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Multiple Stenotic Effect on Blood Flow Characteristics in Presence of Slip Velocity

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Abstract The aim of the present analysis is to study the effect of slip velocity on blood flow through an arterial tube in presence of multiple stenosis. The effects of length of stenosis, shape parameter, parameter γ on resistance to flow and shear stress have been incorporated here. The results have been shown in graphical form and discussed.

Keywords: *resistance to flow, wall shear stress, stenosis, shape parameter, Herschel-Bulkley flui.*

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1. Introduction

As blood is a suspension of fluid particles, such as erythrocytes, leukocytes and platelets in an aqueous solution, blood behaves neither Newtonian nor non-Newtonian fluid. The behaviour of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and then blood behaves as a non-newtonian fluid [1,2]). Many biomedical researchers (Eckstein et. al. [3], Choung et. al. [4], Fung [5], Sparks [6], Tilles et. al. [7], Goldsmith [8], Carr [9]) have analysed various types of mathematical models to study the blood flow characteristics for better understanding the physiological systems of human body. A large number of researchers (Feng et. al. [10], Jilma [11], Haldar [12], Liepsch [13]) have presented various types of mathematical models by considering blood as Newtonian or non-Newtonian fluid to study the flow characteristics of blood. Presently in many cases it has been noticed that cardiovascular diseases are responsible for the death of the people. From various types of medical literature it is clear to us that around 80% of total death of people are due to the malfunction of blood flow characteristics. Blood flow characteristics mainly depends on the arterial diseases, such as stenosis and aneurysm. Stenosis is a serious arterial disease causes serious cardiovascular disorders, by reducing the supply of blood. Though actual formation of stenosis is somewhat unclear to us, it is well-known that deposition of fatty substances like lipids, cholesterol in the inner wall of the artery and unnatural growth of the connective tissue may be responsible for the formation of stenosis. Many biomedical researchers (Jung et. al. [14], Mousa [15], Sugihara Seki [16], Cokelet [17], Das et. al. [18]) have studied the non-Newtonian behaviour of blood to discuss the various aspects of blood flow. Various mathematical models have been developed by many Mathematicians (Young [19], Shukla [20], RamchandraRao and Devanathan [21], Hall [22], Manton [23], Smith [24], Duck [25]) to study the steady and unsteady flow of blood through channels with variable cross section. The steady flow of blood through stenotic arterial tube has been studied by many Mathematicians (Misra and Chakraborty [26], Padmanavan [27], Mehrotha et. al. [28]) by considering blood as non-Newtonian fluid, in presence of mild stenosis. In a recent paper Siddiqui [29], Biswas and Laskar [30] have been developed mathematical models to study the blood flow characteristics through a stenosed arterial segment under slip condition. But they have considered the effect of single stenosis. But since stenosis may be developed in series or in overlapping form, in the present analysis I have considered the effect of multiple stenosis on blood flow by considering blood as Herschel-Bulkley type non- Newtonian fluid in presence of slip velocity on the arterial wall.

2. Mathematical Formulation

Let us consider the steady one dimensional laminar axially symmetric and radially non-symmetric constricted artery. The mathematical expression for stenosis is given by

$$\begin{aligned}\frac{R}{R_0} &= 1-\epsilon\Big[L_0^{(s-1)}\{\gamma z-nd-(n-1)L_0\} \\ &\qquad -\{\gamma z-nd-(n-1)L_0\}^s\Big] \qquad (1)\\ &\qquad : n(d+L_0)-L_0\le \gamma z\le n(d+L_0)\\ &= 1, \quad \text{otherwise}\end{aligned}$$

Where R is the radius of the tube in the stenotic region; R_0, the radius of the artery outside the stenotic region; $s(\geq 2)$ is a shape parameter determining stenosis shape; L_0, the length of the stenosis, d indicates its location, n is the number of stenosis in the artery, $\gamma(\geq 1)$ is a parametric constant.

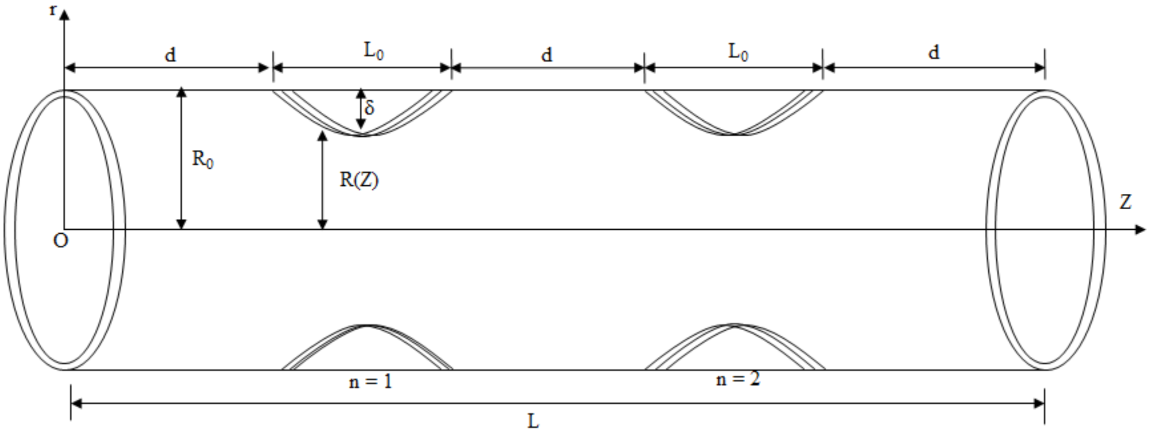

Figure 1. Geometry of arterial stenosis

$$\epsilon = \frac{\delta}{R_0} \frac{s^{s/(s-1)}}{L_0^s (s-1)}. \quad (2)$$

Where δ is the maximum height of the stenosis at

$$z = \frac{nd + (n-1) L_0 + \frac{L_0}{s^{1/(s-1)}}}{\gamma}. \quad (3)$$

The equation governing the flow of blood is given by

$$-\frac{dp}{dz} = \frac{1}{r} \frac{d(r\tau)}{dr}. \quad (4)$$

The boundary conditions are,

(i) τ is finite at $r = 0$ (regular condition)

(ii) $w = w_s$ at $r = R(z)$ (slip condition).

The relationship between shear stress and shear rate for Herschel-Bulkley fluid is given by,

$$\tau = \mu(-\frac{\partial w}{\partial r})^n + \tau_0, \qquad \tau \geq \tau_0$$
$$-\frac{\partial w}{\partial r} = 0, \qquad \tau < \tau_0. \quad (5)$$

For simplicity the Herschel-Bulkley index is taken as $n = 1/2$. Therefore,

$$-\frac{\partial w}{\partial r} = \frac{\tau^2 - 2\tau\tau_0 + \tau_0^2}{\mu^2}. \quad (6)$$

Integrating (4) and using the boundary condition (i) we get,

$$\tau = -\frac{r}{2} \frac{dp}{dz}. \quad (7)$$

The skin friction τ_w is given by

$$\tau_w = -\frac{R}{2} \frac{dp}{dz}, \text{ where } r = R(z). \quad (8)$$

The volumetric flow rate is given by

$$Q = \int_0^R 2\pi r w \, dr$$
$$= 2\pi \left[\frac{w_s R^2}{2} + \frac{1}{\mu^2} \int_0^{\tau_w} \left(\tau^2 - 2\tau\tau_0 + \tau_0^2 \right) \frac{R^3 \tau^2}{\tau_w^3} d\tau \right] \quad (9)$$

when the boundary condition (ii) is used, which can be written as

$$\frac{Q}{\pi} - w_s R^2 = \frac{5R^3 \tau_w^2}{\mu^2} \left[\frac{1}{25} - \frac{\tau_0}{10\tau_w} + \frac{1}{15} \frac{\tau_0^2}{\tau_w^2} \right]. \quad (10)$$

When $\frac{\tau_0}{\tau_w} \ll 1$, replacing $\frac{1}{15}$ by $\frac{1}{16}$ we get,

$$\frac{\mu^2}{5R^3} \left(\frac{Q}{\pi} - w_s R^2 \right) = [\frac{\tau_w}{5} - \frac{\tau_0}{4}]^2. \quad (11)$$

From which we get,

$$\tau_w = \frac{5\tau_0}{4} + \mu \left[\frac{5Q}{\pi R^3} - \frac{5w_s}{R} \right]^{\frac{1}{2}}. \quad (12)$$

Since $\tau_w = -\frac{R}{2} \frac{dp}{dz}$, we get

$$\frac{dp}{dz} = -\frac{5\tau_0}{2R} - 2\mu \left[\frac{5Q}{\pi R^5} - \frac{5w_s}{R^3} \right]^{\frac{1}{2}}. \quad (13)$$

Integrating with respect to z with the condition that $p = p_0$ at $z = 0$ and $p = p_1$ at $z = L$

$$p_0 - p_1$$
$$= \frac{5\tau_0}{2R_0} \int_0^L (\frac{R}{R_0})^{-1} dz + 2\mu \int_0^L [\frac{5Q}{\pi R^5} - \frac{5w_s}{R^3}]^{\frac{1}{2}} dz. \quad (14)$$

Resistance to flow is given by

$$\lambda = \frac{p_0 - p_1}{Q}$$
$$= \frac{5\tau_0}{2R_0 Q} \int_0^L (\frac{R}{R_0})^{-1} dz + \frac{2\mu}{Q} \int_0^L \left[\frac{5Q}{\pi R^5} - \frac{5w_s}{R^3} \right]^{\frac{1}{2}} dz. \quad (15)$$

Let us take

$$f_1 = \frac{5\tau_0}{2R_0 Q}, \; f_2 = \frac{2\mu}{Q}. \quad (16)$$

Thus we can write

$$\lambda = f_1\Bigg[\int_0^{\frac{n(d+L_0)-L_0}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz + \sum_{n=1}^{n_{\max}} \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz + \int_{\frac{n(d+L_0)}{\gamma}}^{L} \left(\frac{R}{R_0}\right)^{-1}dz\Bigg]$$
$$+ f_2\Bigg[\int_0^{\frac{n(d+L_0)-L_0}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz + \sum_{n=1}^{n_{\max}} \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz + \int_{\frac{n(d+L_0)}{\gamma}}^{L} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz \quad (17)$$

$$= (f_1+f_3)\left(L-\frac{L_0}{\gamma}\right) + f_1\int_{\frac{d}{\gamma}}^{\frac{(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz + f_1\int_{\frac{2d+L_0}{\gamma}}^{\frac{2(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz$$
$$+ f_2\int_{\frac{d}{\gamma}}^{\frac{(d+L_0)}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz + f_2\int_{\frac{2d+L_0}{\gamma}}^{\frac{2(d+L_0)}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz. \quad (18)$$

Where

$$f_3 = \frac{2\mu}{Q}\left[\frac{5Q}{\pi R_0^5}-\frac{5w_s}{R_0^3}\right]^{\frac{1}{2}}. \quad (19)$$

If there is no stenosis, i.e., in the normal condition

$$\lambda_N = (f_1+f_3)L. \quad (20)$$

Therefore dimensionless resistance to flow is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1-\frac{L_0}{\gamma L}+\frac{f_1(I_1+I_2)+f_2(I_3+I_4)}{(f_1+f_3)L} \quad (21)$$

Where

$$I_1 = \int_{\frac{d}{\gamma}}^{\frac{(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz;$$

$$I_2 = \int_{\frac{2d+L_0}{\gamma}}^{\frac{2(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1}dz;$$

$$I_3 = \int_{\frac{d}{\gamma}}^{\frac{(d+L_0)}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz;$$

$$I_4 = \int_{\frac{2d+L_0}{\gamma}}^{\frac{2(d+L_0)}{\gamma}} \left[\frac{5Q}{\pi R^5}-\frac{5w_s}{R^3}\right]^{\frac{1}{2}}dz. \quad (22)$$

Now

$$\tau_w = \frac{5\tau_0}{4}+\mu\left[\frac{5Q}{\pi R^3}-\frac{5w_s}{R}\right]^{\frac{1}{2}}. \quad (23)$$

Wall shear stress in normal situation can be written as

$$\tau_N = \mu\left[\frac{5Q}{\pi R_0^3}-\frac{5w_s}{R_0}\right]^{\frac{1}{2}}. \quad (24)$$

Thus the wall shear stress ratio can be obtained as

$$\bar{\tau}_w = \frac{\tau_w}{\tau_N} = \frac{\frac{5\tau_0}{4}+\mu\left[\frac{5Q}{\pi R^3}-\frac{5w_s}{R}\right]^{\frac{1}{2}}}{\mu\left[\frac{5Q}{\pi R_0^3}-\frac{5w_s}{R_0}\right]^{\frac{1}{2}}}. \quad (25)$$

The wall shear stress ratio at the midpoint of stenosis is given by

$$\bar{\tau}_{wm} = \frac{\frac{5\tau_0}{4}+\mu\left[\frac{5Q}{\pi \delta^3}-\frac{5w_s}{\delta}\right]^{\frac{1}{2}}}{\mu\left[\frac{5Q}{\pi R_0^3}-\frac{5w_s}{R_0}\right]^{\frac{1}{2}}}. \quad (26)$$

3. Numerical Discussions

To illustrate the flow behaviour, the results are shown graphically with the help of MATLAB-7.6. The effects of various parameters on resistance to flow and wall shear stress are calculated here.

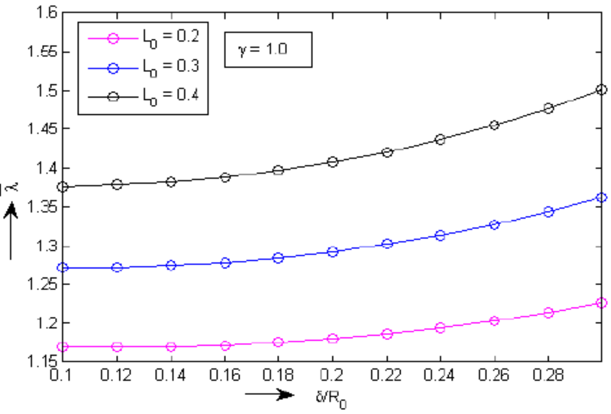

Figure 2. Variation of non-dimensional resistance to flow $\bar{\lambda}$ for different values of stenosis length L_0

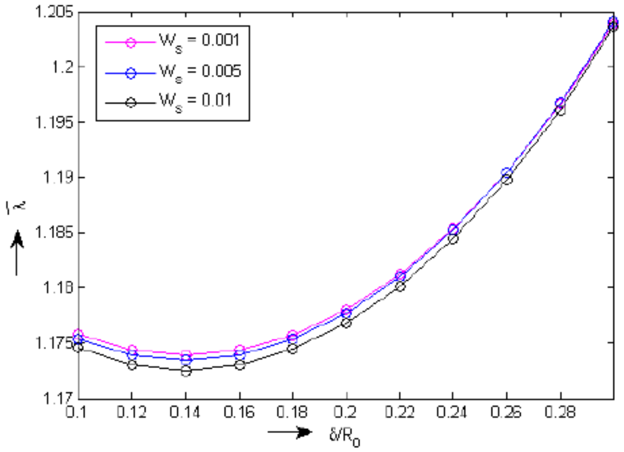

Figure 3. Variation of non-dimensional resistance to flow $\bar{\lambda}$ for different values of slip velocity *ws*

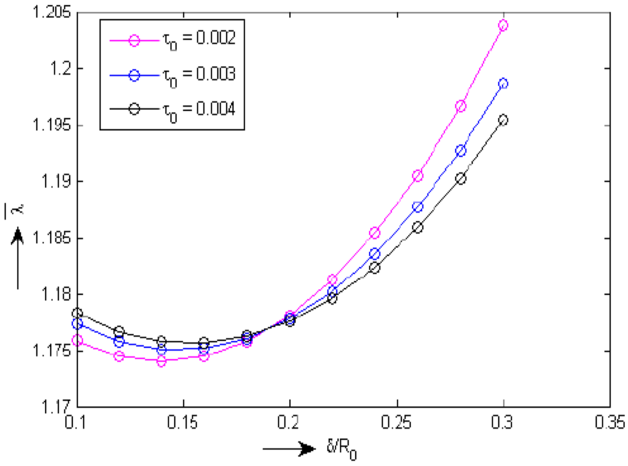

Figure 4. Variation of non-dimensional resistance to flow $\bar{\lambda}$ for different values of yield stress τ_0

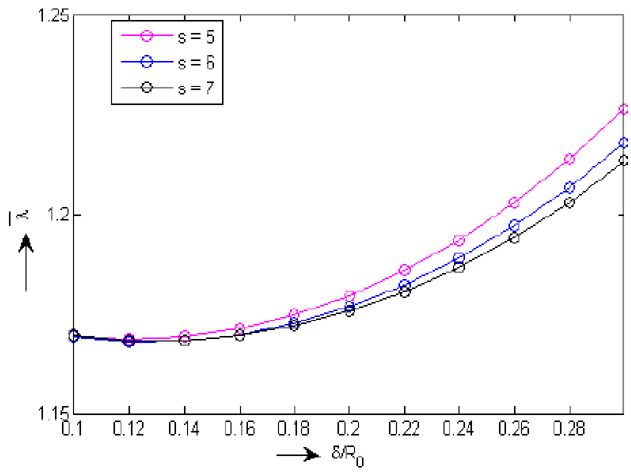

Figure 5. Variation of non-dimensional resistance to flow $\bar{\lambda}$ with the variation of shape parameter *s*

Figure 2- Figure 6 show the variation of resistance to flow for different values of stenosis length, slip velocity, yield stress and shape parameters s, γ. It is observed from the figures that $\bar{\lambda}$ increases with the increase of stenosis size and stenosis length L_0 for all other constant values of the parameters, but the reverse effectoccurs when the slip velocity w_s and shape parameters s and γ increase. It is also clear from the figures that $\bar{\lambda}$ increases with the increase of τ_0 up to the value 0.18 of $\frac{\delta}{R_0}$ and then decreases.

Figure 7-Figure 8 illustrate the effect of slip velocity and yield stress on wall shear stress. It is found that $\bar{\tau}_w$ increases with the increase of slip velocity and yield stress as *z* increases.

Figure 9 reveals that the variation of wall shear stress ratio at the midpoint of stenosis. It is found that $\bar{\tau}_{wm}$ increases with the increase of slip velocity.

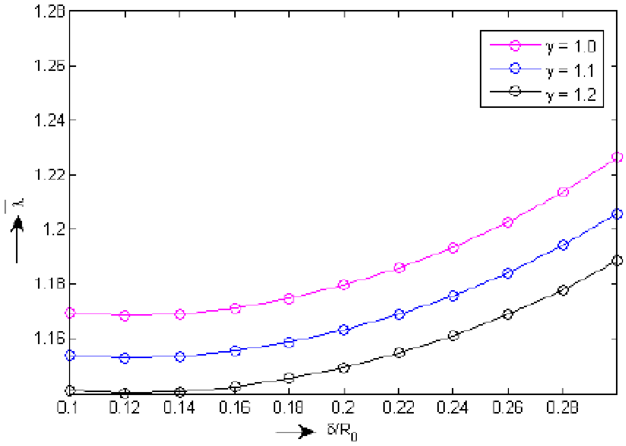

Figure 6. Variation of non-dimensional resistance to flow $\bar{\lambda}_w$ with the variation of parameter γ

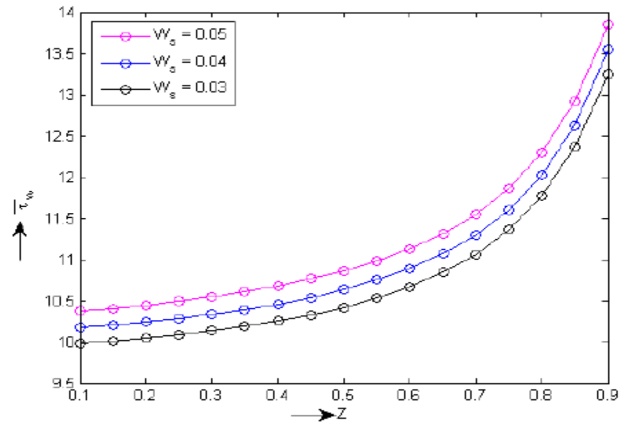

Figure 7. Variation of non-dimensional wall shear stress $\bar{\tau}_w$ for different values of slip velocity w_s

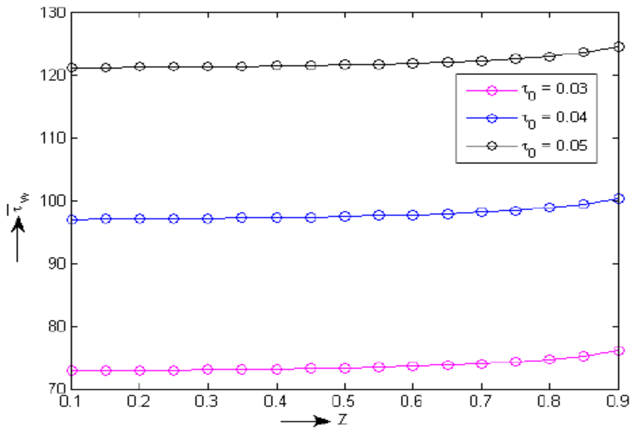

Figure 8. Variation of non-dimensional wall shear stress $\bar{\tau}_w$ for different values of yield stress τ_0

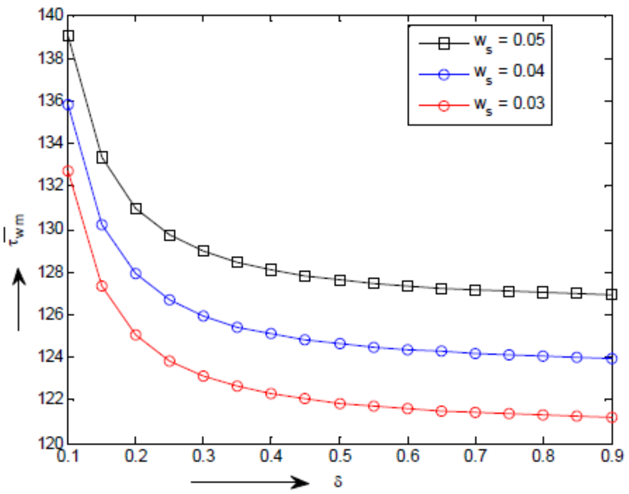

Figure 9. Fluctuation of non-dimensional wall shear stress at the midpoint of stenosis $\bar{\tau}_{wm}$ for different values of slip velocity w_s

4. Conclusions

The analytic expression for resistance to flow with multiple stenosis situated at equal distances has been incorporated here, which has definite effect in various types of cardiovascular diseases, like stroke, hypertension, brain haemorrhage etc. It is observed that growing of γ has small variations for different value of stenosis shape parameter. This study may be effective for invention of new diagnostic tools in medical sciences.

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