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Does Unemployment Induce Crime in Society? A Mathematical Study

Shyam Sundar[1,*], Agraj Tripathi[2], Ram Naresh[3]

[1]Department of Mathematics, Pranveer Singh Institute of Technology, Kanpur, India
[2]Department of Mathematics, AITH, Kanpur, India
[3]Department of Mathematics, School of Basic and Applied Sciences, Harcourt Butler Technical University, Kanpur-208002, India
*Corresponding author: ssmishra15@gmail.com

Abstract Today unemployment has become a global phenomenon which may be instrumental in forcing unemployed persons to earn their livelihood in an illegal manner resulting in a crime. It is possible that unemployed individuals may become more prone to develop a tendency of committing a crime when they come in contact with persons involved in criminal activities but are still unexposed. Further, when unemployed individuals are exposed to have committed a crime, they are captivated and finally awarded imprisonment if the offence charged on them is proved under the existing criminal laws. In this paper, a nonlinear mathematical model is developed to study the role of unemployment in inducing crime by taking into account four dependent variables representing the unemployment class, the employment class, the criminal class and the jail class. The model analysis, using stability theory of ordinary differential equations, provides some local and nonlinear stability conditions regarding stability of equilibrium of the model system. It is inferred that the endemic equilibrium is locally asymptotically stable as well as nonlinearly stable. Numerical simulations of the model system have also been carried to support the analytical findings and showing the effect of certain key parameters on different variables. It is observed that the increase in unemployment rate induces crime in the community leading to increase the burden on jail class.

Keywords: *unemployment, criminals, jail, equilibrium analysis, stability*

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1. Introduction

The increasing human population and associated social factors play a major role in raising unemployment. To cope up with the huge population load, the demand for large numbers of jobs is increasing but the rate of creation of new jobs in public and private sectors is not up to the mark. Further, due to rapid technological advancement, the need of manpower deployment in either sector is continuously declining and hence the opportunities of securing employment are getting reduced causing unemployment.

Moreover, persons with sufficient academic qualifications but not well equipped with requisite skills may not be hired by the public or private sectors also remain unemployed. In addition, most of the public and private sectors are cutting jobs to improve upon their economy according to budgetary allocations. Due to poor economic growth, insalubrious conditions of industries and several ups and downs of trade cycles, employers compel persons to drop the job and thus causing an increase in unemployment [4,6,10].

The individuals belonging to unemployed segment also contribute to lowering the economy of a nation by way of being not able to pay income tax [1]. In India, unemployment has also been a great conundrum during past several years. According to *The Hindu* newspaper report, 15% of all Indians with higher education (e.g. with graduate, postgraduate, technical and Ph. D. degrees) were jobless in 2011 [15]. Among all states of India, Kerala has high literacy rate but in contrary the unemployment rate of the state was found to be more than 30% in 2011. It has also been stated in the report that in India the unemployment rate has risen from 6.8% in 2001 to 9.6% in 2011. It is noted that after getting sufficient knowledge, qualification and expertise some people still remain unemployed due to reduced number of jobs in the public or private sectors as stated earlier. Sometimes unemployed people, registered at employment exchanges, receive a small amount by the government in the form of unemployment allowance thus creating an extra burden on debilitating the economy of a nation. Moreover, long state of unemployment reduces the efficiency of individuals for a job. This reduction in efficiency of individuals together with enhancement in latest technology further lowers their chances of being hired for a job and hence contributes to increase the number of unemployed people. It is worth notable here that it brings frustration among job seekers and a part of such community may develop a tendency to move towards crime either directly or by coming in contacts with existing criminals and finally leading to jail sentence [2,3,17].

Crime is one of the most threatening act affecting widely the economy, prosperity, development, social and political structure of both developed and developing countries. It is a cogent truth that there is a link between economic conditions, unemployment and crime. Thus, a major challenge before nations is to make a comprehensive study on unemployment and consequent crime so as to resolve the problem and overcome such a formidable situation. Literature study reveals that high unemployment rate among the youths of nations leads to significant growth in crime and hence criminal individuals [5,7,12,13,14,16]. Hibbert [5] made a study on unemployment and crime in Jamaica and found that 61% of unemployed youth were ranging between 20 to 24 years old and 96% were between 17 to 24 years. The study reveals that unemployment is a most crucial factor that leads people towards crime. Thus, committing crime by unemployed individuals is a big challenge worldwide as it is closely related to social and economic factors [7]. Due to non-availability of jobs or less opportunities within relevant time, people move towards crime and hence the magnitude of crime has increased over time.

Little attention has been paid to study the linkage of unemployment and criminals using mathematical models [8,9,11]. In this regard, Misra and Singh [8] proposed and analysed a nonlinear mathematical model on unemployment and derived the conditions under which unemployment increases. They further analysed a nonlinear model for the control of unemployment incorporating time delay in creating new vacancies [9].

The present investigation aims to study the link between unemployment and crime. The main objective of the study is to explore the effect of unemployment on crime. For this, a nonlinear mathematical model is developed using ordinary differential equations. In view of the social structure, four dependent variables representing the unemployment class, the employment class, the criminal class and the jailed class are taken into account for modeling unemployment - criminal dynamics.

Rest of the paper is organized as follows: In Section 2, the formulation of the model system is presented whereas Section 3 corresponds to the equilibrium analysis. Section 4 describes the local and global stability of the equilibrium followed by numerical simulation in Section 5. In Section 6, the summary and conclusions of the study are given.

2. Mathematical Model

To formulate the model system for the social structure under consideration, let $U(t)$ be the number of unemployed persons, $E(t)$ the number of employed persons, $C(t)$ the number of criminals and $J(t)$ be the number of criminals incarcerated to jail. It is assumed that the persons, who are fully qualified and eligible for jobs but are jobless, loose their skill proficiency as the time passes and remain unemployed thus increasing the number of unemployed persons with a constant rate Q. The unemployed individuals get employment with a constant rate λ where E_a denotes the total number of available vacancies at time t. It is also assumed that unemployed individuals get employed due to self employment at a rate α. Let μ be the rate at which employed persons get fired from their jobs thus increasing the growth of unemployed class. It is further assumed that the individuals, who remain unemployed for a long time and their opportunity of getting job is almost vanished, move towards crime either by self or by being motivated after coming in contact with existing criminals. It is, therefore, reasonable to assume that the growth of criminal class is directly proportional to the number of unemployed persons as well as the number of existing criminals (i.e. βUC), β being the rate by which unemployed individuals come into contact with criminals to join the criminal community.

Let the constant q be the direct inflow of individuals in the criminal class where it is assumed that employed individuals do not indulge in criminal activities. It is socially feasible that some of the criminal individuals leave the criminal class because of their effective counselling but due to unavailability of employment they remain unemployed and hence increase the growth of unemployed class. On the other hand some of the criminal individuals, depending on the magnitude of offence, are sentenced to jail. Thus, it is assumed that ν is the rate by which criminal individuals leave their class and a fraction of them $\varphi\nu C$ ($0<\varphi\le 1$) are sentenced to jail while remaining $(1-\varphi)\nu C$ join the unemployed class. Further, after completing their term of jail sentence, such individuals may opt to live a good and normal life either by getting employment in any form or by self employment without indulging again in criminal activities. But in real social situations it may not happen so. Therefore, it is reasonable to assume that some of them go for employment to earn their livelihood and others may turn back indulging in criminal activities. Since after getting released from the jail, it may become very difficult or even not possible for an individual to find any job and in such a situation they remain unemployed thus increasing the growth of unemployed class. In the fourth equation, therefore, of the model system, the jailed individuals are assumed to be released at a rate δ after completing their jail sentence term out of which some of them rejoin the criminal class by the reduced rate $\phi\delta$ ($0<\phi\le 1$) and the remaining of them join the unemployed class by rate $(1-\phi)\delta$. The constant d represents the death rate in all the classes.

In view of the above considerations, a model for unemployment - criminal dynamics is proposed as follows,

$$\frac{dU(t)}{dt} = Q-\lambda U(E_a-E)-\beta UC -(\alpha+d)U+\mu E+(1-\varphi)\nu C+(1-\phi)\delta J \tag{1}$$

$$\frac{dE(t)}{dt} = \lambda U(E_a-E)-(\mu+d)E+\alpha U \tag{2}$$

$$\frac{dC(t)}{dt} = q+\beta UC-(\nu+d)C+\phi\delta J \tag{3}$$

$$\frac{dJ(t)}{dt} = \varphi\nu C-(\delta+d)J \tag{4}$$

$$U(0) \geq 0,\ E(0) \geq 0,\ C(0) \geq 0,\ J(0) \geq 0$$

All the coefficients in the model system are taken to be positive.

Remark:

(1) It is noted from equation (1) of model system that if $\mu = 0$ and λ, the transfer rate coefficient of unemployed individuals to the employed class, is very large then dU/dt may become negative. Thus if unemployed individuals have more opportunities to get employment, the possibility of moving towards crime and subsequent jail sentence can be reduced.

(2) It is also noted from equation (3) of model system that if rate of movement of unemployed individuals to criminal class is zero (i.e. $\beta = 0$) and ν is very large with φ as small as possible then dC/dt may become negative. It means that if the migration of unemployed individuals to criminal class is stopped and persuading the existing criminals to quit the network then, in view of remark (1), our society can be made free from crime.

Now the following lemma is stated which establishes the boundaries of variables in the model system (1) - (4):

Lemma 2.1. The region of attraction for all solutions of model system (1) - (4) initiating in the positive octant is given by the set Γ:

$$\Gamma = \left\{ \begin{array}{l} (U,\ E,\ C,\ J) \in R_+^4, \\ 0 \leq U + E + C + J \leq \dfrac{Q+q}{d} \end{array} \right\}. \tag{5}$$

3. Equilibrium Analysis

The model system (1) - (4) has only one non-negative equilibrium $P^*(U^*, E^*, C^*, J^*)$ which is determined from the algebraic equations obtained by putting the right hand side of the model equations to zero.

$$\begin{aligned} &Q - \lambda U(E_a - E) - \beta UC - (\alpha + d)U \\ &+ \mu E + (1-\varphi)\nu C + (1-\phi)\delta J = 0 \end{aligned} \tag{6}$$

$$\lambda U(E_a - E) - (\mu + d)E + \alpha U = 0 \tag{7}$$

$$q + \beta UC - (\nu + d)C + \phi\delta J = 0 \tag{8}$$

$$\varphi \nu C - (\delta + d)J = 0. \tag{9}$$

Using eqs. (8) and (9) in eq. (6) we get,

$$Q - \lambda U(E_a - E) - (\alpha + d)U + \mu E - \frac{(\beta U - c_1)q}{c_2 - \beta U} = 0 \tag{10}$$

where $c_1 = (1-\varphi)\nu + (1-\phi)\dfrac{\varphi\delta\nu}{\delta + d}$ and $c_2 = \nu + d - \dfrac{\varphi\phi\delta\nu}{\delta + d}$.

From eq. (7) we note that,

(i) $E = E_a \Rightarrow U = \dfrac{(\mu + d)E_a}{\alpha}$

(ii) $U = 0 \Rightarrow E = 0$

(iii) $U = -\dfrac{\mu + d}{\lambda}$ is an asymptote parallel to E -axis.

(iv) $E = E_a + \dfrac{\alpha}{\lambda}$ is an asymptote parallel to U -axis

(v) $\dfrac{dE}{dU} = \dfrac{\lambda(E_a - E) + \alpha}{\lambda U + \mu + d} > 0$

From eq. (10), we note that,

(i) $U = 0 \Rightarrow E = -\dfrac{1}{\mu}\left[Q + \dfrac{qc_1}{c_2}\right]$

(ii) $E = E_a \Rightarrow Q - (\alpha + d)U + \mu E_a - \dfrac{q(\beta U - c_1)}{c_2 - \beta U} = 0$

(iii) $U = \dfrac{(\lambda c_2 - \mu\beta) \pm \sqrt{(\lambda c_2 - \mu\beta) + 4\lambda\mu\beta c_2}}{2\lambda\beta}$ are asymptotes parallel to E -axis.

(iv) $E = E_a + \dfrac{\alpha + d}{\lambda}$ is an asymptote parallel to U -axis

(v) $$\frac{dE}{dU} = \frac{\lambda(E_a - E) + (\alpha + d) + \dfrac{q\left\{d + \varphi\nu\left(1 - \dfrac{\phi\delta}{\delta + d}\right)\right\}}{(c_2 - \beta U)^2}}{\lambda U + \mu} > 0.$$

Thus the existence of (U^*, E^*) can be shown as in Figure 1. Finding the values of U^* and E^*, we can find the values of C^* and J^* from eqs. (8) and (9).

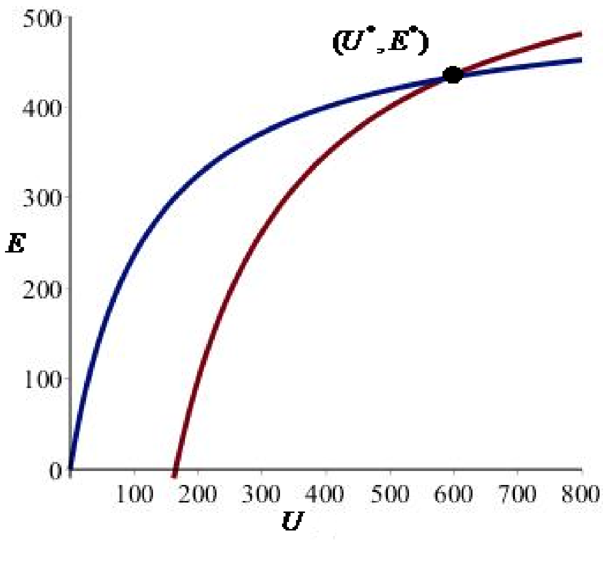

Figure 1. Existence of (U^*, E^*)

In the following, we check the characteristics of various dynamical variables with respect to key parameter.

3.1. Variation of Variables with Respect to Key Parameter

To see the variation of different variables we let,

$$\begin{aligned} &f_1(U, E, \lambda) = Q - \lambda U(E_a - E) \\ &-(\alpha + d)U + \mu E - \frac{(\beta U - c_1)q}{c_2 - \beta U} = 0 \end{aligned} \tag{11}$$

$$f_2(U, E, \lambda) = \lambda U(E_a - E) - (\mu + d)E + \alpha U = 0 \quad (12)$$

3.1.1. Variation of U with λ

We have

$$\frac{dU}{d\lambda} = \frac{\frac{\partial f_1}{\partial \lambda}\frac{\partial f_2}{\partial E} - \frac{\partial f_1}{\partial E}\frac{\partial f_2}{\partial \lambda}}{\frac{\partial f_1}{\partial U}\frac{\partial f_2}{\partial E} - \frac{\partial f_1}{\partial E}\frac{\partial f_2}{\partial U}} \quad (13)$$

$$\frac{\partial f_1}{\partial U} = -\lambda(E_a - E) - (\alpha + d) - \frac{(\beta U - c_1)q\beta}{(c_2 - \beta U)^2} - \frac{q\beta}{c_2 - \beta U},$$

$$\frac{\partial f_1}{\partial \lambda} = -U(E_a - E), \frac{\partial f_1}{\partial E} = \lambda U + \mu,$$

$$\frac{\partial f_2}{\partial U} = \lambda(E_a - E) + \alpha, \frac{\partial f_2}{\partial \lambda} = U(E_a - E),$$

$$\frac{\partial f_2}{\partial E} = -\lambda U - (\mu + d).$$

It can be easily checked that $\frac{\partial f_1}{\partial \lambda}\frac{\partial f_2}{\partial E} - \frac{\partial f_1}{\partial E}\frac{\partial f_2}{\partial \lambda} < 0$ and $\frac{\partial f_1}{\partial U}\frac{\partial f_2}{\partial E} - \frac{\partial f_1}{\partial E}\frac{\partial f_2}{\partial U} > 0$ and hence $\frac{dU}{d\lambda} < 0$. This implies that the number of unemployed individuals decreases in the society as rate of getting employment increases. Similarly we can easily check that $\frac{dC}{d\lambda} < 0$ and $\frac{dJ}{d\lambda} < 0$ showing that the number of criminals and jailed persons decreases with increase in the rate of employment. Thus, if more employment opportunities are generated, the crime rate can be slowed down which will result in reduced number of criminals and subsequent jailed criminals.

4. Stability Analysis

In the following, we discuss the local and nonlinear stability of the equilibrium of the model (1) - (4). The results are stated in the form of following theorems:

Theorem 4.1.

The equilibrium $P^*(U^*, E^*, C^*, J^*)$ is locally asymptotically stable under the following conditions,

$$\varphi\phi\delta\nu < \frac{1}{6}(\nu + d - \beta U^*)(\delta + d) \quad (14)$$

$$\max\{p_1, p_2\} < \min\{l_1, l_2, l_3\} \quad (15)$$

where,

$$p_1 = \frac{5}{2}\frac{(\lambda(E_a - E^*) + \alpha)^2}{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d)}$$

$$p_2 = \frac{15}{4}\frac{(\beta C^*)^2}{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(-\beta U^* + \nu + d)}$$

$$l_1 = \frac{2}{5}\frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d)}{(\lambda U^* + \mu)^2}$$

$$l_2 = \frac{4}{15}\frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(-\beta U^* + \nu + d)}{(-\beta U^* + (1-\varphi)\nu)^2}$$

$$l_3 = \frac{2}{5}\frac{\phi\delta}{\varphi\nu}\frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\delta + d)}{((1-\phi)\delta)^2}$$

(For proof see Appendix - A).

Theorem 4.2.

The equilibrium $P^*(U^*, E^*, C^*, J^*)$ is nonlinearly stable inside the region of attraction Γ if following inequalities hold:

$$\varphi\phi\delta\nu < \frac{1}{6}(\nu + d - \beta U^*)(\delta + d) \quad (16)$$

$$\max\{r_1, r_2\} < \min\{s_1, s_2, s_3\} \quad (17)$$

where, $r_1 = \frac{5}{2}\frac{(\lambda(E_a - E^*) + \alpha)^2}{(\alpha + d)(\mu + d)}$,

$$r_2 = \frac{15}{4}\frac{\left(\frac{Q+q}{d}\right)^2 \beta^2}{(\alpha + d)(\nu + d - \beta U^*)}$$

$$s_1 = \frac{2}{5}\frac{(\alpha + d)(\delta + d)}{(1-\phi)\delta},$$

$$s_2 = \frac{4}{15}\frac{(\alpha + d)(\nu + d - \beta U^*)}{(-\beta U^* + (1-\varphi)\nu)^2},$$

$$s_3 = \frac{2}{5}\frac{(\alpha + d)(\mu + d)}{(\lambda U^* + \mu)^2}$$

(For proof see Appendix - B).

Remark: Above two theorems imply that if $\varphi = 0$ and $\phi = 0$ then the conditions (14) and (16) are satisfied automatically. Along with these assumption if the rate of transfer of unemployed individuals to criminal class is zero (i.e. $\beta = 0$) and $E^* = E_a$ then the possibility of satisfying conditions (15) and (17) is more plausible. This implies that these parameters have destabilizing effect on the model system.

5. Numerical Simulation

To check the feasibility of our analysis regarding the existence of $P^*(U^*, E^*, C^*, J^*)$ and the corresponding stability conditions, we have conducted some numerical simulation of the model (1) - (4) using MAPLE18 by choosing the following set of parameter values:

$$Q = 1000, q = 100, E_a = 500, \lambda = 0.01,$$
$$\beta = 0.00022, \alpha = 0.2,$$
$$\mu = 0.2, d = 1, \varphi = 0.6, \nu = 0.6, \phi = 0.2, \delta = 1$$

For the above set of parameter values, the equilibrium values of different variables are obtained as

$U^* = 586.14, E^* = 431.63, C^* = 69.68, J^* = 12.54.$

Eigenvalues of the Jacobian matrix corresponding to the equilibrium $P^*(U^*, E^*, C^*, J^*)$ are found as – 7.9470, –0.9999, –1.3765, –2.1079. Since all the eigenvalues are negative, therefore, the equilibrium P^* is locally asymptotically stable. To see the nonlinear stability behavior, for the above set of parameter values, computer generated graph in $U-E-C$ plane has been drawn in Figure 2 with different initial starts. From this figure, we note that all the trajectories starting at any point always approach to its equilibrium P^*.

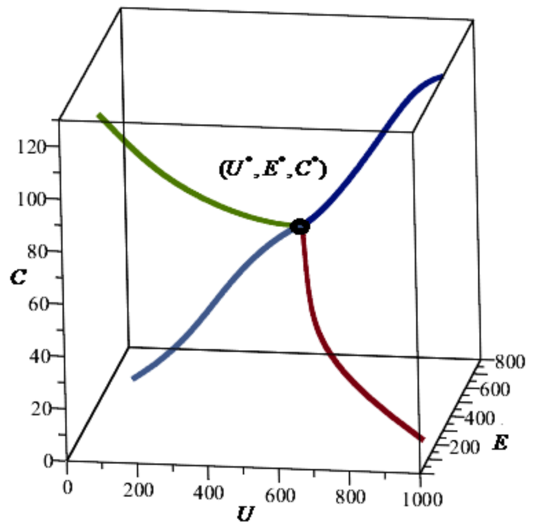

Figure 2. Nonlinear stability in $U-E-C$ plane

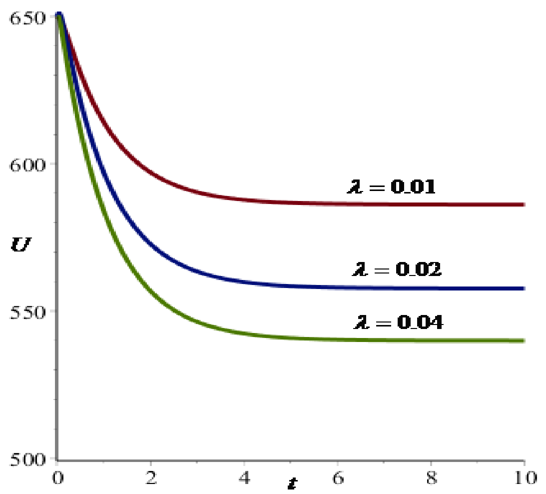

Figure 3. Variation of unemployed class with time 't' for different values of λ, the rate by which unemployed individuals get employment

To see the effect of various parameters on the dependent variables, we have solved the equations of model (1) - (4) and plotted these in Figure 3 - Figure 14. In Figure 3, the variation of unemployed individuals with time 't' is shown for different values of λ, the rate by which unemployed individuals get employment. From this figure, it is seen that on increasing λ, the number of unemployed individuals decreases which in turn increases the number of employed individuals (Figure 4). In Figure 5 - Figure 8, the variation of unemployed individuals, employed individuals, criminals and jailed individuals respectively with time 't' is shown for different values of α, the rate by which unemployed individuals get self employed. From these figures, it is seen that as the rate of transfer of unemployed individuals to employed class due to self employment increases, the number of employed individuals increases and that of unemployed individuals decreases (Figure 5 and Figure 6 respectively). This decrease in number of unemployed individuals increases the number of employed individuals which in turn decreases the number of criminals (Figure 7) and jailed individuals (Figure 8). This implies that in case of shortage of vacancies, unemployed individuals should be motivated for self employment so that the possibility of unemployed individuals indulging in criminal activities would be eliminated. The variation of individuals in criminal class with time 't' for different values of the transfer rate of unemployed individuals to criminal class (i.e. β) is shown in Figure 9. It is found that as the transfer rate β from unemployed class increases, the individuals in criminal class increase leading to increase the individuals in jailed class (Figure 10). Thus, in order to reduce the number of criminals and subsequent jailed individuals, more employment opportunities should be created either through new vacancies or through self employment so that unemployed individuals are not attracted towards crime.

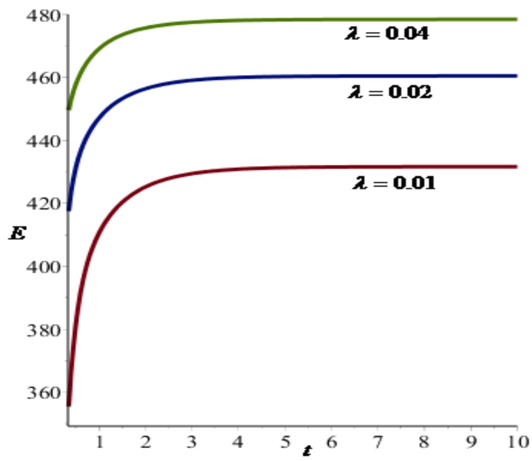

Figure 4. Variation of employed class with time 't' for different values of λ, the rate by which unemployed individuals get employment

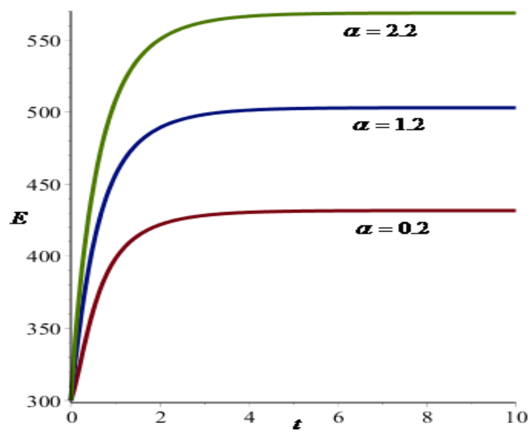

Figure 5. Variation of employed class with time 't' for different values of α, the rate by which unemployed individuals get self employed

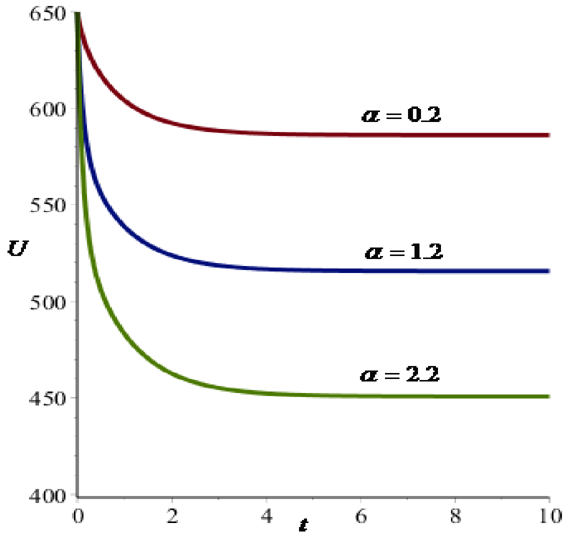

Figure 6. Variation of unemployed class with time ' t ' for different values of α , the rate by which unemployed individuals get self employed

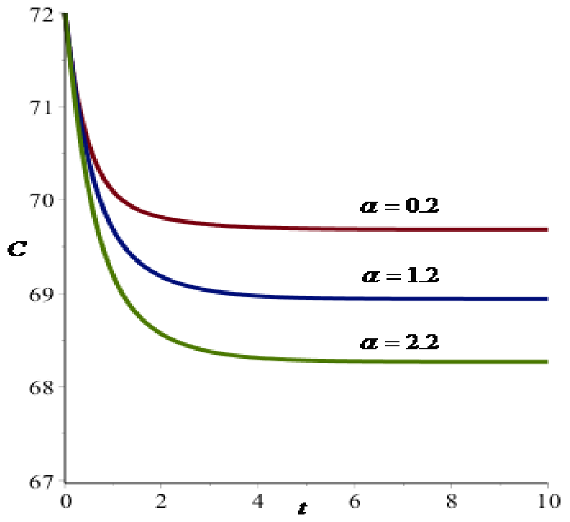

Figure 7. Variation of criminal class with time ' t ' for different values of α , the rate by which unemployed individuals get self employed

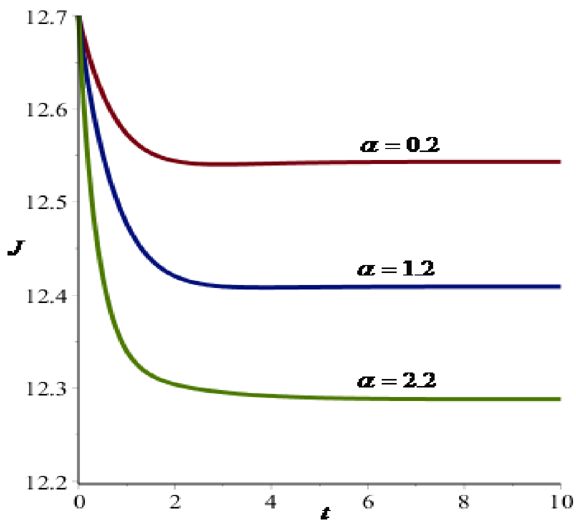

Figure 8. Variation of individuals in jail class with time ' t ' for different values of α , the rate by which unemployed individuals get self employed

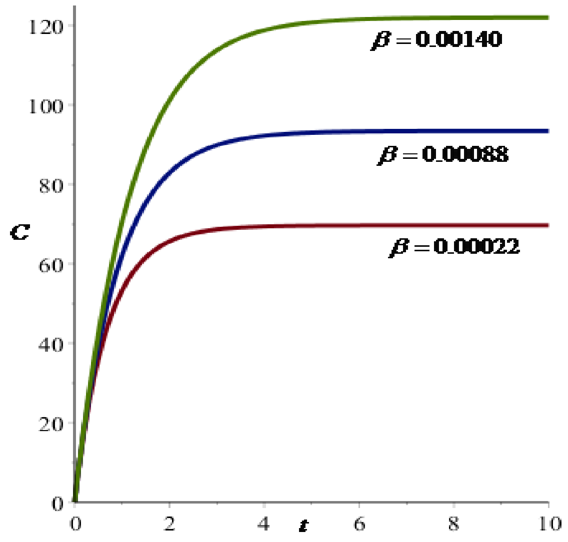

Figure 9. Variation of criminal class with time ' t ' for different values of β , the transfer rate of unemployed individuals to criminal class

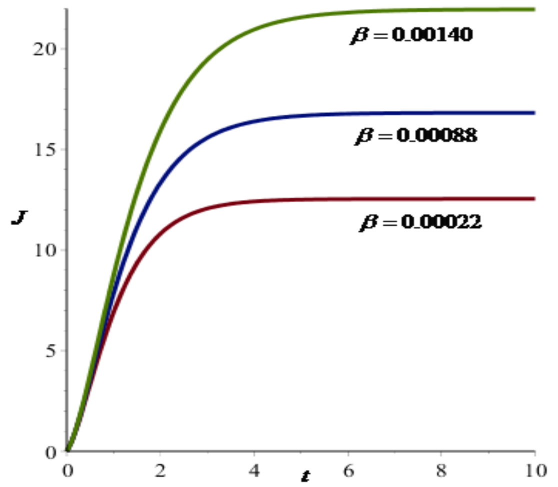

Figure 10. Variation of individuals in jail class with time ' t ' for different values of β , the transfer rate of unemployed individuals to criminal class

In the absence of proper counselling or self motivation, individuals in criminal class continue to indulge in criminal activities and when identified they are sentenced to jail depending on the magnitude of their offence, if proved. In such a situation, if the rate of transfer of criminal individuals to jail class (i.e. φ) increases, the individuals in jail class increase enhancing an extra burden on jail and hence on the economy of a country (Figure 11). Thus, if this rate (i.e. φ) decreases then the individuals in unemployed class increases (Figure 12). It is, therefore, speculated that if proper counselling is given to criminal individuals to leave the criminal activity, only a few would be sentenced to jail while the increased number of remaining fraction joining the unemployed class can be convinced or motivated for self employment. Further, it is worth mentioning here that after being released from the jail if these criminal individuals re-enter into the criminal class and indulge in criminal activities then definitely it will create an alarming situation for the society and the country. This aspect is shown in Figure 13 where the variation of criminal individuals with time ' t ' for different values of ϕ, the rate of transfer of jailed individuals to criminal class, is plotted.

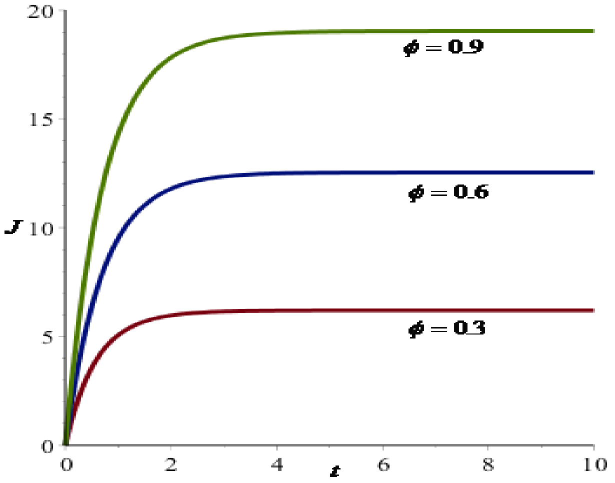

Figure 11. Variation of individuals in jail class with time 't' for different values of φ, the rate of transfer of criminal individuals to jail class

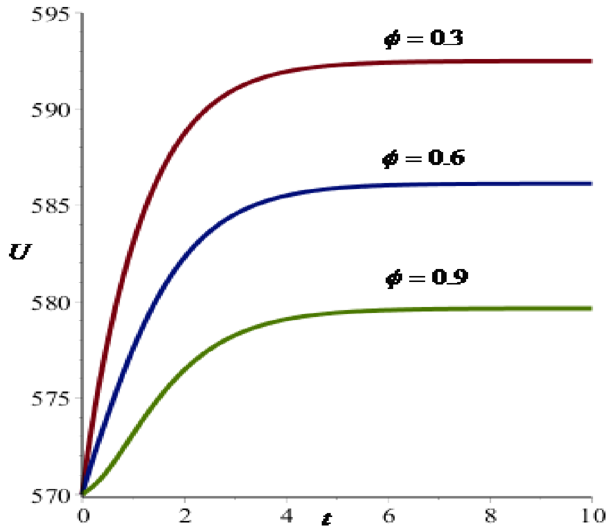

Figure 12. Variation of unemployed class with time 't' for different values of φ, the rate of transfer of criminal individuals to jail class

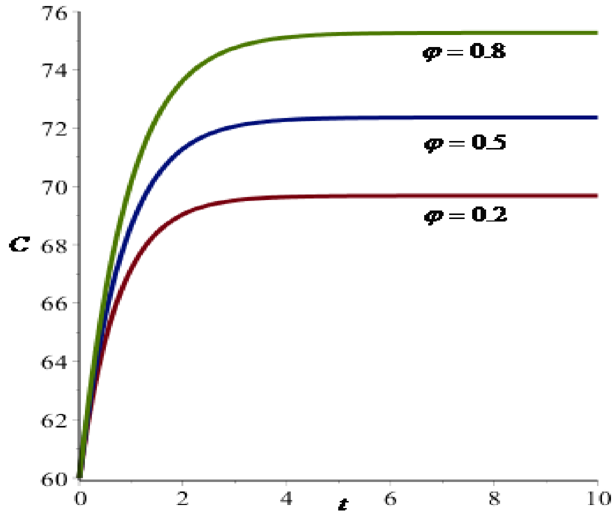

Figure 13. Variation of criminal class with time 't' for different values of ϕ, the rate of transfer of jailed individuals to criminal class

It is seen from this figure that the number of criminal individuals increases with increase in the transfer rate of jailed individuals to criminal class. This implies that crime is bound to rise in society if the jailed individuals, after completing their jail sentence, again indulge in criminal activities. Decrease in the transfer rate of jailed individuals to criminal class (ϕ) implies increase in the number of unemployed individuals (Figure 14). This further signifies that if the individuals released from jail do not indulge in criminal activities, the crime can be under control. On the other hand resulting higher number of unemployed individuals can be motivated to search for a job or get self employed.

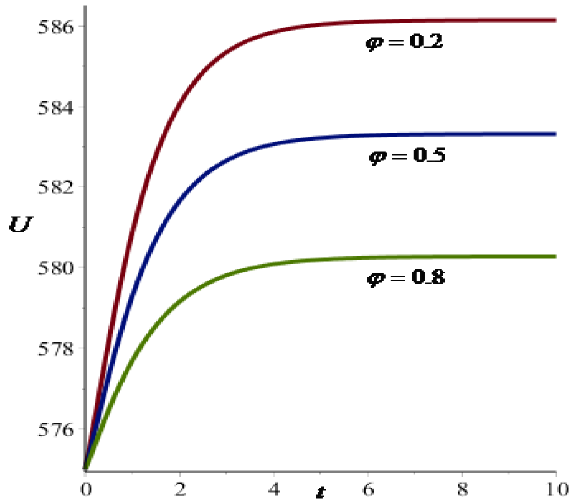

Figure 14. Variation of unemployed class with time 't' for different values of ϕ, the rate of transfer of jailed individuals to criminal class

6. Conclusions

In this paper, a nonlinear mathematical model is proposed and analyzed to study the effect of unemployment on crime. For modeling unemployment-criminal dynamics, we have considered four dependent variables namely, the unemployment class, the employment class, the criminal class and the jail class. The proposed model has been analysed using stability theory of differential equations and numerical simulation. Some inferences have been drawn regarding stability of equilibrium and it is found that a unique non-trivial equilibrium exists which is locally as well as nonlinearly asymptotically stable under certain conditions.

From the model analysis, it is found that as the rate of transfer of unemployed individuals to employed class due to self employment or by way of getting employment increases, the number of unemployed individuals decreases. This decrease in the number of unemployed individuals i.e. the increase in the number of employed individuals decreases the number of criminals and jailed individuals and hence reduces crime. The increased interaction of unemployed individuals with criminal class induces the unemployed individuals to join criminal class. Thus, crime can be reduced by reducing the interaction of unemployed individuals with criminal population and it would be possible by proper counselling of such persons and by creating new jobs in public/private sectors for getting them employed. Crime can also be reduced, if the individuals released from jail after completing their jail sentence do not indulge in criminal activity. Moreover, such individuals can be properly counselled and motivated to search for a job or get self employed. This ultimately will be helpful in reducing the burden of crime in the society.

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Appendix- A

Proof of the Theorem 4.1

To establish the local stability behavior of $P^*(U^*, E^*, C^*, J^*)$, we have considered the following positive definite function,

$$V = \frac{1}{2}k_1u^2 + \frac{1}{2}k_2e^2 + \frac{1}{2}k_3c^2 + \frac{1}{2}k_4 j^2 \tag{A1}$$

where $k_i\,(i = 1...4)$ are positive constants to be chosen appropriately and u, e, c, j are small perturbations about P^* i.e. $U = U^* + u, E = E^* + e, C = C^* + c, J = J^* + j$.

Differentiating (A1) with respect to *t* along the solutions of system (1) - (4) and after a simple algebraic manipulations we get,

$$\begin{aligned}\frac{dV}{dt} &= -k_1(\lambda(E_a - E^*) + \beta C^* + \alpha + d)u^2 - k_2(\lambda U^* + \mu + d)e^2 \\ &\quad - k_3(\nu + d - \beta U^*)c^2 - k_4(\delta + d)j^2 + k_1(\lambda U^* + \mu)ue \\ &\quad + k_2(\lambda(E_a - E^*) + \alpha)ue + k_1(-\beta U^* + (1-\varphi)\nu)uc \\ &\quad + k_3\beta C^* uc + k_1(1-\phi)\delta\, uj + (k_3\phi\delta + k_4\varphi\nu)cj.\end{aligned} \tag{A2}$$

Now $\frac{dV}{dt}$ will be negative definite provided the following inequalities are satisfied:

$$k_1(\lambda U^* + \mu)^2 < \frac{2}{5}k_2(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d) \tag{A3}$$

$$k_2(\lambda(E_a - E^*) + \alpha)^2 < \frac{2}{5}k_1(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d) \tag{A4}$$

$$k_1(-\beta U^* + (1-\varphi)\nu)^2 < \frac{4}{15} k_3((\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\nu + d - \beta U^*) \quad \text{(A5)}$$

$$k_3(\beta C^*)^2 < \frac{4}{15} m_1(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\nu + d - \beta U^*) \quad \text{(A6)}$$

$$k_1((1-\phi)\delta)^2 < \frac{2}{5} k_4(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\delta + d) \quad \text{(A7)}$$

$$(k_3\phi\delta + k_4\varphi\nu) < \frac{2}{3} k_3 k_4 (\nu + d - \beta U^*)(\delta + d) \quad \text{(A8)}$$

Now re-writing eq. (A8) as,

$$(k_3\phi\delta - k_4\varphi\nu) + 4\phi\delta\varphi\nu < \frac{2}{3} k_3 k_4 (\nu + d - \beta U^*)(\delta + d)$$

and choosing $k_3 = 1$ and $k_4 = \frac{\phi\delta}{\varphi\nu}$, it reduces to

$$\varphi\phi\delta\nu < \frac{1}{6}(\nu + d - \beta U^*)(\delta + d). \quad \text{(A9)}$$

Now choosing $k_2 = 1$, remaining equations reduce for k_1 , from which we get,

$$\max\{p_1, p_2\} < \min\{l_1, l_2, l_3\}$$

where,

$$p_1 = \frac{5}{2} \frac{(\lambda(E_a - E^*) + \alpha)^2}{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d)}$$

$$p_2 = \frac{15}{4} \frac{(\beta C^*)^2}{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(-\beta U^* + \nu + d)}$$

$$l_1 = \frac{2}{5} \frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\lambda U^* + \mu + d)}{(\lambda U^* + \mu)^2}$$

$$l_2 = \frac{4}{15} \frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(-\beta U^* + \nu + d)}{(-\beta U^* + (1-\varphi)\nu)^2}$$

$$l_3 = \frac{2}{5} \frac{\phi\delta}{\varphi\nu} \frac{(\lambda(E_a - E^*) + \beta C^* + \alpha + d)(\delta + d)}{((1-\phi)\delta)^2}.$$

Thus, $\frac{dV}{dt}$ will be negative definite under the conditions (14) and (15) given in the theorem.

Appendix- B

Proof of the Theorem 4.2

To prove this theorem, we consider following positive definite function about P^*,

$$M = \frac{m_1}{2}(U - U^*)^2 + \frac{m_2}{2}(E - E^*)^2 + \frac{m_3}{2}(C - C^*)^2 + \frac{m_4}{2}(J - J^*)^2 \quad \text{(B1)}$$

where $m_i (i = 1...4)$ are positive constants, to be chosen appropriately. Differentiating above equation with respect to t along the solutions of system (1) - (4) and after simple algebraic manipulations we get,

$$\begin{aligned}\frac{dM}{dt} &= -m_1(\lambda(E_a - E) + \beta C + \alpha + d)(U - U^*)^2 - m_2(\lambda U + \mu + d)(E - E^*)^2 \\ &\quad - m_3(\nu + d - \beta U^*)(C - C^*)^2 - m_4(\delta + d)(J - J^*)^2 \\ &\quad + m_1(\lambda U^* + \mu)(U - U^*)(E - E^*) + m_2(\lambda(E_a - E^*) + \alpha)(U - U^*)(E - E^*) \\ &\quad + m_1(-\beta U^* + (1-\varphi)\nu)(U - U^*)(C - C^*) + m_3\beta C^*(U - U^*)(C - C^*) \\ &\quad + m_1(1-\phi)\delta(U - U^*)(J - J^*) + (m_3\phi\delta + m_4\varphi\nu)(C - C^*)(J - J^*).\end{aligned} \tag{B2}$$

Now $\frac{dM}{dt}$ will be negative definite inside the region of attraction provided the following inequalities are satisfied,

$$m_1(\lambda U^* + \mu)^2 < \frac{2}{5} m_2(\lambda(E_a - E) + \beta C + \alpha + d)(\lambda U + \mu + d) \tag{B3}$$

$$m_2(\lambda(E_a - E^*) + \alpha)^2 < \frac{2}{5} m_1(\lambda(E_a - E) + \beta C + \alpha + d)(\lambda U + \mu + d) \tag{B4}$$

$$m_1(-\beta U^* + (1-\varphi)\nu)^2 < \frac{4}{15} m_3(\lambda U + \mu + d)(\nu + d - \beta U^*) \tag{B5}$$

$$m_3(\beta C^*)^2 < \frac{4}{15} m_1(\lambda U + \mu + d)(\nu + d - \beta U^*) \tag{B6}$$

$$m_1((1-\phi)\delta)^2 < \frac{2}{5} m_4(\lambda(E_a - E) + \beta C + \alpha + d)(\delta + d) \tag{B7}$$

$$(m_3\phi\delta + m_4\varphi\nu) < \frac{2}{3} m_3 m_4(\nu + d - \beta U^*)(\delta + d) \tag{B8}$$

Again the last equation can be written as,

$$(m_3\phi\delta - m_4\varphi\nu) + 4\phi\delta\varphi\nu < \frac{2}{3} m_3 m_4(\nu + d - \beta U^*)(\delta + d)$$

Now choosing $m_3 = 1$ and $m_4 = \frac{\phi\delta}{\varphi\nu}$, it reduces to

$$\varphi\phi\delta\nu < \frac{1}{6}(\nu + d - \beta U^*)(\delta + d) \tag{B9}$$

Now maximizing LHS and minimizing RHS and choosing $m_2 = 1$, remaining equations reduce for m_1, from which we get,

$$\max\{r_1, r_2\} < \min\{s_1, s_2, s_3\}$$

where, $r_1 = \frac{5}{2}\frac{(\lambda(E_a - E^*) + \alpha)^2}{(\alpha + d)(\mu + d)}$, $r_2 = \frac{15}{4}\frac{\left(\frac{Q+q}{d}\right)^2 \beta^2}{(\alpha + d)(\nu + d - \beta U^*)}$ $s_1 = \frac{2}{5}\frac{(\alpha + d)(\delta + d)}{(1-\phi)\delta}$, $s_2 = \frac{4}{15}\frac{(\alpha + d)(\nu + d - \beta U^*)}{(-\beta U^* + (1-\varphi)\nu)^2}$, $s_3 = \frac{2}{5}\frac{(\alpha + d)(\mu + d)}{(\lambda U^* + \mu)^2}$

Thus, $\frac{dM}{dt}$ will be negative definite under the conditions (16) and (17) of the theorem within the region of attraction.