusions

In this article, the new extension of the generalized and improved $(G'/G)-$expansion method has been applied successfully in the Fisher Equation. The auxiliary equation used in the method that involves many arbitrary parameters and those can take any real values then the NLODE produces many new solutions. The obtained solutions show that the method is effective and gives precise and direct solutions. Therefore, we conclude that this method could be implemented for constructing various types of wave solutions of NLEEs those arise in the application of mathematical field.

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