American Journal of Applied Mathematics and Statistics, 2018, Vol. 6, No. 1, 1-5
Available online at http://pubs.sciepub.com/ajams/6/1/1

DOI:10.12691/ajams-6-1-1

Fixed Point Theorems on Parametric A-metric Space

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Abstract In this paper, we introduce the notion of parametric A-metric space as generalisation of parametric metric space and parametric S-metric space. Further we prove some fixed point theorem of expansive mapping in the setting of parametric A-metric space.

Keywords: *parametric metric space, A-metric space, parametric A-metric space, expansive mappings, fixed point*

Cite This Article: N. Priyobarta, Yumnam Rohen, and Stojan Radenovic, "Fixed Point Theorems on Parametric A-metric Space." *American Journal of Applied Mathematics and Statistics*, vol. 6, no. 1 (2017): 1-5. doi: 10.12691/ajams-6-1-1.

1. Introduction

Fixed point theory and different forms of generalization of metric space is one of the interesting topic for many researchers. This can be witnessed from the vast literature available in this topic. In order to study some forms of generalization of metric space one can see the research papers in [1-14,19-21,23-26] and references there in. As one of the generalization Sedghi et. al. [1] introduced the concept of *S*-metric space. The definition of *S*-metric space is as follows.

Definition 1.1. [1] Let X be a nonempty set. An S-metric on X is a function $S: X^3 \to [0,\infty)$ that satisfies the following conditions,

1. $S(x,y,z) \geq 0,$
2. $S(x,y,z) = 0$ if and if x=y=z,
3. $S(x,y,z) \leq S(x,x,a)+S(y,y,a)+S(z,z,a)$ for each $x,y,z,a \in X$.

The pair (*X*, *S*) is called *S*-metric space.

Further the definition of *S*-metric space is generated by extending to *n*-tuple by Abbas et.al. [2]. The extended structure is called *A*-metric. The definition of *A*-metric space is given as follows.

Definition 1.2. [2] Let X be a nonempty set. A function $A: X^n \to [0,\infty)$ is called an A-metric on X if for any $x_i, a \in X, i = 1,2,....n,$ the following conditions hold:

(A1) $A(x_1,x_2,x_3,.....x_{n-1},x_n) \geq 0,$

(A2) $A(x_1,x_2,x_3,.....x_{n-1},x_n) = 0$ if and only $x_1 = x_2 = x_3 = ... = x_{n-1} = x_n,$

(A3)

$$\begin{aligned}
&A(x_1,x_2,x_3,.....x_{n-1},x_n)\\
&\leq [A(x_1,x_1,x_1,.....(x_1)_{n-1},a)\\
&+A(x_2,x_2,x_2,.....(x_2)_{n-1},a)\\
&+A(x_3,x_3,x_3,.....(x_3)_{n-1},a)\\
&\vdots\\
&+A(x_{n-1},x_{n-1},x_{n-1},.....(x_{n-1})_{n-1},a)\\
&+A(x_n,x_n,x_n,.....(x_n)_{n-1},a)]
\end{aligned}$$

The pair (X, A) is called an A-metric space.

In another achievement towards the generalization of metric space Hussain et. al. [15] introduced the concept of parametric metric space. They proved some fixed point theorems in complete parametric metric space. The concept of parametric metric space is further extended to parametric *S*-metric space by Nihal et. al. [16]. Using some expensive mappings, they proved a fixed point theorem on a parametric *S*-metric space. They also gave examples of parametric *S*-metric which is not generated by any parametric metric space. Hussain et. al. [17] also introduced the concept of parametric *b*-metric space and investigates the existence of fixed points under various contractive conditions in this space. Reshuni et. al. [18] further established some fixed point, common fixed and coincidence point theorems for expansive type mappings in parametric metric space and parametric *b*-metric spaces. We recall following definitions etc.

Definition 1.3. [15] Let X be a nonempty set and let $P: X \times X \times (0,\infty) \to [0,\infty)$ be a function. P is called a parametric metric on X if,

(P1) P (a, b, t) = 0 if and only if a = b,
(P2) P (a, b, t) = P (b, a, t) ,
(P3) P (a, b, t) ≤ P (a, x, t) + P (x, b, t),

for each a, b, x ∈ X and all t > 0.

The pair (X, P) is called a parametric metric space.

Definition 1.4. [15] Let (X, P) be a parametric metric space and let $\{a_n\}$ be a sequence in X:

(1) $\{a_n\}$ converges to x if and only if there exists $n_0 \in N$ such that $P(a_n,x,t) < \varepsilon$, for all $n \geq n_0$ and t > 0; that is, $\lim_{n\to\infty} P(a_n,x,t) = 0$. It is denoted by $\lim_{n\to\infty} a_n = x$.

(2) $\{a_n\}$ is called a Cauchy sequence if, for all t > 0, $\lim_{n,m\to\infty} P(a_n, a_m, t) = 0$.

(3) (X, P) is called complete if every Cauchy sequence is convergent.

Definition 1.5. [16] Let X be a nonempty set and let $P_s : X \times X \times X \times (0,\infty) \to [0,\infty)$ be a function. P_s is called a parametric S-metric on X if,

(PS1) $P_s(a,b,c,t) = 0$ if and only if $a = b = c$,

(PS2) $P_s(a,b,c,t) \le P_s(a,a,x,t) + P_s(b,b,x,t) + P_s(c,c,x,t)$,

for each $a,b,c,x \in X$ and all t > 0. The pair (X, P_s) is called a parametric S-metric space.

Lemma 1.1. [16] Let (X, P_s) be a parametric S-metric space. Then we have $P_s(a,a,b,t) = P_s(b,b,a,t)$ for each $a,b \in X$ and all t > 0.

Lemma 1.2. [16] Let (X, P) be a parametric metric space and let the function $P_S^P : X \times X \times X \times (0,\infty) \to [0,\infty)$ be defined by $P_S^P(a,b,c,t) = P(a,c,t) + P(b,c,t)$, for each $a,b,c \in X$ and all t > 0. Then P_S^P is a parametric S-metric and the pair (X, P_S^P) is a parametric S-metric space.

Lemma 1.3. [16] Let (X, P_s) be a parametric S-metric space. If $\{a_n\}$ converges to x, then x is unique.

Lemma 1.4. [16] Let (X, P_s) be a parametric S-metric space. If $\{a_n\}$ converges to x, then $\{a_n\}$ is Cauchy.

Definition 1.6. Let (X, P_s) be a parametric S-metric space and let $T : X \to X$ be a self-mapping of X. T is said to be a continuous mapping at x in X if $\lim_{n\to\infty} P_S(Ta_n, Ta_n, Tx, t) = 0$, for any sequence $\{a_n\}$ in X and all t > 0 such that $\lim_{n\to\infty} P_S(a_n, a_n, x, t) = 0$.

Motivated by the concepts introduced by Abbas et. al. [2], Hussian et. al. [15] and Nihal et. al. [16], we further investigate and extend the concept of parametric metric space and parametric *S*-metric to parametric *A*-metric space. We also give some properties of parametric *A*-metric space. Further, we prove some fixed point theorems for various expansive mappings in the setting of parametric *A*-metric space.

2. Parametric *A*-Metric Spaces

In this section, we introduce the notion of parametric A-metric space and give some basic properties of this space.

Definition 2.1. Let X be a nonempty set and let $P_A : X^n \times [0,\infty) \to [0,\infty)$ be a function. P_A is called a parametric A-metric on X if,

(PA1) $P_A(a_1, a_2, \ldots a_n, t) = 0$ if and only if $a_1 = a_2 = \ldots a_n$,

(PA2)

$$\begin{aligned} P_A(a_1, a_2, \ldots a_n, t) &\le P_A(a_1, a_1, \ldots (a_1)_{n-1}, t) \\ &+ P_A(a_2, a_2, \ldots (a_2)_{n-1}, t) \\ &\vdots \\ &+ P_A(a_n, a_n, \ldots (a_n)_{n-1}, t) \end{aligned}$$

for each $a_1, a_2, \ldots a_n, x \in X$ and all t ≥ 0. The pair (X, P_A) is called a parametric A-metric space. Now we give the following examples of parametric A-metric spaces.

Example 1. Let $X = \mathbb{R}$ and let the function $P_A : X^n \times [0,\infty) \to [0,\infty)$ be denoted by

$$P_A(a_1, a_2, \ldots a_n, t) = g(t)(|a_1 - a_2| + |a_2 - a_3| + \ldots + |a_n - a_1|)$$

for each $a_1, a_2, \ldots, a_n \in \mathbb{R}$ and all t > 0, where $g : (0,\infty) \to (0,\infty)$ is a continuous function. Then P_A is a parametric A-metric and the pair $(\mathbb{R}, P_A)$ is a parametric A-metric space.

Example 2. Let $X = (\mathbb{R}^+, P_A) \cup \{0\}$ and let the function $P_A : X^n \times [0,\infty) \to [0,\infty)$ be defined by

$$P_A(x_1, x_2, \ldots, x_n, t) = \begin{cases} 0, & if x_1 = x_2 = \ldots = x_n; \\ g(t)\max\{x_1, x_2, \ldots, x_n\}, & otherwise. \end{cases}$$

for each $x_1, x_2, \ldots, x_n \in X$ and all t > 0, where $g : [0,\infty) \to (0,\infty)$ is a continuous function. Then P_A is a parametric A-metric space.

We prove the following Lemma which can be considered as the symmetry condition in a parametric A-metric space.

Lemma 2.1. Let (X, P_A) be a parametric A-metric space. Then we have

$$P_A(a,a,\ldots,a,b,t) = P_A(b,b,\ldots,b,a,t) \quad (1)$$

for each $a,b \in X$ and all a > 0.

Proof. Using the condition (PA2), we obtain

$$\begin{aligned} P_A(a,a,\ldots,a,b,t) &\le (n-1)P_A(a,a,\ldots,a,b,t) \\ &+ P_A(b,b,\ldots,b,a,t) \\ &= P_A(b,b,\ldots,b,a,t) \end{aligned} \quad (2)$$

$$\begin{aligned} P_A(b,b,\ldots,b,a,t) &\le (n-1)P_A(b,b,\ldots,b,a,t) \\ &+ P_A(a,a,\ldots,a,b,t) \\ &= P_A(a,a,\ldots,a,b,t). \end{aligned} \quad (3)$$

From (2) and (3) we have

$$P_A(a,a,\ldots,a,b,t) = P_A(b,b,\ldots,b,a,t).$$

Lemma 2.2. Let (X, P_A) be a parametric A-metric space. If $\{a_n\}$ converges to x, then x is unique.

Proof. Let $\lim_{n\to\infty} a_n = x$ and let $\lim_{n\to\infty} a_n = y$ with $x \ne y$. Then there exists $n_1, n_2 \in \mathbb{N}$ such that

$$P_A(a_n, a_n, \ldots, a_n, x, t) < \frac{\varepsilon}{2(n-1)},$$

$$P_A(a_n, a_n, \ldots, a_n, y, t) < \frac{\varepsilon}{2}$$

for each $\varepsilon > 0$, all t > 0 and $n \ge n_1, n_2$. If we take $n_0 = \max\{n_1, n_2\}$, then using condition (PA2) and Lemma 2.1, we have

$$\begin{aligned}&P_A(x,x,...x,y,t)\le(n-1)P_A(x,x,...,x,a_n,t)\\&+P_A(y,y,...,y,a_n,t)\\&<(n-1)\frac{\varepsilon}{2(n-1)}+\frac{\varepsilon}{2}\\&=\varepsilon\end{aligned}$$

for each $n\ge n_0$. Therefore $P_A(x,x,...x,y,t)=0$ and $x=y$.

Lemma 2.3. Let (X,P_A) be a parametric A-metric space. If $\{a_n\}$ converges to x, then $\{a_n\}$ is Cauchy.

Proof: By similar argument as in Lemma 2.2. One can easily follow the result.

Definition 2.2. Let (X,P_A) be a parametric A-metric space and let $T:X\to X$ be a self-mapping of X. T is said to be continuous mapping at x in X if $\lim_{n\to\infty}P_A(Ta_n,Ta_n,...,Ta_n,Tx,t)=0$ for any sequence $\{a_n\}$ in X and all t > 0 such that

$$\lim_{n\to\infty}P_A(a_n,a_n,...,a_n,x,t)=0.$$

3. Some Fixed Point Results

In this section, we give some fixed-point results for expansive mappings in a complete parametric A-metric space.

Definition 3.1. Let (X,P_A) be a parametric A-metric space and let T be a self-mapping of X. (AP1) There exists real numbers $k_1>1$ and $k_2,k_3\ge0$ such that

$$\begin{aligned}&P_A(Ta,Ta,...,Ta,Tb,t)\\&\ge k_1P_A(a,a,...,a,b,t)\\&+k_2P_A(Ta,Ta,...,Ta,a,t)\\&+k_3P_A(Tb,Tb,...,Tb,b,t)\end{aligned}$$

for each $a,b\in X$ and all t > 0.

Theorem 3.1. Let (X,P_A) be a complete parametric A-metric space and let T be a surjective self-mapping of X. If T satisfies condition (AP1), then T has a unique fixed point in X.

Proof. Using the hypothesis, it can be easily seen that T is injective. Indeed, if we take Ta = Tb, then, using condition (AP1), we get

$$\begin{aligned}&0=P_A(Ta,Ta,...,Ta,t)\ge k_1P_A(a,a,...,a,b,t)\\&+k_2P_A(Ta,Ta,...,Ta,a,t)+k_3P_A(Ta,Ta,...,Ta,b,t)\end{aligned}$$

for all t > 0 and so $P_A(a,a,...,a,b,t)=0$, that is, we have a = b since $k_1>1$. Let us denote the inverse mapping of T by F. Let $a_0\in X$ and define the sequence $\{a_n\}$ as follows:

$$\begin{aligned}&a_1=Fa_0,\\&a_2=Fa_1=F^2a_0,......,\\&a_{n+1}=Fa_n=F^{n+1}a_0.\end{aligned}\tag{4}$$

Suppose that $a_n\ne a_{n+1}$ for all n. Using condition (AP1) and Lemma 2.1, we have

$$\begin{aligned}&P_A(a_{n-1},a_{n-1},...,a_{n-1},a_n,t)\\&=P_A(TT^{-1}a_{n-1},TT^{-1}a_{n-1},...,TT^{-1}a_{n-1},TT^{-1}a_n,t)\\&\ge k_1P_A(T^{-1}a_{n-1},T^{-1}a_{n-1},...,T^{-1}a_{n-1},T^{-1}a_n,t)\\&+k_2P_A(TT^{-1}a_{n-1},TT^{-1}a_{n-1},...,TT^{-1}a_{n-1},TT^{-1}a_{n-1},t)\\&+k_3P_A(TT^{-1}a_n,TT^{-1}a_n,...,TT^{-1}a_n,TT^{-1}a_n,t)\\&=k_1P_A(Fa_{n-1},Fa_{n-1},...,Fa_{n-1},Fa_n,t)\\&+k_2P_A(a_{n-1},a_{n-1},...,a_{n-1},Fa_{n-1},t)\\&+k_3P_A(a_n,a_n,...,a_n,Fa_n,t)\\&=k_1P_A(a_n,a_n,...,a_n,a_{n+1},t)\\&+k_2P_A(a_{n-1},a_{n-1},...,a_{n-1},a_n,t)\\&+k_3P_A(a_n,a_n,...,a_n,a_{n+1},t)\\&=(k_1+k_3)P_A(a_n,a_n,...,a_n,a_{n+1},t)\\&+k_2P_A(a_{n-1},a_{n-1},...,a_{n-1},a_n,t)\end{aligned}$$

which implies that

$$\begin{aligned}&(1-k_2)P_A(a_{n-1},a_{n-1},...a_{n-1},a_n,t)\\&\ge(k_1-k_3)P_A(a_n,a_n,...a_n,a_{n+1},t).\end{aligned}\tag{5}$$

Clearly, we have $k_1+k_3\ne0$. Hence, we have

$$\begin{aligned}&P_A(a_n,a_n,...a_n,a_{n+1},t)\\&\le\frac{1-k_2}{k_1+k_3}P_A(a_{n-1},a_{n-1},...a_{n-1},a_n,t).\end{aligned}\tag{6}$$

If we put $k=\frac{k_2}{k_1+k_3}$, , then we get k > 1, since $k_1+k_2+k_3>1$. Repeating this process in condition (6), we find

$$P_A(a_n,a_n,...a_n,a_{n+1},t)\le k^nP_A(a_0,a_0,...a_0,a_1,t)\tag{7}$$

for all t > 0.

Let $m,n\in\mathbb{N}$ with m > n ≥ 1. Using inequality (7) and condition ($PA2$), we have

$$\begin{aligned}&P_A(a_n,a_n,...a_n,a_m,t)\\&\le\frac{2k_2}{1-k_3}P_A(a_0,a_0,...a_0,a_1,t).\end{aligned}\tag{8}$$

If we take limit for $m,n\to\infty$, we obtain

$$\lim_{m,n\to\infty}P_A(a_n,a_n,...,a_n,a_m,t)=0.\tag{9}$$

Therefore $\{a_n\}$ is Cauchy. Then there exists $y\in X$ such that

$$\lim_{n\to\infty}a_n=y,\tag{10}$$

since (X,P_A) is a complete parametric A-metric space. Using the surjectivity hypothesis, there exists a point $x\in X$ such that $Tx = y$. From condition ($AP1$), we have

$$\begin{aligned}&P_A(a_n,a_n,...a_n,y,t)=P_A(Ta_{n+1},Ta_{n+1},...,Ta_{n+1},Tx,t)\\&\ge k_1P_A(a_{n+1},a_{n+1},...,a_{n+1},x,t)\\&+k_2P_A(a_n,a_n,...,a_n,a_{n+1},t)\\&+k_3P_A(y,y,...,y,x,t)\end{aligned}\tag{11}$$

If we take limit for $n \to \infty$, we obtain

$$0 \geq (k_1 + k_3) P_A(y, y, ..., y, x, t) \quad (12)$$

which implies that $y = x$ and $T y = y$. Now we show the uniqueness of y. Let z be another fixed point of T with $y \neq z$. Using condition ($AP1$) and Lemma 2.1, we get

$$\begin{aligned} &P_A(y, y, ..., y, z, t) = P_A(T_y, T_y, ...T_y, T_z, t) \\ &\geq k_1 P_A(y, y, ..., y, z, t) + k_2 P_A(y, y, ..., y, t) \\ &+k_2 P_A(z, z, ..., z, t) \\ &= k_1 P_A(y, y, ..., y, z, t) \end{aligned} \quad (13)$$

which implies that $y = z$, since $k_1 > 1$.

Consequently, T has a unique fixed point y.

We give some examples which satisfy the conditions of Theorem 3.1.

Example 3. Let $X = \mathbb{R}^+ \cup \{0\}$ be a complete A-metric space with the A-metric defined in Example 2. Let us define the self-mapping $T : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ as

$$Tx = \beta x \quad (14)$$

for all $x \in \mathbb{R}$ with β > 1, and the function $g(t) : (0, \infty) \to (0, \infty)$ as

$$g(t) = t^2 \quad (15)$$

for all $t \in (0, \infty)$ Then T satisfies the conditions of Theorem 3.1 with $k_1 = \beta$ and $k_2 = k_3 = 0$.

Then T has a unique fixed point x = 0 in X.

Example 4. Let $X = \mathbb{R}^+ \cup \{0\}$ be a complete A-metric space with the A-metric defined in Example 2. Let us define the self-mapping $T : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ as

$$Tx = x = \log(x+1) \quad (16)$$

for all $x \in \mathbb{R}$ with β > 1, and the function $g : (0, \infty) \to (0, \infty)$ as

$$g(t) : t^3 + t^2 + t + 1 \quad (17)$$

for all $t \in (0, \infty)$, then T satisfies the conditions of Theorem 3.1 with

$$k_1 = \min\{\log(x+1) : x \neq 0 \in X\}$$

and $k_2 = k_3 = 0$. Then T has a unique fixed point x = 0 in X.

Corollary 3.1. Let (X, P_A) be a complete parametric A-metric space and let T be a surjective self-mapping of X. If there exist real numbers $k_1 > 1$ and $k_2 \geq 0$ such that

$$\begin{aligned} &P_A(Ta, Ta, ..., Ta, Tb, t) \geq k_1 P_A(a, a, ..., a, b, t) \\ &+k_2 \max\{P_A(Ta, Ta, ..., Ta, a, t), \\ &P_A(Tb, Tb, ..., Tb, b, t)\} \end{aligned} \quad (18)$$

for each $a, b \in X$ and all t > 0, then T has a unique fixed point in X.

If we take $k_1 = k$ and $k_2 = k_3 = 0$. and $k_1 = k$ and $k_2 = 0$ in Theorem 3.1 and Corollary 3.1, respectively, then we obtain the following corollaries.

Corollary 3.2. Let (X, P_A) be a complete parametric A-metric space and let T be a surjective self-mapping of X. If there exists a real number k > 1 such that

$$P_A(Ta, Ta, ..., Ta, Tb, t) \geq k P_A(a, a, ..., a, b, t) \quad (19)$$

for each $a, b \in X$ and all t > 0, then T has a unique fixed point in X.

Corollary 3.3. Let (X, P_A) be a complete parametric A-metric space and let T be a surjective self-mapping of X. If there exist a positive integer m and a real number k > 1 such that

$$\begin{aligned} &P_A(T^m a, T^m a, ..., T^m a, T^m b, t) \\ &\geq k P_A(a, a, ..., a, b, t) \end{aligned} \quad (20)$$

for each $a, b \in X$ and all t > 0, then T has a unique fixed point in X.

Proof. From Corollary 3.2, by a similar way used in the proof of Theorem 3.1, it can be easily seen that T^m has a unique fixed point a in X. Also we have

$$Ta = TT^m a = T^{m+1} a = T^m a) \quad (21)$$

and so we obtain that Ta is a fixed point for T^m. We get, Ta = a, since a is the unique fixed point.

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