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New Unified Integral Involving General Polynomials of Multivariable H-function

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Abstract In the present paper, the author establish new unified integral whose integral contains products of H-function of several complex variable [1] and a general polynomials given by Srivastava [2] with general arguments. A large number of integrals involving various simpler functions follow as special cases of this integral.

Keywords: *multivariable H-function, general polynomials, G-function, hypergeometric function*

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1. Introduction

The H-function of several complex variables is defined by Srivastava and Panda [1] as:

$$H\left[z_1,\ldots,z_r\right] = H^{0,\lambda:M^{(1)},N^{(1)};\ldots;M^{(r)},N^{(r)}}_{A,B:P^{(1)},Q^{(1)};\ldots;P^{(r)},Q^{(r)}} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} \left[(a_j;\vartheta_j^{(1)},\ldots,\vartheta_j^{(r)})_{1,A}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P^{(1)}};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P^{(r)}}\right] \\ \left[(b_j;\psi_j^{(1)},\ldots,\psi_j^{(r)})_{1,B}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q^{(1)}};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q^{(r)}}\right] \end{matrix}\right.\right] \quad (1.1)$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\left(\begin{matrix}\phi_1(\xi_1)\ldots\phi_r(\xi_r) \\ \psi(\xi_1,\ldots,\xi_r)\, z_1^{\xi_1}, \\ \ldots, z_r^{\xi_r}\end{matrix}\right) d\xi_1\ldots d\xi_r$$

where $i=\sqrt{-1}$,

$$\phi_i\xi_i = \frac{\left[\prod_{j=1}^{M^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}\xi_i)\right]\left[\prod_{j=1}^{M^{(i)}}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i)\right]}{\left[\prod_{j=M^{(i)}+1}^{Q^{(i)}}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}\xi_{(i)})\right]\left[\prod_{j=N^{(i)}+1}^{P^{(i)}}\Gamma(c_j^{(i)}-\gamma_j^{(i)}\xi_i)\right]} \quad (1.2)$$

for all $i\in\{1,\ldots,r\}$ and

$$\psi(\xi_1,\ldots,\xi_r) = \frac{\prod_{j=1}^{\lambda}\Gamma(1-a_j+\sum_{k=1}^{r}\vartheta_j^{(k)}\xi_k)}{\left[\prod_{j=\lambda+1}^{A}\Gamma(a_j-\sum_{i=1}^{r}\vartheta_j^{(i)}\xi_i)\right]\left[\prod_{j=1}^{B}\Gamma(1-b_j+\sum_{i=1}^{r}\psi_j^{(i)}\xi_i)\right]}. \quad (1.3)$$

The H-function of several complex variables in (1.1) converges absolutely if

$$|\arg(z_i)| < \frac{\pi}{2}T_i \quad (1.4)$$

where

$$T_i = \sum_{j=1+\lambda}^{A}\vartheta_j^{(i)} + \sum_{j=1}^{N^{(i)}}\gamma_j^{(i)} - \sum_{j=1+N^{(i)}}^{C^{(i)}}\gamma_j^{(k)} - \sum_{j=1}^{B}\psi_j^{(i)} + \sum_{j=1}^{M^{(i)}}\delta_j^{(i)} - \sum_{j=1+M^{(i)}}^{Q^{(i)}}\delta_j^{(i)} > 0, \quad (1.5)$$
$$\forall i\epsilon(1,\ldots,r).$$

The general polynomials have been defined and introduced by Srivastava [2] as following

$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[t_1,\ldots,t_s\right] = \sum_{k_1=0}^{n_1/m_1},\ldots,\sum_{k_s=0}^{n_s/m_s}\frac{(-n_1)_{m_1k_1}}{k_1!},\ldots,\frac{(-n_s)_{m_sk_s}}{k_s!} \quad (1.6)$$
$$A\left[n_1,k_1;\ldots;n_s,k_s\right]t_1^{k_1},\ldots,t_s^{k_s}$$

where $n_i = 0,1,2,\ldots,\ \forall i(1,\ldots,s)$; $m_1,\ldots,m_s$ arbitrary positive integers and the coefficient are $A[n_1,k_1;\ldots;n_s,k_s]$ are arbitrary constants, real or complex.

2. Main Result

In this section, we have derived the following integral

$$\int_0^\infty z^{\eta-1}\left[z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right]^{-\mu} H^{0,\lambda:M^{(1)},N^{(1)};\ldots;M^{(r)},N^{(r)}}_{A,B:P^{(1)},Q^{(1)};\ldots;P^{(r)},Q^{(r)}}\left[\begin{array}{c} x_1\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\nu_1}, \\ \ldots, x_r\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\nu_r}\end{array}\right]$$

$$\times S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}\left[\begin{array}{c} y_1\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_1}, \\ \ldots, y_r\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_r}\end{array}\right]dz$$

$$= 2b^{-\nu}\left(\frac{b}{2}\right)^{\eta}\Gamma(2\eta)\sum_{k_1=0}^{n_1/m_1},\ldots,\sum_{k_s=0}^{n_s/m_s}(-n_1)_{m_1k_1},\ldots,(-n_s)_{m_sk_s}$$

$$A[n_1,k_1;\ldots;n_s,k_s]\frac{\left(\frac{y_1}{b^{\beta_1}}\right)^{k_1}}{k_1!},\ldots,\frac{\left(\frac{y_s}{b^{\beta_s}}\right)^{k_s}}{k_s!} H^{0,\lambda+2:M^{(1)},N^{(1)};\ldots;M^{(r)},N^{(r)}}_{A+2,B+2:P^{(1)},Q^{(1)};\ldots;P^{(r)},Q^{(r)}} \tag{2.1}$$

$$\times\left[\begin{array}{c} x_1b^{-\nu_1} \\ \vdots \\ x_rb^{-\nu_r}\end{array}\left|\begin{array}{l}\left[\begin{array}{l}\left(-\mu-\sum_{i=1}^r\beta_ik_i,\nu_1,\ldots,\nu_r\right), \\ \left(1+\eta-\mu-\sum_{i=1}^r\beta_ik_i,\nu_1,\ldots,\nu_r\right), \\ (a_j,\vartheta_j^{(1)},\ldots,\vartheta_j^{(r)})_{1,A}: \\ \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P^{(1)}};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P^{(r)}}\end{array}\right] \\ \left[\begin{array}{l}(b_j,\psi_j^{(1)},\ldots,\psi_j^{(r)})_{1,B}, \\ \left(1-\mu-\sum_{i=1}^r\beta_ik_i,\nu_1,\ldots,\nu_r\right), \\ \left(-\mu-\sum_{i=1}^r\beta_ik_i,\nu_1,\ldots,\nu_r\right): \\ \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q^{(1)}};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q^{(r)}}\end{array}\right]\end{array}\right.\right]$$

provided that $\nu > 0, Re(\eta,\mu,\beta) > 0$ and

$$\nu\min\left[\mathrm{Re}\left(\frac{\beta_j}{n_j}\right)\right]+\sum_{i'=1}^r\nu_i'\min\left[\mathrm{Re}\left(\frac{d'_{j'}}{\delta'_{j'}}\right)\right]>\eta,$$

$$j=1,..,m \text{ and } j'=1,\ldots,u'.$$

Proof: In obtain result (2.1), first we express the H-function of several complex variables in terms of Mellin-Barnes contour integrals using eq. (1.1) and the general polynomial $S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}[t_1,\ldots,t_s]$ in series from given by eq. (1.6). Now interchanging the order of summation and integration which is permissible under the stated conditions, we obtain

$$\int_0^\infty x^{z-1}\left[x+a+\left(x^2+2ax\right)^{\frac{1}{2}}\right]^{-\rho}dx$$

$$=2\rho a^{-\rho}\left(\frac{b}{2}\right)^{z}[\Gamma(1+\rho+z]^{-1}\Gamma(2z)\Gamma(\rho-z) \tag{2.2}$$

$$0<\mathrm{Re}(z)<\rho.$$

Evaluating the above z-integral with the help of a known result given [4] and reinterpreting the result thus obtained in terms of H-function of r-variables, we reach at the desired result.

3. Special Cases

I. Taking $\lambda = A, M^{(i)} = 1, \nu^{(i)} = P^{(i)}$, and $Q^{(i)} = Q^{(i)} + 1\ \forall\ i \in (1,\ldots,r)$ the result in (2.1) reduces to the following integral transformation:

$$\int_0^\infty z^{\eta-1}\left[z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right]^{-\mu} S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}\left[\begin{array}{c} y_1\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_1}, \\ \ldots, y_r\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_r}\end{array}\right] F^{A:N^{(1)};\ldots;N^{(r)}}_{B:Q^{(1)};\ldots;Q^{(r)}}$$

$$\times\left[\begin{array}{c} -x_1\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\nu_1} \\ \vdots \\ -x_r\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\nu_r}\end{array}\left|\begin{array}{l}\left\{\begin{array}{l}\left[1-(a):\vartheta^{(1)},\ldots,\vartheta^{(r)}\right] \\ \left[1-(c)^{(1)}:\gamma^{(1)};\ldots;1-(c)^{(r)}:\gamma^{(r)}\right]\end{array}\right\} \\ \left\{\begin{array}{l}\left[1-(b):\psi^{(1)},\ldots,\psi^{(r)}\right] \\ \left[1-(d)^{(1)}:\delta^{(1)};\ldots;1-(d)^{(r)}:\delta^{(r)}\right]\end{array}\right\}\end{array}\right.\right]dz$$

$$= 2b^{-\nu}\left(\frac{b}{2}\right)^{\eta}\Gamma(2\eta)\sum_{k_1=0}^{n_1/m_1},\ldots,\sum_{k_s=0}^{n_s/m_s}(-n_1)_{m_1k_1},\ldots,$$

$$(-n_s)_{m_sk_s}\ A[n_1,k_1;\ldots;n_s,k_s]\frac{\left(\frac{y_1}{b^{\beta_1}}\right)^{k_1}}{k_1!},\ldots,\frac{\left(\frac{y_s}{b^{\beta_s}}\right)^{k_s}}{k_s!}$$

$$\frac{\Gamma\left(1+\mu+\sum_{i=1}^{s}\beta_ik_i\right)}{\Gamma\left(\mu+\sum_{i=1}^{s}\beta_ik_i,\right)}\frac{\Gamma(\mu-\eta+\sum_{i=1}^{s}\beta_ik_i)}{\Gamma(1+\mu-\sum_{i=1}^{s}\beta_ik_i)}$$

$$F^{A+2:M^{(1)},N^{(1)};\ldots;M^{(r)},N^{(r)}}_{B+2:P^{(1)},Q^{(1)};\ldots;P^{(r)},Q^{(r)}}$$

$$\times\left[\begin{array}{c} x_1b^{-\nu_1}\\ \vdots \\ x_rb^{-\nu_r}\end{array}\left|\begin{array}{l}\left\{\begin{array}{l}\left[1+\mu+\sum_{i=1}^{s}\beta_ik_i,\nu_1,\ldots,\nu_r\right],\\ \left[\mu-\eta+\sum_{i=1}^{s}\beta_ik_i,\nu_1,\ldots,\nu_r\right],\\ \left[1-(a):\vartheta^{(1)},\ldots,\vartheta^{(r)}\right]:\\ {[1-(c)^{(1)}:\gamma^{(1)};\ldots;1-(c)^{(r)}:\gamma^{(r)}]}\end{array}\right\}\\ \left\{\begin{array}{l}\left[1-(b):\psi^{(1)},\ldots,\psi^{(r)}\right]\\ \left[\mu+\sum_{i=1}^{s}\beta_ik_i,\nu_1,\ldots,\nu_r\right],\\ \left[1+\mu+\sum_{i=1}^{s}\beta_ik_i,\nu_1,\ldots,\nu_r\right]:\\ {[1-(d)^{(1)}:\delta^{(1)};\ldots;1-(d)^{(r)}:\delta^{(r)}]}\end{array}\right\}\end{array}\right.\right]$$

II. When we put $\lambda = A = B = 0$ in (2.1) we get the following transformation

$$\int_0^{\infty} z^{\eta-1}\left[z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right]^{-\mu}$$

$$\prod_{i=1}^{r}H^{M^{(i)},N^{(i)}}_{P^{(i)},Q^{(i)}}\left[x_i\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\nu_i}\right]$$

$$\times S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}\left[\begin{array}{l} y_1\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_1},\\ \ldots,y_r\left\{z+b+\left(z^2+2bz\right)^{\frac{1}{2}}\right\}^{-\beta_r}\end{array}\right]dz$$

$$= 2b^{-\nu}\left(\frac{b}{2}\right)^{\eta}\Gamma(2\eta)\sum_{k_1=0}^{n_1/m_1},\ldots,\sum_{k_s=0}^{n_s/m_s}(-n_1)_{m_1k_1},\ldots,(-n_s)_{m_sk_s}$$

$$A[n_1,k_1;\ldots;n_s,k_s]\frac{\left(\frac{y_1}{b^{\beta_1}}\right)^{k_1}}{k_1!},\ldots,\frac{\left(\frac{y_s}{b^{\beta_s}}\right)^{k_s}}{k_s!}$$

$$H^{0,2:M^{(1)},N^{(1)};\ldots;M^{(r)},N^{(r)}}_{2,2:P^{(1)},Q^{(1)};\ldots;P^{(r)},Q^{(r)}}$$

$$\times\left[\begin{array}{c} x_1b^{-\nu_1}\\ \vdots \\ x_rb^{-\nu_r}\end{array}\left|\begin{array}{l}\left[\begin{array}{l}\left(-\mu-\sum_{i=1}^{r}\beta_ik_i,\nu_1,\ldots,\nu_r\right),\\ \left(1+\eta-\mu-\sum_{i=1}^{r}\beta_ik_i,\nu_1,\ldots,\nu_r\right):\\ \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P^{(1)}};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P^{(r)}}\end{array}\right]\\ \left[\begin{array}{l}\left(1-\mu-\sum_{i=1}^{r}\beta_ik_i,\nu_1,\ldots,\nu_r\right),\\ \left(-\mu-\sum_{i=1}^{r}\beta_ik_i,\nu_1,\ldots,\nu_r\right):\\ \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q^{(1)}};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q^{(r)}}\end{array}\right]\end{array}\right.\right]$$

III. If $\mu_{(i)} = \mu$, $\vartheta^{(i)} = \vartheta$, $\psi^{(i)} = \psi$, $M^{(i)}N^{(i)} = P^{(i)}Q^{(i)} = c^{(i)}\gamma^{(i)} = d^{(i)}\delta^{(i)} = 0$ and $k_{(i)} = k$, $\frac{n_{(i)}}{m_{(i)}} = \frac{N}{M}$, $\frac{z_{(i)}}{a^{\alpha(i)}} = \frac{z}{a^{\alpha}}$, $\forall i \in (1,\ldots,r)$ the result in (2.1) reduces to the known result with a small modification derived by Garg and Mittal [6].

4. Conclusion

Finally we conclude with the remark that results and the operators proved in this paper appear to be new and likely to have useful applications to a wide range of problems of mathematics, statistics and physical sciences.

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