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On Theoretical Methodology of Nuclear Reactions and Outcome of Certain Physical Phenomena from Them. II. To the Space-time Description of Cross Sections and Durations of the Neutron-nucleus Scattering Near 1-2 Resonances in the C- and L-systems

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Abstract It is already known the appearance of time advance (due to distortion by the non-resonant background) instead of the expected time delay in the region of a compound-nucleus resonance in the center-of-mass (C-) system. Here at the same conditions we study cross sections and durations of the neutron-nucleus scattering in the laboratory (L-) system. Here it is shown that such time advance is a virtual paradox but in the L-system the time-advance phenomenon does not occur and only the trivial time delay is observed. At the same time the transformations from C-system into the L-system appeared to be different from the standard kinematical transformations because in the C-system the motion of a compound nucleus is absent but it is present in the L-system. We analyze the initial wave-packet motion (after the collision origin) and the cross section in the laboratory (L-) system. Also here (as physical revelations of profound general methodic and in very good consistent accordance with the experiment) several results of the calculated cross sections for the neutron-nucleus in comparison with the experimental data in the L-system at the range of one or two overlapped compound resonances are presented. It is shown in the space-time approach that the standard kinematical transformations of cross sections from the C-system to the L-system are not valid because it is necessary to consider the center-of-mass motion in the L-system. Finally on a correct self-consistent base of the space-time description of the nuclear processes in the laboratory system with 3 particles in the final channel, it is shown the validity of the former approach, obtained for the space-time description of the nuclear processes with 2-particle channels earlier.

Keywords: *space-time approach to nuclear collision; time delay, time advance, transformations of cross sections from the C-system to the L-system, interference phenomena, recoil system, direct and sequential processes*

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1. The Pre-history of the Problem

It was found in [1-7] the phenomenon of time advance instead of expected time delay in the C-system. This phenomenon is usually accompanied by a cross section minimum for almost the same energy. Then naturally the question had arisen if this advance manifested also in the L-system?

Then in [8,9,10] it was found that the standard formulas of cross section transformations from the L- to C- system are inapplicable in the cases of two (and more) collision mechanisms. Usually the delay-advance phenomenon appears for nucleon-nucleus elastic scattering near a resonance, distorted by the non-resonant background, in the *C*-system. Usually (see, for instance, [1,2,3]) the amplitude $F_C\,(E,\theta)$ for the elastic scattering of nucleons by spherical nuclei near an isolated resonance in the *C*-system can be written as

$$F_C(E,\theta) = f\,(E,\theta) + f_{l,\,res}\,(E,\theta), \qquad (1)$$

where

$$f_{l,res}(E,q) = (2ik)^{-1}(2l+1)P_l(\cos q)$$
$$[\exp(2id_l)\frac{E-E_{res}-i\Gamma/2}{E-E_{res}+i\Gamma/2}-1],$$

$$f(E,q) = (2ik)^{-1}\sum_{\lambda\neq l}(2l+1)P_l(\cos q)[\exp\,(2id_l)-1],$$

here E , E_{res} and Γ are the excitation energy, the resonance energy and the width of the compound nucleus, respectively; we neglect the spin-orbital interaction and consider a comparatively heavy nucleus.

Rewriting (1) in the form

$$F^C(E,\theta) = [A(E^* - E^*_{res}) + iB\Gamma/2](E^* - E^*_{res} + i\Gamma/2)^{-1} \quad (1a)$$

where

$$A = f(E,\theta) + (k)^{-1}(2l+1)P_l(\cos\theta)\exp(i\delta_l^b)\sin\delta_l^b,$$

$$B = f(E,\theta) + (ik)^{-1}(2l+1)P_l(\cos\theta)\exp(i\delta_l^b)\cos\delta_l^b,$$

we obtain the following expression for the scattering *duration* $\tau^C(E,\theta)$:

$$\tau^C(E,\theta) = 2R/v + \hbar\partial \arg F/\partial E \equiv 2R/v + \Delta\tau^C(E,\theta) \quad (2)$$

in case of the quasi-monochromatic particles which have very small energy spreads $\Delta E << \Gamma$. Formula (2) was obtained in [1]. In formula (2), $v=\hbar k/\mu$ is the projectile velocity, R is the interaction radius, and $\Delta\tau^C$ is

$$t^C(E,q) = -(\hbar \mathrm{Re}a/2) \left[(E^* - E^*_{res} - \mathrm{Im}a/2)^2 + (\mathrm{Re}a)^2/4\right]^{-1} + Dt_{res}, \quad (3)$$

with

$$\tau_{res} = (\hbar\Gamma/2)[(E^* - E^*_{res})^2 + \Gamma^2/4]^{-1}, \quad \alpha = \Gamma B/A. \quad (4)$$

From (3) one can see that, if $0<\mathrm{Re}\alpha<\Gamma$, the quantity $\Delta\tau(E,\theta)$ appears to be *negative* in the energy interval ~ $\mathrm{Re}\alpha$ around the center at the energy $E^*_{res}+\mathrm{Im}\alpha/2$. When $0<\mathrm{Re}\alpha/\Gamma<<1$ the minimal delay time can obtain the value $-2\hbar/\mathrm{Re}\alpha<0$. Thus, when $\mathrm{Re}\alpha \to 0^+$, the interference of the resonance and the background scattering can bring to *as much as desired large of the advance* instead of the delay! Such situation is mathematically described by the zero $E^*_{res}+i\alpha/2$, besides the pole $E^*_{res}-i\Gamma/2$, of the amplitude $F^C(E,\theta)$ (or the correspondent T-matrix) in the lower unphysical half-plane of the complex values for energy E. We should notice that a very large advance can bring to the problem of causality violation (see, for instance the note in [2]). The *delay-advance phenomenon* in the C-system was studied in [1-3] for the nucleon-nucleus elastic scattering.

For two overlapped resonances the amplitude for an elastic scattering can be written in center-of-mass system also in form (1):

$$F^C(E,\theta) = f(E,\theta) + f_{l,res}(E,\theta)$$

where

$$f(E,\theta) = f_{coul}(E,\theta) + (2ik)^{-1} \sum (2\lambda+1)P_\lambda(\cos\theta)\left[\exp\left(2i\delta_\lambda^b\right) - 1\right] \quad (5)$$

and already

$$f_{l,res}(E,\theta) = (2ik)^{-1}(2l+1)P_l(\cos\theta)\exp\left(2i\delta_l^b\right) \left[\left(\frac{E - E_{res,1} - i\Gamma/2}{E - E_{res,1} + i\Gamma/2}\right)\left(\frac{E - E_{res,2} - i\Gamma/2}{E - E_{res,2} + i\Gamma/2}\right) - 1\right], \quad (6)$$

we obtain the following expression for the total scattering *duration* $\tau^C(E,\theta)$

$$\tau^C(E,\theta) = 2R/v + \hbar\partial \arg F/\partial E \equiv 2R/v + \Delta\tau^C(E,\theta)$$

for the quasi-monochromatic particles which have very small energy spreads $\Delta E << \Gamma$, when one can use *the method of stationary phase* for approaching the group velocity of the wave packet.

At Figure 1 and Figure 2 we can see the energy dependence of $\Delta t^C(E,\theta)$ for two couples of overlapped resonances in neutron-nucleus elastic scattering [7].

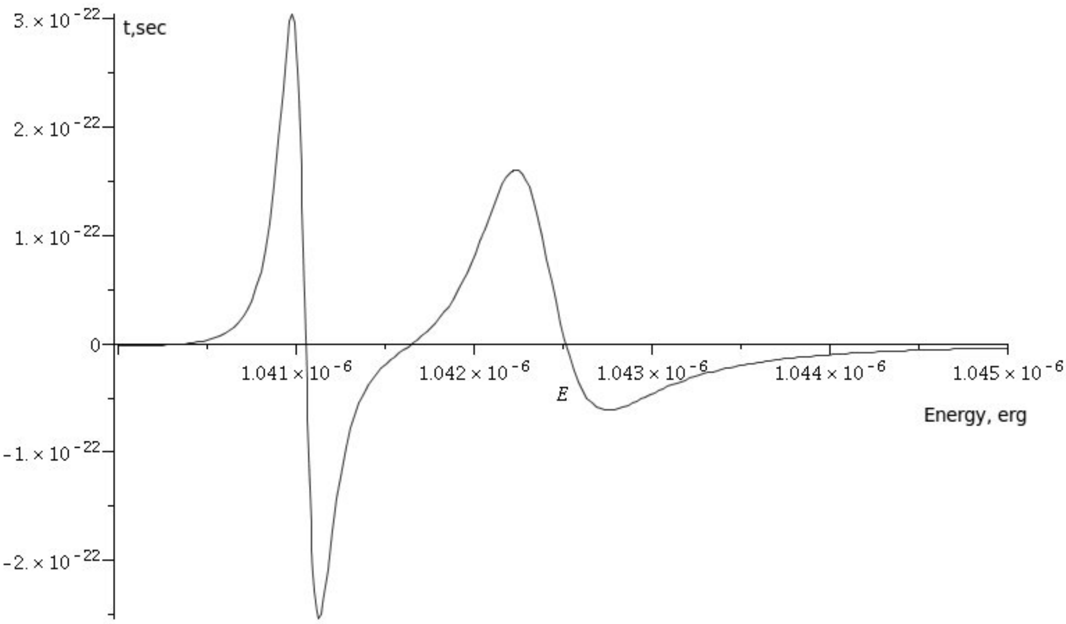

Figure 1. Energy dependence of $\Delta t^C(E,\theta)$ near two overlapped resonances ^{58}Ni $E_1 = 649.8 keV; \Gamma_1 = 0.168 keV$ and $E_2 = 650.6 keV; \Gamma_2 = 0.521 keV$

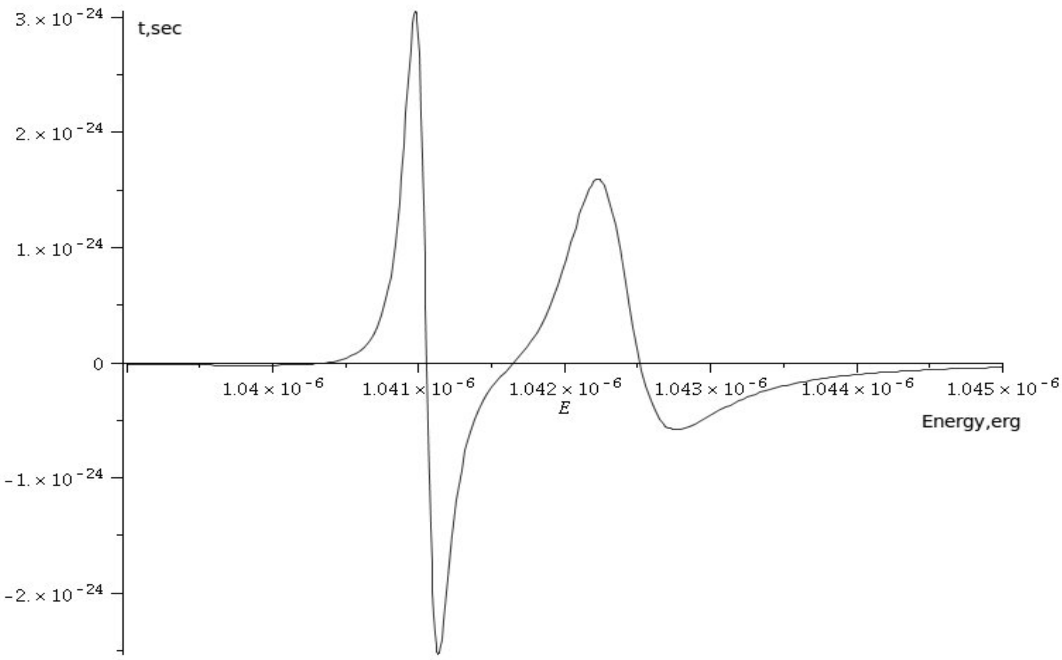

Figure 2. Energy dependence of $\Delta t^C(E,\theta)$ near two overlapped resonances ^{58}Ni $E_3 = 745.6keV; \Gamma_3 = 0.7keV$ and $E_4 = 746{,}5keV; \Gamma_4 = 0{,}8keV$

2. The Collision-process Diagram with 2 Mechanisms (Direct Process and Collision with the Formation of a compound Nucleus)

In Figure 3a, Figure 3b these two processes in the L-system are pictorially presented. They represent a prompt (direct) and a delayed compound-resonance mechanism of the emitting ***y*** particle and ***Y*** nucleus, respectively. The both mechanisms are *macroscopically* schematically indistinguishable but they are *microscopically* different processes

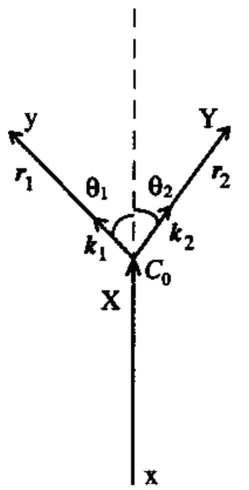

Figure 3(a). Diagram of the direct process

Figure 3(a) represents the direct process of a prompt emission of the final products from the collision point C_0 with a very small time duration τ_{dir}, while Figure 3(b) represents the motion of a compound-resonance nucleus Z from point C_0 to point C_1, where it decays by the final products ***y*** + ***Y*** after traveling a distance between C_0 and C_1 (which is equal to $\sim V_C \Delta\tau_{res}$) before its decay. Here V_C is the compound-nucleus velocity, equal to the center-of-mass velocity, and $\Delta\tau_{res} = =\hbar\Gamma/2) / [(E_Z - E_{res,Z})^2 + \Gamma^2/4]$ is the mean time of the nucleus Z motion before its decay [8,9,10,11] for the case of one compound resonance, the energy spread ΔE of the incident particle ***x*** being very small in comparison with the resonance width Γ, $E_Z = E^*$, $E_{res,Z} = E^*_{res}$. For the clarity of the difference between both processes in time, we impose the evident practical condition

$$\tau_{dir} << \Delta\tau_{res}(E_Z) \text{ near } (E_Z - E_{res,Z})^2 \approx \Gamma^2/4. \qquad (7)$$

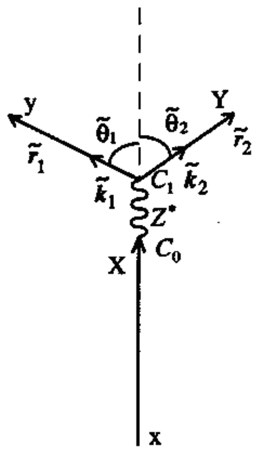

Figure 3(b). Diagram of process with the compound nucleus

For the *macroscopically* defined cross sections, in the case of very large macroscopic distances r_1 (near the detector of the final particle ***y***) with very small angular and energy resolution ($\Delta\theta_1 << \theta_1$ and $\Delta k_1 << k_1$), the angles θ_1 and $\tilde{\theta}_1$, as well as momentums k_1 and $\tilde{k}_1$, can be considered as practically coincident. Really, $\theta_1 - \tilde{\theta}_1 \sim \Delta r_1 / r_1$ and $k_1 - \tilde{k}_1 \sim \Delta r_1 / r_1$ with $|\Delta r_1| = |r_1 - \tilde{r}_1|$. Using the *usual* *macroscopic definition of the cross section* with the help of some transformations for the exit asymptotic wave packet of the system $\boldsymbol{y} + \boldsymbol{Y}$, in [4] it was obtained the following expression for the cross section σ of reaction (4) in the L-system:

$$\sigma = \sigma_0^{(incoh)} + \sigma_1^{(interf)}, \tag{8}$$

where

$$\sigma_0^{(incoh)} \cong \left| f_{dir}^{(L)} \right|^2 + \frac{J_{C\to L} \left| \gamma_{Z^\bullet}^{(C)} \right|^2}{(E_Z - E_{res,Z})^2 + \Gamma^2/4}, \tag{9}$$

$$f_{dir}^{(C)} = \frac{1}{2ik_1^C} \sum_{l' \neq l} (2i+1) P_{l'}(\cos\theta_1^C)(e^{2i\delta_i} - 1), \tag{10}$$

$$\sigma_1^{(\text{interf})} = 2 f_{dir}^{(L)} \cdot \frac{J_{C\to L}^{1/2} \gamma_Z^{(C)}}{E_Z - E_{res,Z} + i\Gamma/2} \cos\Phi, \tag{11}$$

$$\frac{\gamma_Z^{(L)}(E_1, E_2)}{E_Z - E_{res,Z} + i\Gamma/2} = f_{l,res}(E_1^C, \theta_1^C)$$

$$= \frac{\gamma_Z^{(L)}}{2ik_1^C}(2l+1)P_l(\cos\theta_1^C) \tag{12}$$

$$\left\{ \exp(2i\delta_l) \frac{E^C - E_{res}^C - i\Gamma/2}{E^C - E_{res}^C + i\Gamma/2} - 1 \right\},$$

$$\Phi = \chi + \beta + \varphi,\ \chi = \arg(J_{C\to L}^{1/2} \gamma_{Z^\bullet}^{(L)}) - \arg(f_{dir}^{(L)}),$$

$$\beta = \arg(E_Z - E_{res.Z}) + i\Gamma/2)^{-1} \tag{13}$$

$$\varphi = k_1 \Delta r_1 + k_2 \Delta r_2,\ \Delta r_{1,2} = V_{\perp(1,2)} \Delta\tau_{res},$$

$V_{1,2}$ is the projection of the Z-nucleus velocity to the direction of $\vec{k}_{1,2}$, δ_l is the l-wave scattering background phase shift. Formulas (8)-(11) were obtained for a quasi-monochromatic incident beam ($\Delta E << E$) and a very small angular and energy resolution ($\Delta\theta_1 << \theta_1$, $\Delta E << \Gamma$) of the final-particle detector.

For the simplicity we neglect here the spin-orbital coupling and we suppose also that the absolute values of all differences $r_n/v_n - r_p/v_p$ ($n \neq p = 1,2$) are much less than the time resolutions. Here $J_{C\to L}$ is the standard Jacobian of pure cinematic transformations from the C-system to the L-system.

We underline that formulas (8)-(13) for the cross section σ, obtained in [8,9,10,11] and defined by the usual *macroscopic* way, take into account a real *microscopic* motion of the compound nucleus. So, the formulas (8)-(13) differ from the standard kinematical transformation of $\sigma^C(E,\theta) = |F^C(E,\theta)|^2$ from the C-system into the L-system, considering only the kinematical transformations of the energies and angles from the C-system (with $\varphi = 0$) to the L-system. Such difference arises because the formal expression for $\sigma^C(E,\theta)$ as taken *without consideration* of the microscopic difference between the processes in Figure 3a and Figure 3b, and thus *without consideration* of the parameter $\varphi = k_1 \Delta r_1 + k_2 \Delta r_2$, $\Delta r_{1,2} = V_{\perp(1,2)} \Delta\tau_{res.}$

3. The Lack of Time Advance near Compound-resonances in the L-system

We underline that formulas (8)-(13) for the cross section σ, obtained here, are defined by the usual *macroscopic* way and also consider the real *microscopic* motion of the compound nucleus which strongly differ them from the standard cinematic transformation $\sigma^C(E,\theta) = |F^C(E,\theta)|^2$ from C-system into L-system namely by the interference of the amplitudes $f_{dir}^{(L)}$ and $\dfrac{J_{C\to L}^{1/2} \gamma_{Z_\bullet}^{(C)}}{E_Z - E_{res,Z} + i\Gamma/2} \cdot \exp(i\varphi)$, $\varphi = k_1 \Delta r_1 + k_2 \Delta r_2$ (where $\Delta r_{1,2} = V_{proj1,2}\, \Delta\tau_{res}$). The parameter φ reflects the influence of the compound-nucleus motion.

In the first my works (for instance, in [1,2,3]) usually the analysis of the amplitudes, cross sections and durations of the elastic scattering performed on the base of formulas (1) $\to$ (1a) in C-system, in which the compound-nucleus motion in L-system did not taken into account. But taking in account the motion of the decaying compound nucleus in L-system, the expressions for the amplitude of the collision process, which is going on with the formation of excited compound nucleus in the region of a resonance in C- and L-systems, differ not only by the standard cinematic transformations $\{E^C, \theta^C\} \leftrightarrow \{E^L, \theta^L\}$. It is necessary take into account also the motion of the decaying compound nucleus along the distance $V_C \Delta\tau_{res}$, as it was shown in Figure 3a, Figure 3b. In [1,2,3] formulas (1) and (1a) were written in C-system and are described the coherent sum of the interfering terms for the both of cross section $\sigma^C(E,\theta) = |F^C(E,\theta)|^2$ and the time delay $\Delta\tau^C(E,\theta)$ without the microscopic motion of the decaying compound nucleus from point C_0 till point C_1. It is possible to evaluate the general duration of collision in L-system, taking the superposition of the wave packets of the direct scattering and of the scattering, going on with the formation of the intermediate compound nucleus (in the correspondence with diagrams 1a and 1b, respectively), which was obtained in [8], and in the asymptotic range (for $r \to \infty$) after all the simplifications, considering the conservation of energy-impulse, receives the form

$$\Psi^{r_1\to\infty} \approx \text{const}\cdot \exp(-iE_f^0 t/\hbar)\cdot\exp(ik_1^0 r_1 + ik_2^0 r_2)$$
$$\times\{f_{dir}^{(L)} \cdot \exp[-\Delta E\,[\,(t - t_i - \frac{r_1}{V_1^0}) + (t - t_i - \frac{r_2}{V_2^0})]/\hbar]$$
$$+ \frac{J_{C\to L}^{1/2}}{E_Z^\bullet - E_{res,Z} + i\Gamma_Z/2} \times \exp[-\Delta E\,[\,(t - t_i - \Delta\tau_{res} - \frac{\tilde{r}_1}{V_1^0}) \tag{14}$$
$$+ (t - t_i - \Delta\tau_{res} - \frac{\tilde{r}_2}{V_2^0})]/\hbar] \times \exp[ik_1^0 \Delta r_1 + ik_2^0 \Delta r_2]\} \text{ for } \begin{cases} t > t_i + r_1/V_1^0 \\ t > t_i + \Delta\tau_{res} + \tilde{r}_1/V_1^0 \end{cases},$$

where $V_{1,2}^0 = \hbar k_{1,2}^0 / m_{1,2}$, $\Delta r_{1,2} = V_{\perp(1,2)} \Delta\tau_{res}$, $V_{\perp(1,2)}$ is the projection of the nucleus Z^* motion velocity on the $\boldsymbol{k}_{1,2}$ direction, t_i is the initial time moment, defined by the amplitude phase of the initial weight factor g_i, chosen for the simplicity in the Lorentzian form [const/$(E_1 - E_1^0 + i\Delta E)$] with the very small of the energy spread $\Delta E << \Gamma$; $E_l = \hbar^2 k_l^2 / 2m_l$ is the kinetic energy of the l-th particle with mass m_l ($l = 1,2$), correspondent to particles y and Y, respectively. Then, utilizing the general approach from [12] for the mean collision duration

$$< t_{\mathbf{general}} > = \frac{\int_{t_{\min}}^{\infty} t\Psi^{\bullet}_{r_1\to\infty}\hat{j}_1\Psi_{r_1\to\infty}dt}{\int_{t_{\min}}^{\infty} \Psi^{\bullet}_{r_1\to\infty}\hat{j}_1\Psi_{r_1\to\infty}dt} - < t_{\mathbf{initial}} > \approx \hbar/2DE \qquad (15)$$

(with $<t_{initial}> \approx t_i$ for quasi-monochromatic particles), we obtain after all the simplifications, mentioned in [8] and utilized here, the result, which consists in that, that the general time delay corresponds to the time-energy uncertainty relation $<\tau_{general}>\Delta E\sim\hbar$ for quasi-monochromatic particles (for which $\Delta E <<\Gamma$ and $\Delta\tau_{res}\,\Delta E<<1$).

Thus, we obtain the trivial mean time delay in the approximation $\Delta E <<\Gamma$ and $\Delta\tau_{res}\cdot\Delta E<<1$ for *L*-system *without any advance, caused by "virtual unmoving" compound nucleus in C*-system. Formulas (8)-(13) are the result of the self-consistent approach to the realistic analyze of the experimental data on the cross sections of nucleon-nucleus scattering in L-system. And any attempt to describe the experimental data of the nucleon-nucleus-scattering cross sections near an isolated resonance, distorted by the non-resonance background, in L-system on the simple base of formula (1) in C-system with the further use of the standard cinematic relations $\{E^C, \theta^C\}\leftrightarrow\{E^L, \theta^L\}$ in L-system does not have any practical physical sense. And the reason of it is connected with that we neglect the real motion of the compound nucleus.

For the case of two overlapped resonances [13] we have to calculate the wave function quite similarly to the case of one resonance before:

$$\Psi_{r_1\to\infty} \approx 0,$$

when

$$t < t_i + \frac{r_1}{V_1^0}, t < t_i + \tau + \frac{\tilde{r}_1}{V_1^0},$$

$$\begin{aligned}\psi_{r_1\to\infty} &\approx const\cdot e^{-iE_f t/\hbar} e^{i\left(k_1^0 r_1 + k_2^0 r_2\right)} \\ &*\left(f_{dir}^L \exp\left[-\Delta E\left(\left(t-t_i-\frac{r_1}{V_1^0}\right)+\left(t-t_i-\frac{r_2}{V_2^0}\right)\right)/\hbar\right]\right. \\ &+\frac{J^{1/2}_{C\to L}\gamma_Z}{\left(E_Z-E_{res,Z1}+i\Gamma_{Z1}/2\right)\left(E_Z-E_{res,Z2}+i\Gamma_{Z2}/2\right)} * \\ &\exp\left[-\Delta E\left[\left(t-t_i-\tau-\frac{\tilde{r}_1}{V_1^0}\right)+\left(t-t_i-\tau-\frac{\tilde{r}_2}{V_2^0}\right)\right]/\hbar\right] \\ &\left.\exp\left[ik_1^0\Delta r_1+ik_2^0\Delta r_2\right]\right),\end{aligned} \qquad (16)$$

when

$$t > t_i + \frac{r_1}{V_1^0}, t > t_i + \tau + \frac{\tilde{r}_1}{V_1^0}..$$

Here $V_{1,2}^0 = \hbar k_{1,2}^0/m_{1,2}$, $\Delta r_{1,2} = V_{1,2}\,\Delta\tau_{res}$, where $V_{1,2}$ is the projection of the speed of nucleus Z^* on the vectors $\vec{k}_{1,2}$, t_i is initial moment of time.

To calculate the time of delay in the L-system we have to use this formula:

$$\left\langle \tau_{general}\right\rangle = \frac{\int_{t_{min}}^{\infty} tj_i dt}{\int_{t_{min}}^{\infty} j_i dt} - t_{initial} \approx \frac{\hbar}{4\Delta E}, \qquad (17)$$

where $j_i = \mathrm{Re}\left[\psi^+ \frac{\hbar}{im}\frac{\partial\psi}{\partial x}\right]$ is the initial current. So, if we will take into account the movement of the compound-nucleus *the advanced time vanishes* also here.

4. On Cross Sections of Neutron-Nucleus Scattering near a Couple of Overlapped Compound-Nucleus Resonances in the C- and the L-system

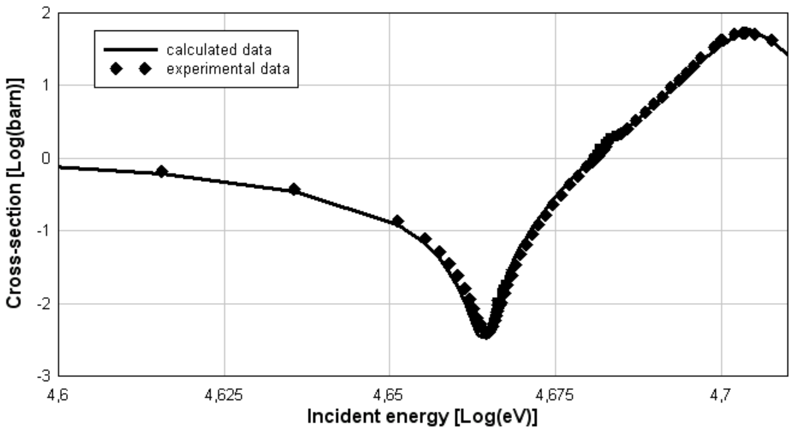

Figure 4. The excitation function for $^{52}Cr(n,n)$.

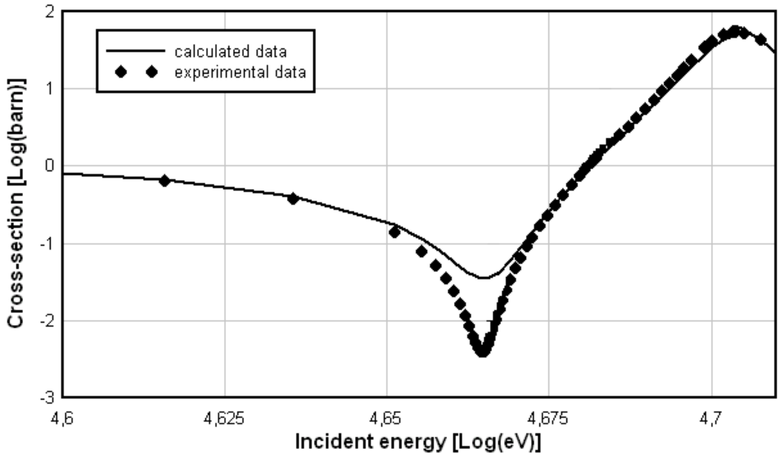

Figure 4a. The excitation function for $^{52}Cr(n,n)$ with $\varphi \equiv 0$.

We have calculated the excitation functions $\sigma(E)$ for the low-energy elastic scattering of neutrons by nuclei ^{52}Cr and ^{56}Fe and in the region of distorted isolated resonances E_{res}=50,5444 keV and Γ=1,81 keV, E_{res}=27.9179 keV and 0.71 keV, respectively. The values of the parameters for the amplitudes of the direct and resonance scattering *separately* in *C*-system for l=0 (and, naturally, without the Coulomb phases) in formulas (8)-(13) were selected with the help of the standard procedure. The fitting parameter χ was chosen to be equal to 0.68π or $0.948\,\pi$ or $0.956\,\pi$ or π, respectively.

The calculation results were obtained with the help of formulas (8)-(13) in the comparison with the experimental data, given from [14]. They are represented in Figure 4-Figure 7, respectively. And the results of calculations performed by the *standard cinematic formulas* from C- into L-system (i.e. by the formulas (8)-(13) but with $\varphi \equiv 0$, that is without diagram, depicted in Figure 3b) are represented in Figure 4a-Figure 5a. One can see that for $\varphi \equiv 0$ the minima are not totally filled.

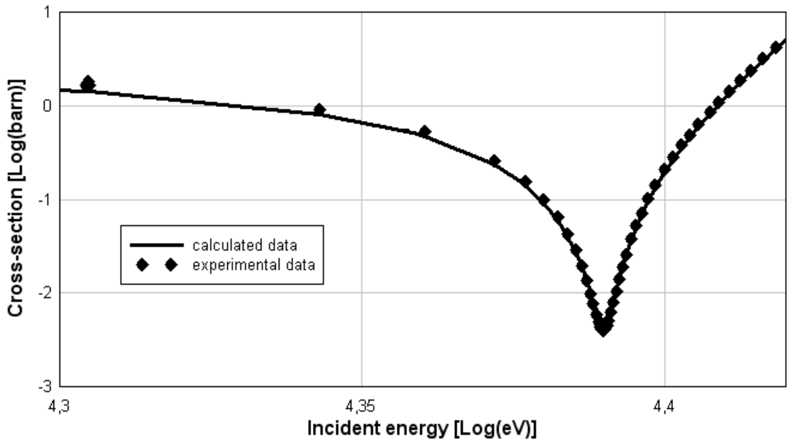

Figure 5. The excitation function for $^{56}Fe(n,n)$

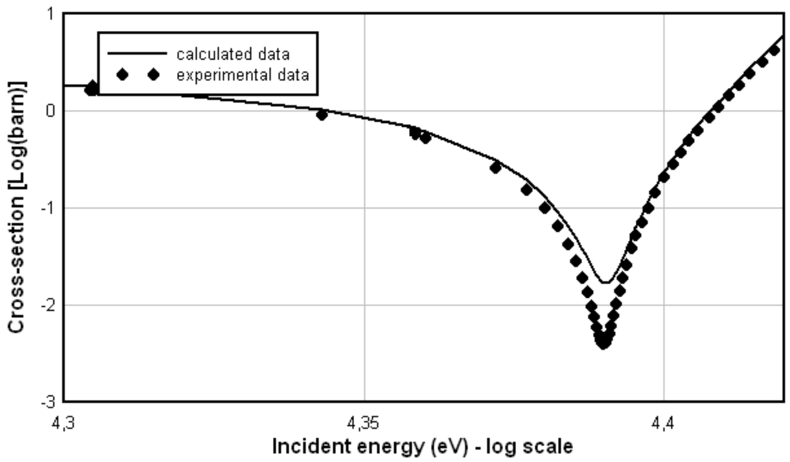

Figure 5a. The excitation function for $^{56}Fe(n,n)$ with $\varphi \equiv 0$

5. The Cross Sections of the Neutron-Nucleus Scattering with Two Overlapped Resonances

If we want to take into consideration the moving of the compound nucleus, we have to use another formula for cross section:

$$\sigma(\theta) = \int dt \int dr_2 \psi^{+}_{r_1\to\infty} \hat{j}_1 \psi_{r_1\to\infty} \approx \int dt \int dr_2 \left|\psi_{r_1\to\infty}\right|^2 = \sigma_{0(incoh)} + \sigma_{1(interf)} \quad (18)$$

where

$$\sigma_0 = \left| f^{(L)}_{dir} \right|^2 + \frac{J_{C\to L}\left|\gamma^{(C)}_{Z^+}\right|^2}{\left(\left(E_Z^+ - E_{res,Z1}\right)^2 + \Gamma_{Z1}^2/4\right)\left(\left(E_Z^+ - E_{res,Z2}\right)^2 + \Gamma_{Z2}^2/4\right)}, \quad (19)$$

$$\sigma_1 = 2\left| f^{(L)}_{dir} \frac{J^{1/2}_{C\to L}\gamma^{(C)}_{Z^+}}{\left(E_Z^+ - E_{res,Z1} + i\Gamma_{Z1}/2\right)\left(E_Z^+ - E_{res,Z2} + i\Gamma_{Z2}/2\right)} \right| \cos\Phi. \quad (20)$$

We can calculate phase Φ the same way, as in the case with the one resonance.

Other values can be found this way:

$$f^{(L)}_{dir} = \sqrt{J_{C\to L}}\, f^{(C)}_{dir} = \sqrt{J_{C\to L}}\, f_b\left(E_1^C, \theta_1^C\right), \quad (21)$$

$$\frac{\gamma^{(C)}_{Z^+}\left(E_1, E_2\right)}{\left(E_Z^+ - E_{res,Z1} + i\Gamma_{Z1}/2\right)\left(E_Z^+ - E_{res,Z1} + i\Gamma_{Z1}/2\right)} = f_{l,res}\left(E_1^C, \theta_1^C\right) \quad (22)$$

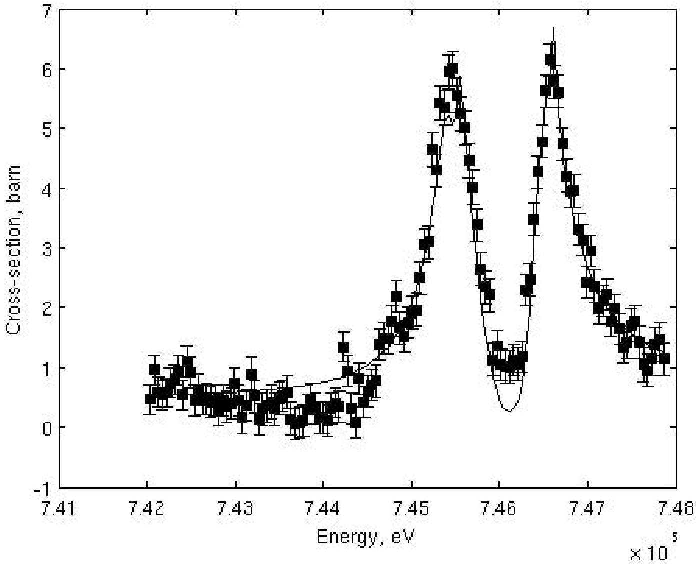

Figure 6. The excitation function for [58]Ni near two overlapped resonances with $E_3 = 745.6keV; \Gamma_3 = 0.7keV$ and $E_4 = 746,5keV; \Gamma_4 = 0,8keV$

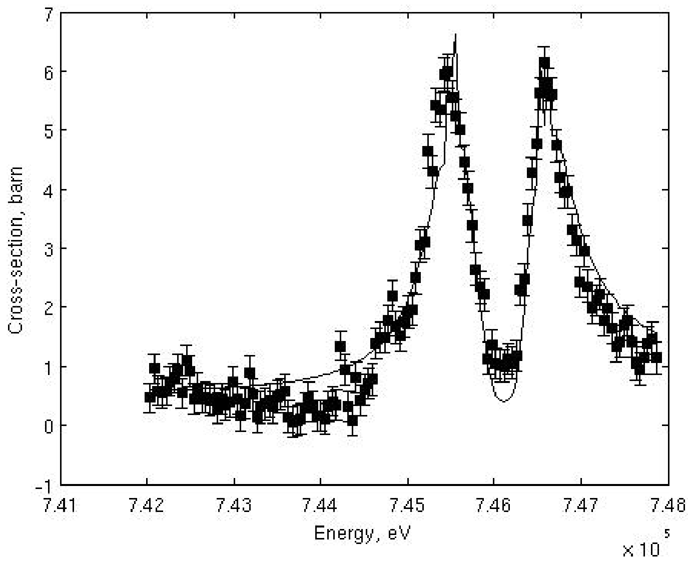

Figure 6a. The excitation function for [58]Ni with φ=0 near two overlapped resonances with $E_3 = 745.6keV; \Gamma_3 = 0.7keV$ and $E_4 = 746,5keV; \Gamma_4 = 0,8keV$

At Figure 6, Figure 6a we can see theoretical function according to (18)-(22) and experimental data. The method of least squares was used to fit the function and experimental data. Experimental data where taken from [15]. After approximation we had such values of the parameters δ_i : $\delta_0 = 2.88$, $\delta_1 = 5.59$, $\delta_3 = 4.1$, $\delta_4 = 2.34$, $\delta_5 = 2.6$, $\delta_6 = 4.75$.

After approximation we had such values of the parameters δ_i : $\delta_0 = 3.72$, $\delta_1 = 0.51$, $\delta_2 = 3.01$, $\delta_3 = 3.13$, $\delta_4 = 3.17$, $\delta_5 = 0.43$, $\delta_6 = 3.13$.

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6. The Space-Time Description of direct And Sequential (via Compound-Nucleus) Processes in the Laboratory System of Nuclear Reactions with 3 particles in the Final Channel

We shall study the interference phenomena in the laboratory system when two particles are simultaneously detected (in a sense that will be specified below) in the nuclear reactions with three nuclei (particles) in the final channel.

The original idea was presented by Podgoretskij and Kopylov [18] for the two-particle emission (evaporation) from heavy nuclei. Here we consider the interference between prompt direct and delayed resonance processes in reaction of the type

$$\mathrm{x} + \mathrm{X} \rightarrow \mathrm{y} + \mathrm{z} + \mathrm{U}. \tag{23}$$

In Figure 7a, Figure 7b two possible mechanisms for reaction (1) are pictorially represented

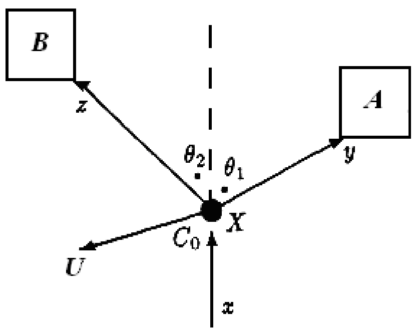

Figure 7a. Direct process reaction channel

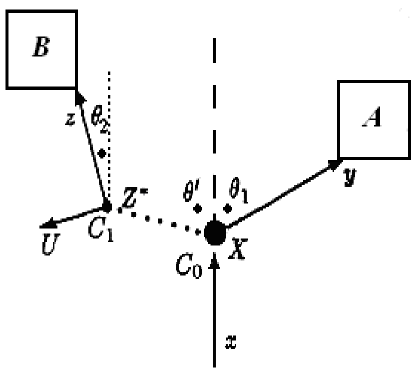

Figure 7b. Sequential process reaction channel

The symbols ***A*** and ***B*** enclosed in boxes stand for detectors located at macroscopic distances r_1 and r_2 from the scattering point C_0 . In Figure 7a the direct (like quasi-free or so called one and two step direct) process of simultaneous prompt emission at point C_0 of all the three final particles is described. Figure 7b presents delayed successive decay process with emission of particle *y* and formation of an intermediate excited nucleus *Z** which subsequently decays into *z* and *U* at point C_1, according the reaction

$$\mathrm{x} + \mathrm{X} \rightarrow \mathrm{y} + \mathrm{Z}^*,\ \mathrm{Z}^* \rightarrow \mathrm{z} + \mathrm{U} \tag{24}$$

In Figure 7c the superposition of the direct and the sequential emission of one from the final particle is displayed in the same picture. For macroscopic distances and under the condition specified below angles θ_2 and $\tilde{\theta}_2$ as well as impulses ***k*** and $\boldsymbol{k_2}$ can be considered practically coincident.

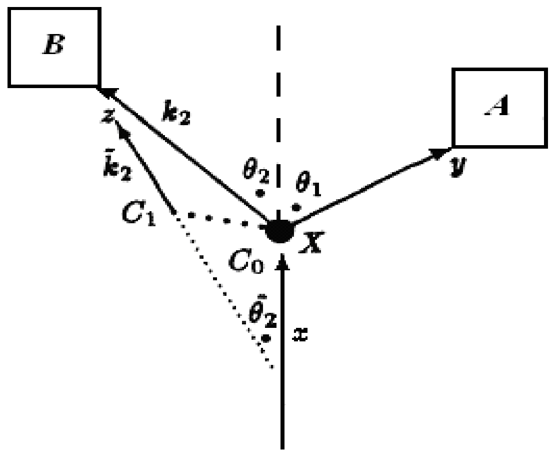

Figure 7c. Simultaneous representation of direct and sequential processes

The asymptotic wave packet, near detectors ***A*** and ***B*** can be described by the following expression:

$$\Psi_{ab}(r_1, r_2 \rightarrow \infty)$$

$$\rightarrow C \times \int d\,\mathrm{k}_x\, g_i(\mathrm{k}_x) \int d\,\mathrm{k}_2\, g_{f,2}(\mathrm{k}_2) \int d\,\mathrm{k}_1\, f_{f,1}(\mathrm{k}_1)$$

$$\times \int d\,\mathrm{k}_3\, \delta(E_i - E_f)\, \delta(\mathrm{K}_i - \mathrm{K}_f)$$

$$\left[f_{dir}^{(L)}(E_1, E_2, E_3, \theta_1, \theta_2, \theta_3)\, e^{\left(\sum_{j=1}^{j=3} i k_j r_{jC_0}\right)} + \frac{f_{Z^*}^{(L)}(E_1, E_2, E_3, \theta_1, \theta_2, \theta_3)}{\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_Z / 2}\, e^{\left(i k_1 r_{C_0} + \sum_{j=2}^{j=3} i k r_{jC_1}\right)} \right] e^{-iE_j t/\hbar}. \tag{25}$$

In this equation C is a normalization constant, g_i, $g_{f,1}$, $g_{f,2}$ are amplitude weight factors describing the impulse spread of the incident particle x and that of the final particles y and z due to detectors resolution,

$$f_{dir}^{(L)} = \sqrt{J_{C \rightarrow L}}\, f_{dir}^{(C)} \tag{26}$$

and

$$f_{Z^*}^{(C)} = \sqrt{J_{R \rightarrow C} J_{C \rightarrow L}}\, f_{xy}^{(C)} \Gamma_Z^{(C)} \tag{27}$$

are the amplitudes for direct and sequential processes (the subscriptions L and C refer to laboratory and center of mass system, respectively), $f_{xy}^{(C)}$ and $\Gamma_Z^{(C)}$ being the amplitude of the first step direct process $x + X \to y + Z^*$ and the reduced-width amplitude of the decay process $Z^* \to z + U$ respectively; ε_Z^*, $\varepsilon_{res,Z}$ and Γ_Z are the excitation energy, the energy and total width of the resonant state of the nucleus Z^*; $J_{R\to C}$ and $J_{C\to L}$ are the Jacobians of the coordinate transformations from the Recoil system to the C-system and from the C-system to the L-system, respectively; r_{km} are the distances from points m ($m = C_0$,Ci) to particles k (with k = 1,2,3 corresponding to y, z, U); E_i, $\boldsymbol{k}_i$ and E_f, $\boldsymbol{k}_f$ the total energies and impulses in the initial and final channels respectively; $E_j = \hbar^2 k_j^2 / 2m_j$ is the kinetic energy of j-th particle, θ_j and $\boldsymbol{k}_j$ being the angle of motion (relative to beam, i.e. incident particle x, direction) and the wave vector of particle j, respectively. In expression (25) $\delta(E_i - E_f)$ and $\delta(\mathrm{K}_i - \mathrm{K}_f)$ take care of energy and impulse conservation. Expression (25) is written on the base of the general formalism described in [19] with application of the asymptotic stationary functions introduced in [16,17] and taking into account particle U explicitly. For the sake of simplicity the factor $r_{1C_0}^{-1} r_{2C_0}^{-1} r_{3C_0}^{-1}$ has been omitted as well as spin and internal coordinates.

The factor $e^{-iE_f t/\hbar}$ can be rewritten as

$$e^{-i(E_1+E_2+E_3)\frac{t}{\hbar}} e^{-iE'_f \frac{t}{\hbar}} \tag{28}$$

and the first three factors of the expression (25), combined with the factor (28), can be formally put in the integrals of eq. (25) as follows:

$$\int d\,\mathrm{k}_1 g_{f,1} e^{ik_1 r_{1m} - iE_1 \frac{i}{k}} \ldots$$

$$\int d\,\mathrm{k}_2 g_{f,2i} e^{ik_i r_{2m} - iE_2 \frac{i}{k}} \ldots$$

$$\int d\,\mathrm{k}_3 g_{f,3i} e^{ik_i r_{3m} - iE_3 \frac{i}{k}} \ldots$$

In order to perform the previous integrals a transformation from variables $k_{1,2,3}$ to variables

$$\boldsymbol{y}_{1,2,3} = \left(\frac{i\hbar t}{m_{1,2,3}}\right)^{1/2} \left(\mathrm{k}_{1,2,3}^0 - \frac{m_{1,2,3} \mathrm{r}_{1,2,3}}{\hbar t}\right) \tag{29}$$

is useful. Here only projections of $\boldsymbol{k}_{1,2,3}$ over the mean vectors $\mathrm{k}_{1,2,3}^0 \equiv \langle \boldsymbol{k}_{1,2,3} \rangle$ are taken, the components of $\boldsymbol{k}_{1,2,3}$ remaining in other parts of (25). The factor $g_{f1,2}$ can be assumed to have the form

$$g_{f1,2} \approx \frac{c_{1,2}}{E_1 - E_{1,2}^0 - i\Delta E} \tag{30}$$

and ΔE to be very small ($\Delta E \ll \Gamma_Z$), as well as the energy spread of the incident particle x. Using a known result for a similar calculation (see, for instance, [20,21]), the wave function becomes

$$\Psi_{ab} \approx 0 \tag{31}$$

for

$$\begin{gathered} t < t_i + \frac{r_{1C_0}}{v_1^0}, t < t_i + \frac{r_{2C_0}}{v_2^0}, t < t_i + \frac{r_{3C_0}}{v_3^0}, \\ t < t_i + \tau + \frac{r_{2C_1}}{v_2^0}, t < t_i + \tau + \frac{r_{3C_1}}{v_3^0} \end{gathered} \tag{32}$$

and

$$\begin{aligned} &\Psi \propto C \times e^{-iE_f^0 t/\hbar} \times e^{i\sum_j k_j^0 r C_0} \\ &\times \left[\begin{aligned} & f_{dir}^L e^{-\Delta E[(t-t_i-\frac{r_{1C_0}}{v_1^0})+(t-t_i-\frac{r_{2C_0}}{v_2^0})+(t-t_i-\frac{r_{3C_0}}{v_3^0})]/\hbar} \\ & + \frac{f_{Z^*}^L}{\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_Z/2} \\ & \times e^{-\Delta E[(t-t_i-\frac{r_{1C_0}}{v_1^0})+(t-t_i-\frac{r_{2C1}}{v_2^0})+(t-t_i-\frac{r_{3C3}}{v_3^0})]/\hbar} \end{aligned} \right] \\ &\times e^{ik_2^0 \Delta r_2 + ik_3^0 \Delta r_3} \end{aligned} \tag{33}$$

for

$$\begin{gathered} t > t_i + \frac{r_{1C_0}}{v_1^0}, t > t_i + \frac{r_{2C_0}}{v_2^0}, t > t_i + \frac{r_{3C_0}}{v_3^0}, \\ t > t_i + \tau + \frac{r_{2C_1}}{v_2^0}, t > t_i + \tau + \frac{r_{3C_1}}{v_3^0}. \end{gathered} \tag{34}$$

Here $\mathrm{v}_{1,2,3}^0 = \hbar \mathrm{k}_{1,2,3}^0 / m_{1,2,3}$, the initial time t_i is defined by the phase of the amplitude weight factor g_i; and the mean time τ of the nucleus Z^* motion before its decay is given by the well known expression:

$$\tau = \frac{\hbar \Gamma_Z / 2}{(\varepsilon_Z^* - \varepsilon_{res,Z})^2 + \Gamma_Z^2 / 4} \tag{35}$$

and

$$\Delta r_{2,3} = V_{\perp(2,3)} \tau, \tag{36}$$

$V_{\perp(2,3)}$ being the projection of the velocity of the nucleus Z^* onto the direction of $\boldsymbol{k}_{2,3}$. The energy spread for particle U is of the order ΔE , according to energy-impulse conservation.

Interference phenomena can occur only in case of simultaneous arrival (within the time resolution of the detectors) of particles y and z on A and B. The coincidence-rate intensity is described by a time integration of

$$\Psi_{ab}^* \hat{j}_1 \hat{j}_2 \Psi_{ab}$$

($\hat{j}_{1,2}$ being the flux probability density operator for particles y and z) over a time interval ΔT, which is great with respect to the time extension of the wave packets, and a spatial integration over particle U coordinates, i.e.:

$$P \approx \int_{t_{\min}}^{\infty} dt \int_{r_{3\min}}^{r_{3\max}} dr_3 \Psi^*_{ab} \hat{j}_1 \hat{j}_2 \Psi_{ab}$$
$$\propto \int_{t_{\min}}^{\infty} dt \int_{0}^{v_3^0(t-t_i-\frac{r_{3C_0}}{v_3^0})} dr_3 |\Psi_{ab}|^2, \quad (37)$$

where t_{min} is the smallest value among

$$t_i + \frac{r_{1C_0}}{v_1^0}, t_i + \frac{r_{2C_0}}{v_2^0}, t_i + \frac{r_{3C_0}}{v_3^0}, t_i + \tau + \frac{r_{2C_1}}{v_2^0}, t_i + \tau + \frac{r_{3C_1}}{v_3^0}$$

r_{3max} is the maximum between $v_3^0(t - t_i - (\frac{r_{3C_1}}{v_3^0}))$ and $v_3^0(t - t_i - \tau - \frac{r_{3C_1}}{v_3^0})$, $r_{3min} \to 0$ for ordinary small wave packets.

Under the standard experimental conditions, i.e. when

$$\Delta E \tau / \hbar << 1 \quad (38)$$

and

$$\delta t = \frac{r_l}{v_l^0} - \frac{r_m}{v_m^0} << \Delta T, (l, m = 1, 2, 3, l \neq m) \quad (39)$$

(ΔT is the time resolution of the coincidence scheme), it is possible to write

$$P = P_0 + P_1, \quad (40)$$

$$P_0 = \left| f^L_{dir} \right|^2 + \frac{\left| f^L_{Z^*} \right|^2}{(\varepsilon^*_Z - \varepsilon_{res,Z})^2 + \Gamma^2_Z / 4} \quad (41)$$

and

$$P_1 = 2 \left| f^L_{dir} \frac{f^L_{Z^*}}{\varepsilon^*_Z - \varepsilon_{res,Z} + i\Gamma_Z / 2} \right| \cos\Phi \quad (42)$$

(in arbitrary units), where

$$\Phi = \delta + \beta + \phi, \quad (43)$$

$$\delta = \arg(f^L_{Z^*}) - \arg(f^L_{dir}),$$

$$\beta = \arg(\varepsilon^*_Z - \varepsilon_{res,Z} + i\Gamma / 2)^{-1},$$

$$\phi = k_2^0 \Delta r_2 + k_3^0 \Delta r_3,$$

$\Delta r_{2,3}$ being defined by (36).

The obtained results (40)-(43), with the incoherent sum P_0, the interference term P_1 and the phase Φ, do evidently generalize the results for the L-system, obtained somewhat earlier by us in [9] for collisions with two-particle channels. Comparing these results with that obtained in a stationary model [16,17], the latter ones are confirmed by the present self-consistent space-time approach in the limit $E << \Gamma_Z$. The same conclusion is valid for the cases in which two intermediate excited nuclei are formed, i.e.

$$x + X \to \begin{cases} y + Z^* \to y + z + U \\ z + Y^* \to z + y + U \end{cases},$$

under the conditions $\Delta E << \Gamma_y$, and $\Delta E << \Gamma_Z$.

Conclusions: The results (40)-(43) are firstly obtained in the space-time description of the interference between different (direct and sequential, containing the decaying compound-nucleus) mechanisms with three nuclei in the final channel. They are the clear generalization of the results for nucleon-nucleus and nucleus-nucleus collisions with two-particle channels, presented in [3,8], and can be easily generalized for the cases in which two intermediate excited (compound) nuclei are formed. Moreover, in the limit $\Delta E/\Gamma_Z \to 0$ they factually pass to the correspondent stationary-model results as presented in [16,17].

Finally, it is rather perspective and really topical to develop the much more complete approach to interference phenomena between the direct and various sequential processes in complex nuclear reactions.

7. Conclusions and Perspectives

Presented here time analysis of experimental data on nuclear processes permits to make the following conclusions and perspectives:

1. The simple application of time analysis of quasi-monochromatic scattering of neutrons by nuclei in the region of isolated resonances, distorted by the non-resonance background, brings in C-system to the delay-advance paradoxical phenomenon near a resonance in any two-particle channel. Such phenomenon of the time-transfer delay in the time advance is usually connected with a minimum in the cross section, or zero in analytic plane of scattering amplitude (apart from the resonance pole) near the positive semi-axis of kinetic energies in lower non-physical semi-plane of the Riemann surface. Here this paradox is eliminated by the thorough space-time analysis in L-system with moving C-system.

2. Moreover, it is also revealed that the *standard formulas of transformations from L*-system *into C-system are in-suitable in the presence of two (and more) collision mechanisms – quick (direct or potential) process when the center-of-mass is practically not displaced in the collision and the delayed process when the long-living compound nucleus is moving in L-system.* And revealed by our group the additional change of the amplitude phase in C → L transformations now agree with the elimination of the paradox of passing the usual time delay in the time advance. The obtained analytic transformations of the cross section from C-system into L-system are illustrated by the calculations of excitation functions for examples of the elastic scattering of neutrons by nuclei ^{52}Cr, ^{56}Fe and ^{58}Ni near the distorted resonances in L-system.

3. The presented here results of time analysis for the quasi-monochromatic neutron-nucleus scattering near the isolated resonances, distorted by the non-resonance background, can be easily generalized to the scattering nucleons by nuclei near two-three overlapped resonances.

4. Of course, new formulas (8)-(13) and (18)-(22) can be also used for *the improvement of the existing general methods of analyzing resonance nuclear data for the two-particle channels* in nucleon-nucleus collisions in L-system and, moreover, can be generalized for more complex collisions.

5. Applying time analysis to elastic nucleon-nucleus with 2-3 overlapping compound-resonances, it is possible also to obtain the paradoxical phenomenon of transition decay in advance in C-system. But the behavior of amplitudes and durations can be certainly more complex than for an isolated resonance. Therefore the study of such cases can be more complicated that for an isolated resonance, and it has to be rather interesting and perspective.

6. It is rather interesting the perspective to apply the results of the space-time description of direct and sequential (via compound-nucleus) processes in the L-system of nuclear reactions with 3 particles in the final channel for concrete investigations, elaborations and calculations of many concrete nuclear collisions.

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