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Reliability and Profit Analysis of a Two-unit Non-identical Standby System in Snowstorm Weather Conditions

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Abstract This paper express reliability measures of a cold standby system which have two units. In cold standby system one unit operative and other unit kept as a spare. In the system both the unit kept as non-identical. The each operative unit fails due to snowstorm with different failure rate. The system completely failed when the both two units are failed. The failed unit cannot be operative directly by the repairman. The failed unit under the snow, first digging out from the snow then hospitalize (repair) the unit after that the unit becomes operative. Some properties of reliability system such as mean time to system failure, availability and profit have been computed. At last particular cases have been taken to explain the model.

Keywords: *non-identical units, snowstorms, digging out*

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1. Introduction

Today in modern industry, reliability has an important role for the system. Reliability models of two-unit standby systems have been analysed by various research including [1,2,3]. Taneja and Tuteja [4,5] discussed various systems with different types of failure and repair rates. Comparative analysis of two-unit standby systems are studied by Singh and Taneja [6] and Malhotra and Taneja [7]. Chandrasekhar P. et al. [8] focused on two unit system with erlangian repair time. Manocha and Taneja [9] worked on such systems with arbitrary distributions. Reliability of a system most affected by abnormal weather conditions such as heavy rain snowstorm, dense fog, high temperature etc. Therefore, many researchers including Goel, Sharma and Gupta [10], Gupta and Goel [11] and Goel, Kumar and Rastogi [12] have explained reliability measures of systems with various weather conditions. Singh et al. [13] analyzed availability of warm standby systems failure due to heavy rain. Nailwal and Singh [14] analysed reliability and sensitivity in different weather conditions.

In this paper, we consider the two-unit non-identical units system.The system has only one repairman as a rescue team. The operative unit failed due to snow storm and some people or systems were trapped under the snow. In such situation repair of the system is very difficult. So after the snowstorm is over, first the failed unit digging out by the repairman. after the digging out then hospitalize the such unit and after that the unit becomes operative.

This paper describes the following subsections and sections.

- Model with mean sojourn times and transition probabilities
- Mean time to system failure
- Steady state availability
- Repairman’s busy period analysis during digging out
- Repairman’s busy period analysis during hospitalization
- Repairman’s expected visits
- Cost-Benefit analysis
- Special case
- Graphical Explanation

2. Notations

λ_1: failure rate of first unit due to snow storm

λ_2: failure rate of second unit due to snow storm

$G_1(t)$, $G_2(t)$: comulative density function of first unit as repair rate of digging out and hospitilization of failed unit respectively.

$G_3(t)$, $G_4(t)$: comulative density function of second unit as repair rate of digging out and hospitilization of failed unit respectively.

$g_1(t)$, $g_2(t)$: probability density function of first unit as repair rate of digging out, and hospitilization of failed unit respectively.

$g_3(t)$ $g_4(t)$: probability density function of second unit as the repair rate of digging out and hospitilization of failed unit respectively.

Op: operative unit

cs: spare unit or cold standby unit

Fd: failed unit is under digging out

FD: failed unit is under digging out continuing on the unit

Fh: failed unit is under hospitalization after snow removing

FH: failed unit is under hospitalization continuing after snow removing

Fwd: waiting for digging out

2.1. Model and Transition Probability:

In the state transition diagram (Figure 1) states 0, 1, 2, 4, 5, 8,9 are regenerative states and 3,6,7,10 and non-regenerative states.

p_{01} =1,

$p_{12}=g_1^*(\lambda_2)$, $p_{13}=$ (1- $g_1^*(\lambda_2)$), p_{14}^{3}=(1- $g_1^*(\lambda_2)$)

$p_{20}=g_2^*(\lambda_2)$, $p_{26}=$ (1- $g_2^*(\lambda_2)$), p_{25}^{6}=(1- $g_2^*(\lambda_2)$)

p_{45}=1, $p_{90}=g_4^*(\lambda_1)$, p_{91}^{10}=(1- $g_4^*(\lambda_1)$)

p_{81} = 1, $p_{59}=g_3^*(\lambda_1)$), p_{58}^{7}=(1- $g_3^*(\lambda_1)$).

By these transition probabilities, it can be verified that

$p_{01}=1$

$p_{12}+p_{14}^{(3)}=p_{12}+p_{13}=1=p_{59}+p_{58}^{(7)}$

$p_{20}+p_{25}^{(6)}=p_{20}+p_{26}=1=p_{90}+p_{91}^{(10)}$

$p_{45}=1=p_{81}$.

If T represents the sojourn then mean sojourn time (μ_i) at the regenerative state 'i' discussed as:

μ_i=E(T)=Pr(T>t)

$$\mu_0=\frac{1}{\lambda_1}$$

$$\mu_1=\frac{1}{\lambda_2}\{1-g_1^*(\lambda_2)\}$$

$$\mu_2=\frac{1}{\lambda_2}\left\{1-g_2^*(\lambda_2)\right\}$$

$$\mu_5=\frac{1}{\lambda_1}\left\{1-g_3^*(\lambda_2)\right\}$$

$$\mu_9=\frac{1}{\lambda_1}\left\{1-g_4^*(\lambda_2)\right\}.$$

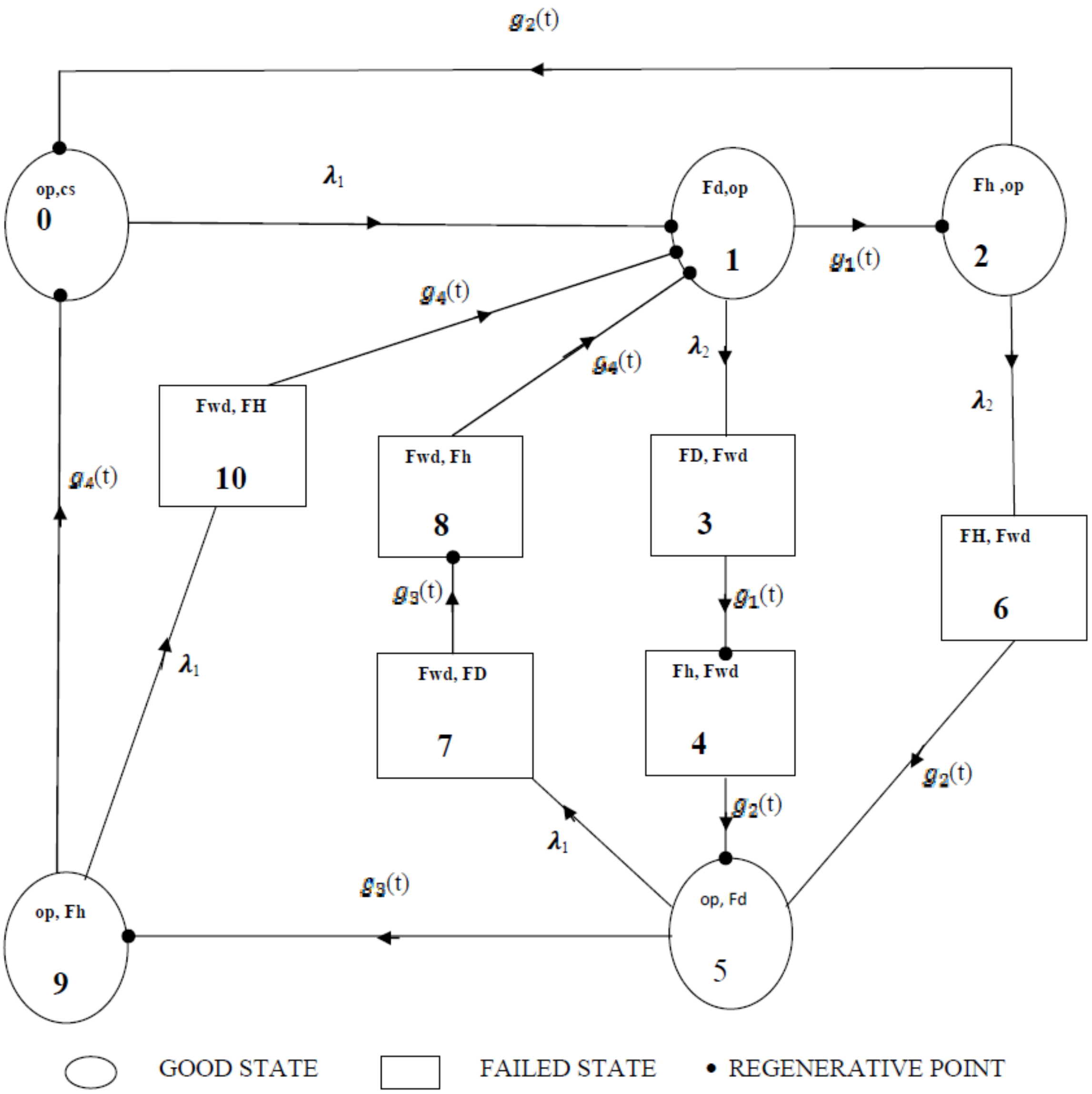

Figure 1.

The unconditional mean time m_{ij} mathematically defined as

$$m_{ij} = \int_0^\infty t\, q_{ij}(t)dt = -q_{ij}^{*'}(0)$$

$$m_{01} = \mu_0$$

$$m_{12} + m_{14}^3 = -g_1^{*'}(0) = k_1(\text{say}), m_{12} + m_{14} = \mu_1$$

$$m_{20} + m_{25}^6 = -g_2^{*'}(0) = k_2(say), m_{20} + m_{26} = \mu_2$$

$$m_{45} = k_2, m_{59} + m_{58}^7 = -g_3^{*'}(0) = k_3(\text{say}),$$

$$m_{81} = k_4, m_{90} + m_{91}^{10} = -g_4^{*'}(0) = k_4(\text{say}).$$

3. Mean Time to System Failure

Mean time to system failure (MTSF) regarding the failed states (i=3, 4, 6, 7, 8, 10) as absorbing states and applying arguments for regenerative process, we get the recursion relation for $\pi_i(t)$,

$$\pi_0(t) = Q_{01}(t) \circledS \pi_1(t),$$

$$\pi_1(t) = Q_{13}(t) + Q_{12}(t) \circledS \pi_2(t)$$

$$\pi_2(t) = Q_{26}(t) + Q_{20}(t) \circledS \pi_0(t).$$

By applying Laplace-Stieltjes transform on these relations and solving for $\pi_i^{**}(s) = \frac{N(S)}{D(S)}$.

Where, $N(S) = Q_{01}^{**}(s)(Q_{13}^{**}(s) + Q_{26}^{**}(s)\, Q_{12}^{**}(s))$

$D(S) = 1 - Q_{01}^{**}(s)\, Q_{20}^{**}(s)\, Q_{12}^{**}(s)$.

When the system begin from the state '0' the mean time to system failure is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \lim_{s\to 0} \frac{1-\frac{N(S)}{D(S)}}{s}$$

$$= \lim_{s\to 0} \frac{D(s)-N(s)}{sD(s)} = \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D}$$

Where $N = \mu_0 + p_{12}\mu_2 + \mu_1$

And

$$D = 1 - p_{12}p_{20}.$$

4. Availability Analysis

System availability is the probability that it is in operation and gives service when we want.

By using the theory of regenerative point process, availability $A_i(t)$ as the probability in the state 'i' at t=0 is seen to satisfy these recursive relation are obtained.

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$

$$A_1(t) = M_1(t) + q_{12}(t) © A_2(t) + q_{15}^4(t) © A_5(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{25}^6(t) © A_5(t)$$

$$A_4(t) = q_{45}(t) © A_5(t)$$

$$A_5(t) = M_5(t) + q_{59}(t) © A_9(t) + q_{58}^7(t) © A_8(t)$$

$$A_8(t) = q_{81}(t) © A_1(t)$$

$$A_9(t) = M_9(t) + q_{90}(t) © A_0(t) + q_{91}^{10}(t) © A_1(t)$$

Where $M_0(t) = e^{-\lambda_1 t}dt$, $M_1(t) = e^{-\lambda_2 t}\overline{G_1}(t)dt$, $M_2(t) = e^{-\lambda_2 t}\overline{G_2}(t)dt$, $M_5(t) = e^{-\lambda_1 t}\overline{G_3}(t)dt$ and $M_9(t) = e^{-\lambda_1 t}\overline{G_4}(t)dt$.

By applying Laplace-Transformations on these relation and solving for $A_0^*(s)$ we get,

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

availability of system in steady state is

$$A_0 = \lim_{s\to 0}\left(sA_0^*(s)\right) = \lim_{s\to 0}\left(s\frac{N_1(s)}{D_1(s)}\right) = \frac{N_1(0)}{D_1'(0)} = \frac{N_1}{D_1}$$

$$N_1 = \mu_0(1 - p_{12}p_{25}^6 p_{91}^{10} - p_{12}p_{25}^6 p_{58}^7 - p_{14}^3 p_{91}^{10} - p_{14}^3 p_{58}^7) + \mu_1 + \mu_2 p_{12} + (\mu_5 + \mu_5)(p_{12}p_{25}^6 + p_{14}^3)$$

$$D_1 = k_1 + k_2 + \mu_0(p_{12}p_{20} + p_{12}p_{25}^6 p_{59}p_{90} + p_{14}^3 p_{59}p_{90}) + (k_3 + k_4)\left(p_{12}p_{25}^6 + p_{14}^3\right)$$

Where, k_1, k_2, k_3 and k_4 is already defined.

5. During Digging out Repairman Busy Period Analysis

$B_i^D(t)$=The system entered from regenerative state 'i'at time t=0 is under repair during digging out.

$$B_0^D(t) = q_{01}(t) © B_1^D(t)$$

$$B_1^D(t) = W_1(t) + q_{12}(t) © B_2^D(t) + q_{15}^4(t) © B_4^D(t)$$

$$B_2^D(t) = q_{20}(t) © B_0^D(t) + q_{25}^6(t) © B_5^D(t)$$

$$B_4^D(t) = q_{45}(t) © B_5^D(t)$$

$$B_5^D(t) = W_5(t) + q_{59}(t) © B_9^D(t) + q_{58}^7(t) © B_8^D(t)$$

$$B_8^D(t) = q_{81}(t) © B_1^D(t)$$

$$B_9^D(t) = W_9(t) + q_{90}(t) © B_0^D(t) + q_{91}^{10}(t) © B_1^D(t)$$

Where $W_2(t) = e^{-\lambda_2 t}\overline{G_2}(t)dt + \lambda_2\ e^{-\lambda_2 t}\overline{G_2}(t)dt$, $W_9(t) = e^{-\lambda_1 t}\overline{G_4}(t)dt + \lambda_1\ e^{-\lambda_1 t}\overline{G_9}(t)dt$, $W_8(t) = \overline{G_3}(t)dt$ and $W_4(t) = \overline{G_2}(t)dt$.

By using Laplace Transforms then solving system of equation for $B_0^{*H}(s)$ we get,

$$B_0^{*H}(s) = \frac{N_3(s)}{D_1(s)}$$

Where

$$N_3(s) = q_{01}^*(s)q_{14}^{*(3)}(s)W_4^*(s) + q_{01}^*(s)q_{12}^*(s)W_2^*(s) + q_{01}^*(s)q_{14}^{*(3)}(s) + q_{45}^*(s)q_{58}^{*(7)}(s)W_8^*(s) + q_{01}^*(s)q_{14}^{*(3)}(s) + q_{45}^*(s)q_{59}^*(s)W_9^*(s) + q_{01}^*(s)q_{12}^*(s)q_{25}^{*(6)}(s)q_{58}^{*(7)}(s)W_8^*(s) + q_{01}^*(s)q_{12}^*(s)q_{25}^{*(6)}(s)q_{59}^*(s)W_9^*(s).$$

where $D_1(s)$ defined already.

During digging out the total time for which the system is under repaired.

$$B_0^{*H} = \frac{N_3(0)}{D_1^{'}(0)} = \frac{N_3}{D_1}$$

Where,

$$N_2 = W_1 + (p_{12}p_{25}^{6} + p_{14}^{3}p_{45})W_5$$

Where $W_1 = W_1^*(0)$ and D_1 is already defined.

6. During Hospitalization Repairman Busy Period Analysis

B_i^H(t)=The system entered from regenerative state 'i' at time t=0 is under repair during digging out.

$$B_0^H(t) = q_{01}(t) © B_1^H(t)$$
$$B_1^H(t) = q_{12}(t) © B_2^H(t) + q_{15}^4(t) © B_4^H(t)$$
$$B_2^H(t) = W_2(t) + q_{20}(t) © B_0^H(t) + q_{25}^6(t) © B_5^H(t)$$
$$B_4^H(t) = W_4(t) + q_{45}(t) © B_5^H(t)$$
$$B_5^H(t) = q_{59}(t) © B_9^H(t) + q_{58}^7(t) © B_8^H(t)$$
$$B_8^H(t) = W_8(t) + q_{81}(t) © B_1^H(t)$$
$$B_9^H(t) = W_9(t) + M_9(t) + q_{90} © B_0^H(t) + q_{91}^{10}(t) © B_1^H(t)$$

Where $W_1(t) = e^{-\lambda_2 t}\overline{G_1}(t)dt + \lambda_2 \; e^{-\lambda_2 t}\overline{G_2}(t)dt$, $W_5(t) = e^{-\lambda_1 t}\overline{G_3}(t)dt + \lambda_1 \; e^{-\lambda_1 t}\overline{G_3}(t)dt$

By using Laplace Transforms then solving system of equation for $B_0^{*D}(s)$ we get,

$$B_0^{*D}(s) = \frac{N_2(s)}{D_1(s)}$$

Where

$$N_2(s) = q_{01}^*(s)W_1^*(s) + \begin{pmatrix} q_{01}^*(s)q_{12}^*(s)q_{25}^{*(6)}(s) \\ +q_{01}^*(s)q_{14}^{*(3)}(s)q_{45}^*(s) \end{pmatrix} W_5^*(s)$$

where $D_1(s)$ is already defined.

During hospitalization the total time for which the system is under repaired.

$$B_0^{*H} = \frac{N_3(0)}{D_1^{'}(0)} = \frac{N_3}{D_1}$$

Where,

$$N_3 = W_4 p_{14}^{(3)} + p_{12}W_2(t) + p_{14}^{(3)}p_{58}^{(7)}W_8 + p_{14}^{(3)}p_{59}W_9 + p_{12}p_{25}^{(6)}p_{58}^{(7)}W_8 + p_{12}p_{25}^{(6)}p_{59}W_9$$

Where $W_2 = W_2^*(0)$, $W_4 = W_4^*(0)$, $W_8 = W_8^*(0)$, $W_9 = W_9^*(0)$ and D_1 is defined already.

7. Expected Number of Visits by the Repairman

When the system started from the regenerative state 'i' at t=0, V_0(t) denotes the expected number of visits by the repair mean in [0,t].

$$V_0(t) = q_{01}(t) Ⓢ (1 + V_1(t))$$
$$V_1(t) = q_{12}(t) Ⓢ V_2(t) + q_{15}^4(t) Ⓢ V_5(t)$$
$$V_2(t) = q_{20}(t) Ⓢ V_0(t) + q_{25}^6(t) Ⓢ V_5(t)$$
$$V_4(t) = q_{45}(t) Ⓢ V_5(t)$$
$$V_5(t) = q_{59}(t) Ⓢ V_9(t) + q_{58}^7(t) Ⓢ V_8(t)$$
$$V_8(t) = q_{81}(t) Ⓢ V_1(t)$$
$$V_9(t) = q_{90}(t) Ⓢ V_0(t) + q_{91}^{10}(t) Ⓢ V_1(t).$$

By using Laplace-Stieltjes Transformations and solving system of equations for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)}$$

Where

$$N_4(s) = Q_{01}^{**}(s) - Q_{12}^{**}(s)Q_{25}^{**(6)}(s)Q_{59}^{**}(s)Q_{91}^{**(10)}Q_{01}^{**}(s) - Q_{12}^{**}(s)Q_{25}^{**(6)}Q_{58}^{**(7)}Q_{81}^{**}(s)Q_{01}^{**}(s) - Q_{14}^{**(3)}Q_{45}^{**}(s)Q_{59}^{**}(s)Q_{91}^{**(10)}Q_{01}^{**}(s) - Q_{14}^{**(3)}Q_{45}^{**}(s)Q_{58}^{**(7)}(s)Q_{81}^{**}(s)Q_{01}^{**}(s).$$

And $D_1(s)$ is already specified.

$$V_0 = \frac{N_4(0)}{D_1^{'}(0)} = \frac{N_4}{D_1}$$

Where,

$$N_4 = 1 - p_{12}p_{25}^{(6)}p_{59}p_{91}^{(10)} - p_{12}p_{25}^{(6)}p_{58}^{(7)} - p_{14}^{(3)}p_{59}p_{91}^{(10)} - p_{14}^{(3)}p_{58}^{(7)}.$$

And D_1 is defined already.

8. Cost-benefit Analysis

The total profit of the system in steady state is given by

$$P = C_0A_0 - C_{11}B_0^D - C_{12}B_0^H - C_2V_0$$

C_0=Expected revenue in up time(o,t]
C_{11}=Expected total repair cost when repairman is busy under digging out.
C_{12}=Expected total repair cost when repairman is busy under hospitalization.
C_2= Per visit cost of the repairman.

9. Particular Cases

Numerical result for the particular cases the following case is considered:

$$g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t},$$
$$g_3(t) = \alpha_3 e^{-\alpha_3 t}, \text{ and } g_4(t) = \alpha_4 e^{-\alpha_4 t}$$

$$p_{01}=1,\ p_{12}=\frac{\alpha_1}{\lambda_2+\alpha_1},\ p_{14}=\lambda_2/(\lambda_2+\alpha_1),\ p_{14}^{(3)}=\frac{\lambda_2}{\lambda_2+\alpha_1}$$

$$p_{20}=\alpha_2/(\lambda_2+\alpha_2),\ p_{26}=\lambda_2/(\lambda_2+\alpha_2),$$

$$p_{25}^{(6)}=\frac{\lambda_2}{\lambda_2+\alpha_2},\ p_{45}=1.p_{59}=\alpha_3/(\lambda_1+\alpha_3),\ p_{58}^{(7)}=\frac{\lambda_1}{\lambda_1+\alpha_2},$$

$$p_{81}=1,\ p_{90}=\alpha_4/(\lambda_1+\alpha_4),\ p_{91}^{(10)}=\frac{\lambda_1}{\lambda_1+\alpha_4}$$

10. Grapical Interpretation

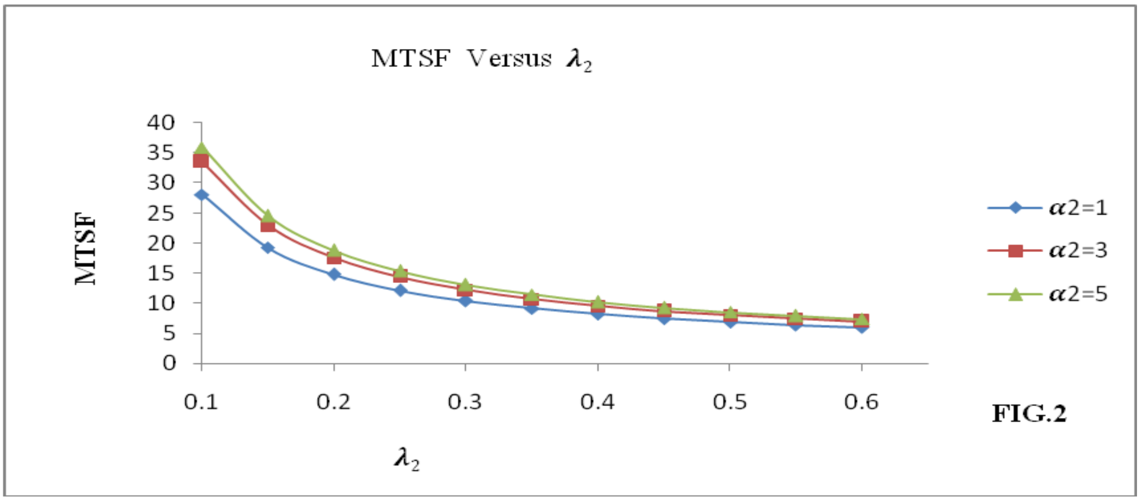

Figure 2.

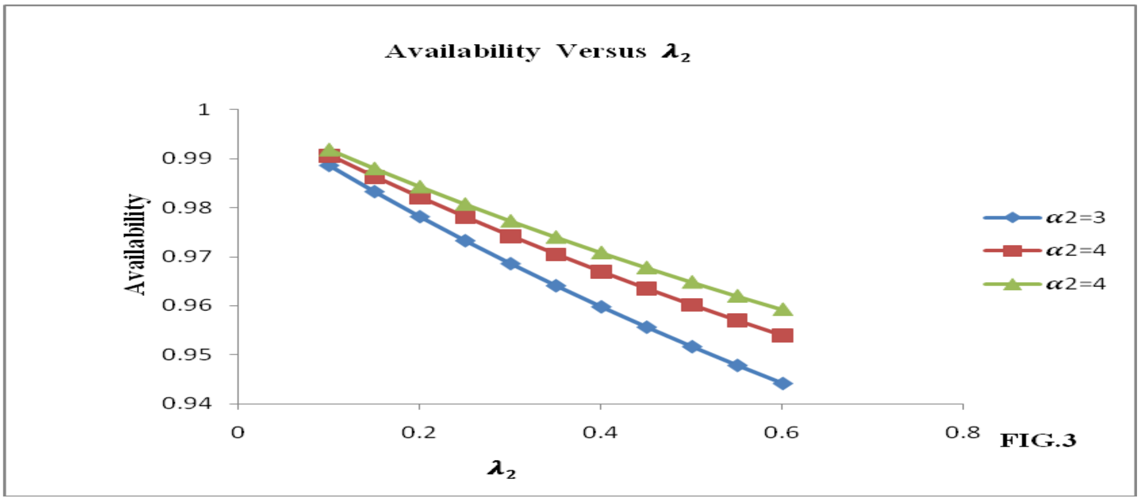

Figure 3.

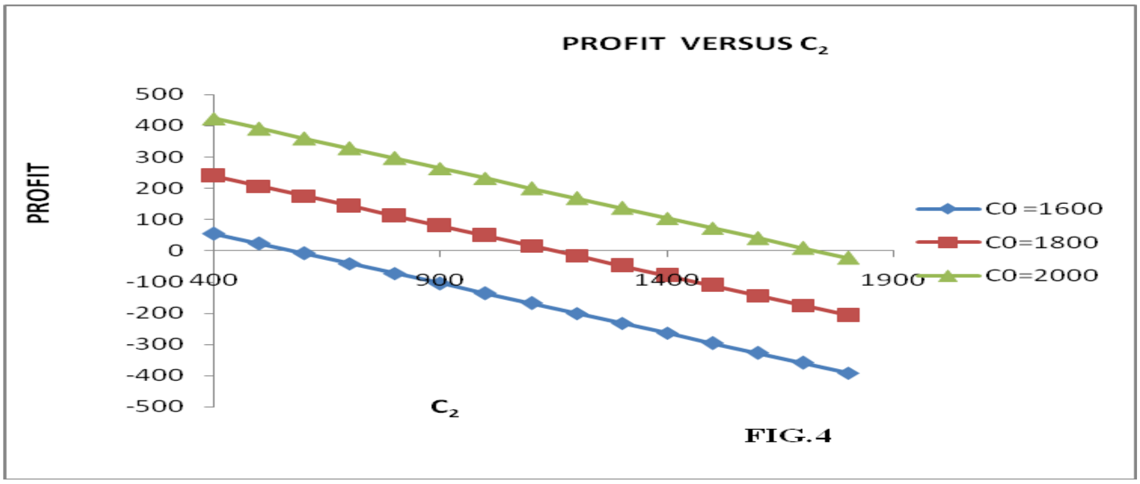

Figure 4.

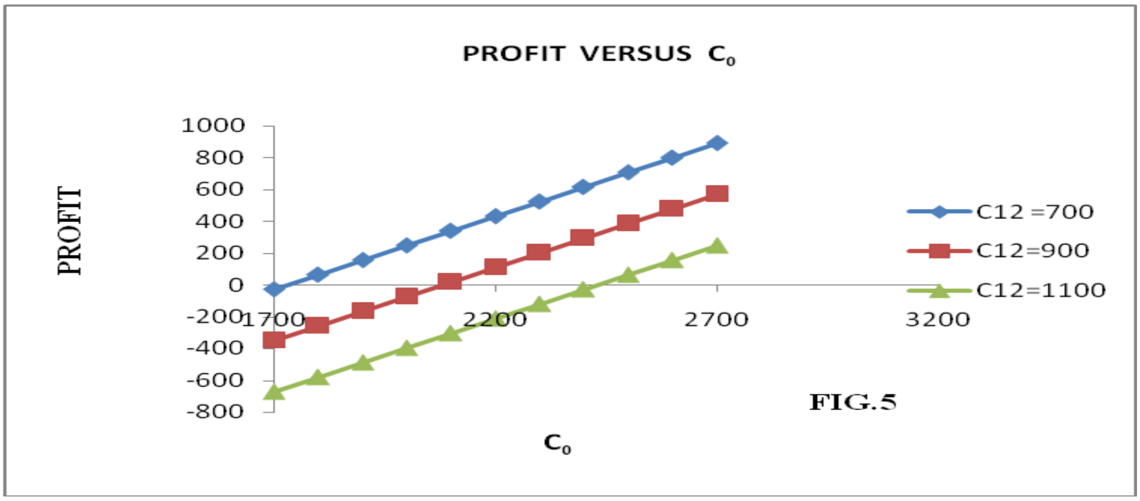

Figure 5.

11. Conclusion

For the particular case discussed above when the system is fails due to snow storm the reliability measures of the system such as mean time to system failure, availability, profit are computed. For the particular case discussed above the graphical interpretation are drawn in figures [2-4]. From the Figure 2 and Figure 3 it is observed the the MTSF and availability decreases as the failure rate increases respectively. Also from the Figure 4 profit is decreases as per visit repair rate of the repairman is increases and from the Figure 5 profit is increases as the revenue cost of the per unit is increases.

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