American Journal of Applied Mathematics and Statistics, 2015, Vol. 3, No. 3, 105-110
Available online at http://pubs.sciepub.com/ajams/3/3/4

DOI:10.12691/ajams-3-3-4

Semi-Markov Model of a Standby System with General Distribution of Arrival and Failure Times of Server

R.K. Bhardwaj[1,*], Ravinder Singh[2]

[1]Department of Statistics, Faculty of Physical Sciences, Punjabi University Patiala-147002 (Punjab), India
[2]Department of Statistics, Punjabi University Patiala-147002 (Punjab), India
*Corresponding author: rkb_mstates@rediffmail.com

Received May 29, 2015; Revised June 16, 2015; Accepted June 25, 2015

Abstract The aim of this paper is to develop a stochastic model of a cold standby system consisting of two identical units and a service facility, called server. This paper considers the failure of server during operation. The semi-Markov approach is explored to develop the probabilistic model and regenerative point technique is used to derive expressions for system's performance measures such as mean time to system failure, availability, profit etc. In the model the server takes some time to arrive at the system. The server, while on job, may fail. Upon failure it goes for treatment and rejoins thereafter. Both the arrival and failure times follow general distribution with different probability distribution functions. The numerical illustration, for a particular case, points out that both the server arrival and failure times significantly affect the system performance.

Keywords: *semi-markov model, standby system, server arrival time, server failure*

Cite This Article: R.K. Bhardwaj, and Ravinder Singh, "Semi-Markov Model of a Standby System with General Distribution of Arrival and Failure Times of Server." *American Journal of Applied Mathematics and Statistics,* vol. 3, no. 3 (2015): 105-110. doi: 10.12691/ajams-3-3-4.

1. Introduction

In many manufacturing, production or some other engineering processes the systems fail due to failure of either unit or component. Such incidents undesirably affect not only the production and revenue but also dilute company's repute in the competitive market. Providing competent service facility and spares (standbys) for these units/ components has been a common practice to avoid or diminish such losses. The standby unit switches into operation whenever the operative unit fails. The failed unit is then taken by the server for required corrective measures. These systems can be studied by developing their probabilistic models.

The semi-Markov approach facilitates performance modeling of continuous time repairable systems under the assumption that the future evolution of the process is independent of the sequence of states visited prior to the current state and independent of the time spent in each of the previously visited states([1,2,3,4]).Exploring semi-Markov theory, the stochastic models of standby systems, concerning failure and repair of operative units, have been widely discussed in the literature ([5-10]).The failure of standby units is also examined by researchers ([11,12,13]).

In repairable reliability models the server plays very decisive role in bringing the failed unit back into operation. But the server, too can fail while working on some assignment. For instances, anelectrician may experience electric shock while handling some fault in supply line, a mechanic may injured during repairing or even checking for some defects in a machine etc. So with the failure of server it become challenging to sustain good level of reliability, availability and hence profit of a system. This fact shows the high practical importance of the issue. Despite of all these things the systems with server failure have not been adequately studied in the past. However, some studies highlight the issue of server failure ([14,15]). Though, they explore the idea with basic single unit models but completely ignore the significance of redundancy.

Recently, [16,17] extended the previously discussed basic models of standby systems with the possibility of server failure incorporating the proviso of standby. They assumed that the server is instantly available in the system when needed. But this assumption seems impractical. Because the server may take some time to arrive at the system because of engagement in pre-assignment, information delays, mishaps or distance, etc.

Keeping these facts in view, this paper investigates a stochastic model of a cold standby system, taking a broad view over the work reported so far. The model has a unit in operation and another identical as standby. At the failure of operating unit, the one in standby instantly switches into operation. The failed unit waits for repair until the arrival of server. Upon arrival the server takes the unit under repair. The server may fail while on work. Subsequently, it goes directly under treatment. The failure, repair, arrival and treatment times follow general probability distributions with different probability densities. All the random variables are statistically independent. The repairs and treatments are perfect i.e. after each repair (treatment) the unit (server) is as good as new. The semi-Markov process and regenerative point technique are used to obtain different performance indices. A particular case is discussed for numerical illustration of the study.

2. Notations

$E / \overline{E}$: Set of regenerative states/non regenerative staes.
O : The unit is operative and in normal mode.
FW_r / FW_R : Failed unit waiting for repair/ continuously from previous state.
SG : The server is good.
FU_r / FU_R : Failed unit under repair/ continuously from previous state.
SFU_t / SFU_T : Failed server under treatment/ continuously from previous state.
$z(t) / Z(t)$: pdf/ cdf of failure time of the unit.
$u(t) / U(t)$: pdf / cdf of failure time of the server.
$g(t) / G(t)$: pdf / cdf of repair time of the failed unit.
$h(t) / H(t)$: pdf / cdf of the treatment time of the server.
$v(t) / V(t)$: pdf / cdf of the arrival time of the server.
$q_{i,j}(t) / Q_{i,j}(t)$: pdf / cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state.
$q_{i,j.k}(t) / Q_{i,j.k}(t)$: pdf / cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$q_{i,j.k,r}(t) / Q_{i,j.k,r}(t)$: pdf / cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0,t].
$q_{i,j.k,r,s}(t) / Q_{i,j.k,r,s}(t)$: pdf / cdf of first passage time from regenerative state i to a regenerative state jor to a failed state j visiting state k, r and s once in (0,t].
$M_i(t)$: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state.
$W_i(t)$: Probability that the server is busy in the state $S_i$upto time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
$m_{i,j}$: contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state j.
$(s) / (c)$: Stieltjes convolution / Laplace convolution.
$\sim / *$: Laplace Stieltjes Transform (LST) / Laplace Transform(LT).

The following are the possible states of the system model

The regenerative states (E) :

$S_0 = (O, CS), S_1 = (FW_r, O), S_2 = (FU_r, O, SG),$
$S_3 = (FW_r, O, SFU_t)$

Non-regenerative states $(\overline{E})$:

$S_4 = (FW_R, FW_r), S_5 = (FU_R, FW_r, SG),$
$S_6 = (FW_R, FW_r, SFU_T),$
$S_7 = (FW_r, FW_R, SFU_t), S_8 = (FU_r, FW_R, SG)$

3. The Model Formulation

3.1. Transition Probabilities

Using basics of probabilistic theory, [18] we obtain the following expressions for the non- zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

as we get

$$p_{0,1} = \int_0^\infty z(t)dt,\ p_{1,2} = \int_0^\infty v(t)\overline{Z}(t)dt,$$
$$p_{1,4} = \int_0^\infty z(t)\overline{V}(t)dt,\ \ p_{1,2.4,(8,7)^n} = p_{1,4}(c)p_{4,8}(c)p_{8,2},$$
$$p_{2,0} = \int_0^\infty g(t)\overline{Z}(t)\overline{U}(t)dt, p_{2,3} = \int_0^\infty u(t)\overline{Z}(t)\overline{G}(t)dt,$$
$$p_{2,5} = \int_0^\infty z(t)\overline{G}(t)\overline{U}(t)dt, p_{2,2.5} = p_{2,5}(c)p_{5,2},$$
$$p_{2,2.5,(7,8)^n} = p_{2,5}(c)p_{5,7}(c)p_{7,8}(c)p_{8,2},$$
$$p_{3,2} = \int_0^\infty h(t)\overline{Z}(t)dt,\ p_{3,6} = \int_0^\infty z(t)\overline{H}(t)dt$$
$$p_{3,2.6,(8,7)^n} = p_{3,6}(c)p_{6,8}(c)p_{8,2},$$
$$p_{4,8} = \int_0^\infty v(t)dt, p_{5,2} = \int_0^\infty g(t)\overline{U}(t)dt,$$
$$p_{5,7} = \int_0^\infty u(t)\overline{G}(t)dt, \quad p_{6,8} = \int_0^\infty h(t)dt,\ p_{7,8} = \int_0^\infty h(t)dt,$$
$$p_{8,2} = \int_0^\infty g(t)\overline{U}(t)dt,\ p_{8,7} = \int_0^\infty u(t)\overline{G}(t)dt$$

The above results show that

$$p_{0,1} = p_{1,2} + p_{1,4} = p_{1,2} + p_{1,2.4,(8,7)^n}$$
$$= p_{2,0} + p_{2,3} + p_{2,5} = p_{2,0} + p_{2,3} + p_{2,2.5}$$
$$+ p_{2,2.5,(8,7)^n} = p_{3,2} + p_{3,6} = p_{3,2} + p_{3,2.6,(8,7)^n}$$
$$= p_{4,8} = p_{5,2} + p_{5,7} = p_{6,8} = p_{7,8}$$
$$= p_{8,2} + p_{8,7} = 1$$

3.2. Mean Sojourn Times

The Mean sojourn time μ_i in state S_i are given by

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt$$

For $i = 0,1,2,3$ we get ,

$$\mu_0 = \int_0^\infty \overline{Z}(t)dt, \mu_1 = \int_0^\infty \overline{V}(t)\overline{Z}(t)dt,$$
$$\mu_2 = \int_0^\infty \overline{Z}(t)\overline{U}(t)\overline{G}(t)dt, \mu_3 = \int_0^\infty \overline{H}(t)\overline{Z}(t)dt$$

The unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j is stated as:

$$m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$$

$$m_{0,1} = \mu_0, m_{1,2} + m_{1,4} = \mu_1, m_{1,2} + m_{1,2.4,(8,7)^n}$$
$$= \mu_1^{'}, m_{2,0} + m_{2,3} + m_{2,5} = \mu_2,$$
$$m_{2,0} + m_{2,3} + m_{2,2.5} + m_{2,2.5,(8,7)^n} = \mu_2^{'},$$

$$m_{3,2}+m_{3,6}=\mu_3, m_{3,2}+m_{3,2.6,(8,7)^n}=\mu_3^{'},$$

$$m_{4,8}=\mu_4, m_{5,2}+m_{5,7}=\mu_5,$$

$$m_{6,8}=\mu_6, m_{7,8}=\mu_7, m_{8,2}+m_{8,7}=\mu_8$$

4. Performance Analysis

4.1. Mean Time to System Failure

Let $\varphi_i(t)$ be the c.d.f of the first passage time from regenerative state S_i to a failed state. Assuming the failed state as absorbing, we get the following recursive relations for $\varphi_i(t)$:

$$\begin{aligned}
&\varphi_0(t)=Q_{0,1}(t)(s)\varphi_1(t)\\
&\varphi_1(t)=Q_{1,2}(t)(s)\varphi_2(t)+Q_{1,4}(t)\\
&\varphi_2(t)=Q_{2,0}(t)(s)\varphi_0(t)+Q_{2,3}(t)(s)\varphi_3(t)+Q_{2,5}(t)\\
&\varphi_3(t)=Q_{3,2}(t)(s)\varphi_2(t)+Q_{3,6}(t)
\end{aligned}\qquad (1)$$

Taking LST of equation (1) and solving for $\tilde{\varphi}_0(s)$, we have

$$R^*(s)=\frac{1-\tilde{\varphi}_0(s)}{s} \qquad (2)$$

The reliability R(t) can be obtained by taking inverse Laplace transform of (2) and MTSF is given by

$$\begin{aligned}
MTSF &= \lim_{s\to 0} R^*(s)=\frac{\{1-\tilde{\varphi}_0(s)\}}{s}\\
&=\frac{[\mu_0+\mu_1][1-p_{2,3}p_{3,2}]+p_{1,2}[\mu_2+\mu_3 p_{2,3}]}{1-p_{2,3}p_{3,2}-p_{1,2}p_{2,0}}
\end{aligned}\qquad (3)$$

4.2. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at any instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are as follows:

$$\begin{aligned}
&A_0(t)=M_0(t)+q_{0,1}(t)(c)A_1(t)\\
&A_1(t)=M_1(t)+q_{1,2}(t)(c)A_2(t)+q_{1,2.4,(8,7)^n}(t)(c)A_2(t)\\
&A_2(t)=M_2(t)+q_{2,0}(t)(c)A_0(t)+q_{2,2.5}(t)(c)A_2(t)\\
&+q_{2,2.5,(7,8)^n}(t)(c)A_2(t)+q_{2,3}(t)(c)A_3(t)\\
&A_3(t)=M_3(t)+q_{3,2}(t)(c)A_2(t)+\\
&q_{3,2.6,(8,7)^n}(t)(c)A_2(t)
\end{aligned}\qquad (4)$$

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0=\int_0^\infty \overline{Z}(t)dt, M_1=\int_0^\infty \overline{V}(t)\overline{Z}(t)dt,$$

$$M_2=\int_0^\infty \overline{Z}(t)\overline{U}(t)\overline{G}(t)dt, M_3=\int_0^\infty \overline{H}(t)\overline{Z}(t)dt$$

Taking LT of equation (4) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\frac{p_{2,0}[\mu_0+\mu_1]+\mu_2+\mu_3 p_{2,3}}{p_{2,0}[\mu_0+\mu_1^{'}]+\mu_2^{'}+\mu_3^{'} p_{2,3}} \qquad (5)$$

4.3. Busy Period Analysis for the Server

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant t given that the system entered regenerative state S_i at t = 0. The recursive relations for $B_i(t)$ are as follows:

$$\begin{aligned}
&B_0(t)=q_{0,1}(t)(c)B_1(t)\\
&B_1(t)=q_{1,2}(t)(c)B_2(t)+q_{1,2.4,(8,7)^n}(t)(c)B_2(t)\\
&B_2(t)=W_2(t)+q_{2,0}(t)(c)B_0(t)+q_{2,2..5}(t)(c)B_2(t)\\
&+q_{2,2.5,(7,8)^n}(t)(c)B_2(t)+q_{2,3}(t)(c)B_3(t)\\
&B_3(t)=q_{3,2}(t)(c)B_2(t)+q_{3,2.6,(8,7)^n}(t)(c)B_2(t)
\end{aligned}\qquad (6)$$

$W_i(t)$ be the probability that the server is busy in state S_i due to repair of the unit up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

$$\begin{aligned}
W_2 &= \overline{Z}(t)\overline{U}(t)\overline{G}(t)+(z(t)\overline{U}(t)\overline{G}(t)(c)1)\overline{G}(t)\\
&+(z(t)\overline{U}(t)\overline{G}(t)(c)u(t)\overline{G}(t)(c)1)\overline{H}(t)\\
&+(z(t)\overline{U}(t)\overline{G}(t)(c)u(t)\overline{G}(t)(c)h(t)(c)1)\overline{G}(t)
\end{aligned}$$

Taking LT, of equation (6) and solving for $B_0^*(s)$, the time for which server is busy due to repair of unit is given by

$$B_0=\lim_{s\to 0} sB_0^*(s)=\frac{W_2^*(0)}{p_{2,0}[\mu_0+\mu_1^{'}]+\mu_2^{'}+\mu_3^{'} p_{2,3}} \qquad (7)$$

4.4. Expected Number of Treatment Given To the Server

Let $T_i(t)$ be the expected number of treatments given to the server in (0,t] given that the system entered regenerative state S_i at t=0. The recursive relations for $T_i(t)$ are as follow:

$$\begin{aligned}
&T_0(t)=Q_{0,1}(t)(s)T_1(t)\\
&T_1(t)=Q_{1,2}(t)(s)T_2(t)+Q_{1,2.4,(8,7)^n}(t)(s)T_2(t)\\
&T_2(t)=Q_{2,0}(t)(s)T_0(t)+Q_{2,2.5}(t)(s)T_2(t)\\
&+Q_{2,2.5,(7,8)^n}(t)(s)T_2(t)+Q_{2,3}(t)(s)T_3(t)\\
&T_3(t)=Q_{3,2}(t)(s)[1+T_2(t)]+Q_{3,2.6,(8,7)^n}(t)(s)T_2(t)
\end{aligned}\qquad (8)$$

Using LT, of equation (8) and solving for $\tilde{T}_0(s)$, the expected number of the treatments given to the server are given by

$$T_0=\lim_{s\to 0} s\tilde{T}_0(s)=\frac{p_{2,3}p_{3,2}}{p_{2,0}[\mu_0+\mu_1^{'}]+\mu_2^{'}+\mu_3^{'} p_{2,3}} \qquad (9)$$

4.5. Expected Number of Repair of the Unit

Let $D_i(t)$ be the expected number of repairs of the unit in (0,t] given that the system entered regenerative state S_i at t=0. The recursive relations for $D_i(t)$ are as follow:

$$\begin{aligned}
&D_0(t) = Q_{0,1}(t)(s)D_1(t) \\
&D_1(t) = Q_{1,2}(t)(s)D_2(t) + Q_{1,2.4,(8,7)^n}(t)(s)[1+D_2(t)] \\
&D_2(t) = Q_{2,0}(t)(s)[1+D_0(t)] + [Q_{2,2.5}(t) \\
&+Q_{2,2.5,(7,8)^n}(t)](s)[1+D_2(t)] + Q_{2,3}(t)(s)D_3(t) \\
&D_3(t) = Q_{3,2}(t)(s)D_2(t) + Q_{3,2.6,(8,7)^n}(t)(s)[1+D_2(t)]
\end{aligned} \tag{10}$$

Using LT, of equation (10) and solving for $\tilde{D}_0(s)$, the expected number of repair of the unit are given by

$$D_0 = \lim_{s\to 0} s\tilde{D}_0(s) = \frac{p_{0,1}p_{1,2.4,(8,7)^n}p_{2,0} + p_{0,1}p_{2,3}[1+p_{3,2.6,(8,7)^n}]}{p_{2,0}[\mu_0 + \mu_1'] + \mu_2' + \mu_3' p_{2,3}} \tag{11}$$

4.6. The Profit

Let X_i (t) denote the measure of i^{th}performance index of the system in (0, t] and C_i be its coefficient then the profit incurred to the system model in (0,t] is given by

$$P(t) = K_0A_0(t) - \sum_{i=1}^{3} C_iX_i(t)$$

For steady state letting $t_{\to\infty}$, we obtain

$$\text{Profit}, P_0 = \lim_{t\to\infty} P(t) = \lim_{t\to\infty}[K_0A_0(t) - \sum_{i=1}^{3} C_i\{X_i(t)\}]$$

$$= K_0A_0 - \sum_{i=1}^{3} C_i\{X_i(\infty)\}$$

where

$$X_i(\infty) = \begin{cases} B_0;\ for,\ i=1 \\ T_0;\ for,\ i=2 \\ D_0;\ for,\ i=3 \end{cases}$$

K_0 = Revenue per unit up time of the system.

C_1 = Cost per unit time for which server is busy.

C_2 = Cost per unit time for the server treatment.

C_3 = Cost per unit time for the repair of the uint.

and A_0, B_0, T_0, D_0 are already defined.

5. Particular Case

Initially various costs and values for different parameters are assumed as given below:

$$C_1 = 500, C_2 = 900, C_3 = 300, K_0 = 20000,$$

$$\lambda = 0.008, \alpha = 0.3, \gamma = 0.02, \psi = 0.08,$$

Substituting, $z(t) = \lambda e^{-\lambda t}, g(t) = \alpha e^{-\alpha t}, u(t) = \gamma e^{-\gamma t},$
$v(t) = \psi e^{-\psi t}, h(t) = \beta e^{-\beta t}$

We obtained the numerical results presented in Table 1 & Table 2. The tables show the effects of repair rate of unit (α), server arrival rate (ψ), failure rate of unit (λ), failure rate of server (γ) and server treatment rate (β). Table 1 shows that as α varies from $\alpha = 0.3$ to $\alpha =$ 0.4, for $\psi = 0.08$, all the performance indices rise for fixed values of other parameters. The trend remains same even when ψ varies from $\psi = 0.08$ to $\psi = 0.09$, for $\alpha = 0.3$ and all remaining parameters kept at same level. On the other side the trend reverses with increase in the values of λ from $\lambda = 0.008$ to $\lambda =$ 0.009 and γ from $\gamma =$ 0.02 to $\gamma =$ 0.04, respectively, as shown in Table 2. Furthermore, both the tables present rising trends for all indices with increasing values of server treatment rate β. Here it is important to note that the numerical results attain most expected trends and show that the system's overall performance rises with increasing α, β and declines with increasing γ, ψ, λ.

Table 1. effect of α and ψ on system performance w.r.t server treatment rate β ($\lambda = 0.008, \gamma = 0.02$)

Performance Index	β	$\alpha = 0.3$, $\psi = 0.08$	$\alpha = 0.4$, $\psi = 0.08$	$\alpha = 0.3$, $\psi = 0.09$
MTSF	0.01	1021.958	1103.566	1082.283
	0.02	1081.749	1157.676	1151.442
	0.03	1113.427	1185.776	1188.378
	0.04	1133.046	1202.986	1211.356
	0.05	1146.390	1214.609	1227.030
Availability	0.01	0.9626	0.9702	0.9644
	0.02	0.9793	0.9827	0.9811
	0.03	0.9838	0.9860	0.9856
	0.04	0.9856	0.9874	0.9874
	0.05	0.9866	0.9881	0.9884
Profit	0.01	19239.55	19393.67	19275.61
	0.02	19573.92	19644.4	19610.19
	0.03	19662.56	19710.06	19698.63
	0.04	19699.89	19737.52	19735.78
	0.05	19719.51	19751.89	19755.27

Table 2. effect of λ and γ **on system performance w.r.t server treatment rate** β **(** $\alpha = 0.3, \psi = 0.08$ **)**

Performance Index	β	$\lambda = 0.008$, $\gamma = 0.02$	$\lambda = 0.009$, $\gamma = 0.02$	$\lambda = 0.008$, $\gamma = 0.04$
MTSF	0.01	1021.958	836.073	892.612
	0.02	1081.749	880.104	979.978
	0.03	1113.427	903.926	1030.223
	0.04	1133.046	918.855	1062.861
	0.05	1146.390	929.087	1085.767
Availability	0.01	0.9626	0.9553	0.9363
	0.02	0.9793	0.9748	0.9692
	0.03	0.9838	0.9801	0.9781
	0.04	0.9856	0.9823	0.9819
	0.05	0.9866	0.9835	0.9839
Profit	0.01	19239.55	19091.59	18713.16
	0.02	19573.92	19481.72	19370.16
	0.03	19662.56	19587.35	19549.2
	0.04	19699.89	19632.29	19625.26
	0.05	19719.51	19656.05	19665.38

6. Conclusion

The paper explores semi-Markov process and regenerative point technique to develop a probabilistic model for a two unit cold standby system. A service facility, called server, takes some time to arrive at the system. The possibility of failure of the server itself during operation is also considered. A particular case is discussed for numerical illustrations. The results, thus obtained, reveal that the reliability and profit of a two unit cold standby system can be enhanced by reducing the arrival time of the server and providing prompt treatment to the server at its failure. Therefore it can be suggested that arrival and failure of the server are the key issues to be focused essentially. The numerical results of the illustration indicate that the probabilistic model is appropriately developed and statistically fit for implementation in a realistic situation.

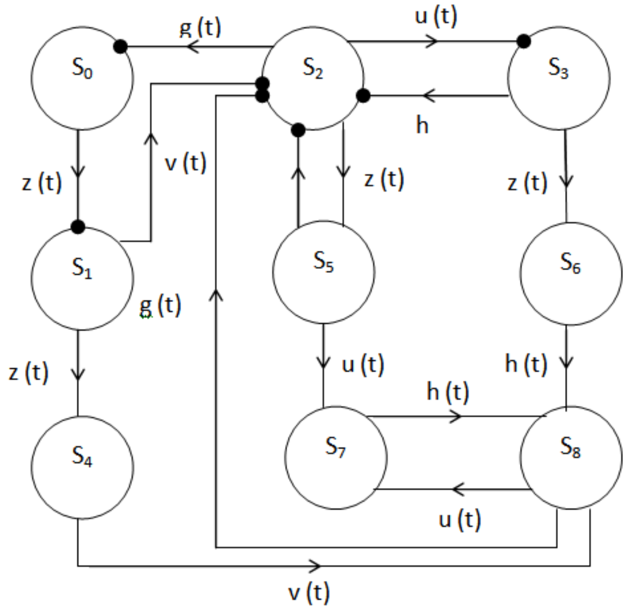

Figure 1. State Transition Diagram

Acknowledgements

1. The work reported in this paper is financially supported by Ministry of Human Resource Development, Government of India, under major research project scheme ofUniversity Grant Commission [F.N. 42-34/ 2013(SR)].
2. The authors are very grateful to the anonym reviewer for giving valuable suggestions.

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