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The Similar Structure Method for Solving the Radial Seepage Model of Fractal Composite Reservoir with Double-porosity

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Abstract This paper introduces the fractal theory into composite reservoir with double-porosity, and establishes the radial seepage model of fractal composite reservoir with double-porosity. Based on the similar structure theory of solutions for the boundary value problem of differential equation, the similar structure expression of solutions can be obtained in Laplace space. The similar structure theory of solution which avoids the complex calculation is used to solve the model, meanwhile, the similar structure expression of solution reflects the influence of different parameters for the bottom pressure.

Keywords: *radial seepage, composite reservoir with double-porosity, laplace transformation, similarity kernel function, similar structure*

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1. Introduction

Fractal theory was born in the field of mathematics, and it is a good tool to describe irregular objects. Chang and Yortsos [1] introduced the fractal theory into the study of reservoir seepage in 1990 for the first time, producing many studies about the application of the fractal theory in well test model. Li [2,3,4] introduced the fractal theory into the dual-porosity reservoir model in well test analysis under three outer boundary condition(infinite, closed, constant pressure), and obtained the solution of dimensionless reservoir's pressure and dimensionless well-bore's pressure in Laplace space. Deng and Li [5,6,7] used the fractal dimension and the fractal index to describe features of the fractured medium and the rock matrix in the fractal composite reservoirs, and established an analysis mathematical model of well test with the well-born storage and the skin effects, and obtained a formula of Laplace space solution on dimensionless reservoir's pressure and dimensionless well-bore's pressure.

In this paper, the fractal theory is introduced into the composite reservoir with double-porosity, establishing the radial seepage model of fractal composite reservoir with double-porosity, and obtaining the solution of the model in Laplace space.

2. SSM the Boundary Value Problem for Composite Modified Bessel Equations

In order to solve the reservoir model, this paper introduces SSM [8] (The Similar Structure Method) for solving the modified composite Bessel equations' boundary value problem:

$$x^2 y_1'' + A_1 x y_1' - B_1 x^{C_1} y_1 = 0, \quad (a < x < c); \tag{1}$$

$$x^2 y_2'' + A_2 x y_2' - B_2 x^{C_2} y_2 = 0, \quad (c < x < b); \tag{2}$$

$$[E y_1 + (1+EF) y_1']_{x=a'} = D; \tag{3}$$

$$y_1|_{x=c} = m\, y_2|_{x=c}; \tag{4}$$

$$y_1'|_{x=c} = n\, y_2'|_{x=c}; \tag{5}$$

$$[G y_2 + H y_2']_{x=b} = 0. \tag{6}$$

Where A_i, B_i, C_i, D, E, F, G, H, a, b, c, m and n are all known as real numbers and satisfy $a < c < b$, $D \neq 0$, $G^2 + H^2 \neq 0$, $B_i > 0$, $C_i \neq 0$ $(i = 1, 2)$.

The SSM steps are presented as follow.

Step 1. Find two linearly independent solutions of $x^2 y_i'' + A_i x y_i' - B_i x^{C_i} y_i = 0$, as follows [9]:

$$x^{\alpha_i} K_{\nu_i}(k_i x^{\beta_i}),\ x^{\alpha_i} I_{\nu_i}(k_i x^{\beta_i}) \quad i = 1, 2 \tag{7}$$

Where $\alpha_i = (1 - A_i)/2$, $\beta_i = C_i/2$, $k_i = 2\sqrt{B_i}/C_i$, $\nu_i = (1 - A_i)/C_i$, and $I_{\nu_i}(\cdot)$, $K_{\nu_i}(\cdot)$ are modified Bessel functions of order n [10].

Step 2. Guide function $\varphi_{0,0}^{i}\left(x,\varepsilon\right)$ is defined by $x^{\alpha_i}K_{v_i}(k_i x^{\beta_i})$ and $x^{\alpha_i}I_{v_i}(k_i x^{\beta_i})$:

$$\varphi_{0,0}^{i}(x,\varepsilon)=(x\varepsilon)^{\alpha_i}\begin{bmatrix}K_{v_i}(k_i x^{\beta_i})\cdot I_{v_i}(k_i\varepsilon^{\beta_i})\\ +I_{v_i}(k_i x^{\beta_i})K_{v_i}(k_i\varepsilon^{\beta_i})\end{bmatrix}. \quad (8)$$

If we define

$$\psi_{h,l}(x,\varepsilon,t)=K_h(xt)I_l(\varepsilon t)+(-1)^{h-l+1}I_h(xt)K_l(\varepsilon t) \quad (9)$$

where h,l are real numbers, $\varphi_{0,0}^{i}\left(x,\varepsilon\right)$ can be rewritten as

$$\varphi_{0,0}^{i}(x,\varepsilon)=(x\varepsilon)^{\alpha_i}\psi_{v_i,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i). \quad (10)$$

Then, calculate respectively the partial derivative of $\varphi_{0,0}^{i}\left(x,\varepsilon\right)$ for x, ε:

$$\begin{aligned}\varphi_{1,0}^{i}(x,\varepsilon)&=\frac{\partial}{\partial x}\varphi_{0,0}^{i}(x,\varepsilon)\\ &=(x\varepsilon)^{\alpha_i}\begin{bmatrix}\dfrac{\alpha_i-\beta_i v_i}{x}\psi_{v_i,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\\ -k_i\beta_i x^{\beta_i-1}\psi_{v_i-1,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\end{bmatrix}\end{aligned} \quad (11)$$

$$\begin{aligned}\varphi_{0,1}^{i}(x,\varepsilon)&=\frac{\partial}{\partial \varepsilon}\varphi_{0,0}^{i}(x,\varepsilon)\\ &=(x\varepsilon)^{\alpha_i}\begin{bmatrix}\dfrac{\alpha_i-\beta_i v_i}{\varepsilon}\psi_{v_i,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\\ +k_i\beta_i \varepsilon^{\beta_i-1}\psi_{v_i,v_i-1}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\end{bmatrix}\end{aligned} \quad (12)$$

$$\begin{aligned}\varphi_{1,1}^{i}(x,\varepsilon)&=\frac{\partial^2}{\partial x\partial \varepsilon}\varphi_{0,0}^{i}(x,\varepsilon)\\ &=(x\varepsilon)^{\alpha_i}\left\{\begin{array}{l}\dfrac{\alpha_i-\beta_i v_i}{\varepsilon}\begin{bmatrix}\dfrac{\alpha_i-\beta_i v_i}{x}\psi_{v_i,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\\ -k_i\beta_i x^{\beta_i-1}\psi_{v_i-1,v_i}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\end{bmatrix}\\ +k_i\beta_i\varepsilon^{\beta_i-1}\begin{bmatrix}\dfrac{\alpha_i-\beta_i v_i}{x}\psi_{v_i,v_i-1}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\\ -k_i\beta_i x^{\beta_i-1}\psi_{v_i-1,v_i-1}(x^{\beta_i},\varepsilon^{\beta_i},k_i)\end{bmatrix}\end{array}\right\}\end{aligned} \quad (13)$$

Step 3. The similar kernel function of right region is defined, as follows:

$$\Phi_2(x)=\frac{G\varphi_{0,0}^{2}(x,b)+H\varphi_{0,1}^{2}(x,b)}{G\varphi_{1,0}^{2}(c,b)+H\varphi_{1,1}^{2}(c,b)} \quad (c<x<b). \quad (14)$$

The similar kernel function of left region is defined, as follows:

$$\Phi_1(x)=\frac{m\Phi_2(c)\varphi_{0,1}^{1}(x,c)-n\varphi_{0,0}^{1}(x,c)}{m\Phi_2(c)\varphi_{1,1}^{1}(a,c)-n\varphi_{1,0}^{1}(a,c)} \quad (a<x<c).(15)$$

Step 4. Obtaining the similar structure formula of solutions, they are

$$y_1=D\cdot\frac{1}{E+\dfrac{1}{F+\Phi_1(a)}}\cdot\frac{1}{F+\Phi_1(a)}\cdot\Phi_1(x) \quad (16)$$

$$\begin{aligned}y_2=D\cdot&\frac{1}{E+\dfrac{1}{F+\Phi_1(a)}}\cdot\frac{1}{F+\Phi_1(a)}\\ &\cdot\frac{\varphi_{0,1}^{1}(c,c)}{m\Phi_2(c)\varphi_{1,1}^{1}(a,c)-n\varphi_{1,0}^{1}(a,c)}\cdot\Phi_2(x)\end{aligned} \quad (17)$$

3. The SSM for Solving the Radial Seepage Model of Fractal Composite Reservoir with Double-porosity.

The dimensionless mathematical model of fractal composite with double-porosity with radial seepage in Appendix A. Taking the Laplace transform for the dimensionless mathematical model as follow:

$$\overline{p}_{ijD}(r_D,z)=\int_0^{\infty}e^{-zt_D}p_{ijD}(r_D,t_D)dt_D,\ i=1,2\ \ j=f,m;$$

$$\overline{p}_{wD}(z)=\int_0^{\infty}e^{-zt_D}p_{wD}(t_D)dt_D;$$

$$\overline{q}_D(z)=\int_0^{\infty}e^{-zt_D}q_D(t_D)dt_D.$$

We can write the model in Laplace space as follows:
The basic seepage equations:

$$\frac{d^2\overline{p}_{1fD}}{dr_D^{\,2}}+\frac{\gamma_1}{r_D}\frac{d\overline{p}_{1fD}}{dr_D}-X_1(z)r_D^{\theta_{1f}}\,\overline{p}_{1fD}=0 \quad (18)$$
$$1<r_D<\beta$$

$$\overline{p}_{1mD}=\frac{\lambda_1}{(1-\omega)zr_D^{\theta_{1m}}+\lambda_1}\overline{p}_{1fD} \qquad 1<r_D<\beta \quad (19)$$

$$\frac{d^2\overline{p}_{2fD}}{dr_D^{\,2}}+\frac{\gamma_2}{r_D}\frac{d\overline{p}_{2fD}}{dr_D}-X_2(z)r_D^{\theta_{2f}}\,\overline{p}_{2fD}=0 \quad (20)$$
$$R_D>r_D>\beta$$

$$\overline{p}_{2mD}=\frac{\lambda_2}{\sigma_m zr_D^{\theta_{2m}}+\lambda_2}\overline{p}_{2fD} \qquad R_D>r_D>\beta \quad (21)$$

$$\left.\begin{bmatrix}-C_Dz\overline{p}_{1fD}(r_D,z)\\ +(1+C_Dz\cdot S)\cdot\dfrac{d\overline{p}_{1fD}(r_D,z)}{dr_D}\end{bmatrix}\right|_{r_D=1}=-\overline{q}_D \quad (22)$$

$$\begin{aligned}&\overline{p}_{1fD}(\beta,z)=\overline{p}_{1fD}(\beta,z),\\ &\left.\frac{d\overline{p}_{1fD}(r_D,z)}{dr_D}\right|_{r_D=\beta}\\ &=\gamma_f\beta^{\beta22-\beta21}\left.\frac{d\overline{p}_{2fD}(r_D,z)}{dr_D}\right|_{r_D=\beta}\end{aligned} \quad (23)$$

$$\overline{p}_{2fD}(\infty,z)=0 \text{ or } \left.\frac{d\overline{p}_{2fD}(r_D,z)}{dr_D}\right|_{r_D=R_D}=0 \quad (24)$$
$$\text{or } \overline{p}_{2fD}(R_D,z)=0$$

Where:

$$X_1(z) = z[\omega + \frac{\lambda_1(1-\omega)r_D^{D^*_{1m}-D^*_{1f}}}{\lambda_1 + (1-\omega)r_D^{\theta_{1m}} z}] \tag{25}$$

$$X_2(z) = \frac{\sigma_f \sigma_m r_D^{\theta_{2m}} z + (\sigma_f + \sigma_m r_D^{D^*_{2m}-D^*_{2f}})\lambda_2}{\sigma_m z r_D^{\theta_{2m}} + \lambda_2} z \tag{26}$$

Equations (18), (20), (22), (23) and (24) comparing with (1)~(6) result in

$$y_i = \overline{p}_{ifD}; x = r_D; A_i = \gamma_i; B_i = X_i(z);$$
$$E = -C_D z; F = -S; D = -\overline{q}_D; m = 1;$$
$$n = \gamma_f \beta^{\beta_{22}-\beta_{21}}; a = 1; c = \beta; i = 1, 2.$$

$G = 1$, $H = 0$, $b = \infty$ or $G = 1$, $H = 0$, $b = R_D$ or $G = 0$, $H = 1$, $b = R_D$ representing there types of outer boundaries, respectively. So we can construct similar structure formula of dimensionless reservoir's pressure in Laplace space, as follows:

$$\overline{p}_{1fD}(r_D, z) = \overline{q}_D \frac{1}{C_D z - \frac{1}{\Phi_1(1,z) - S}} \frac{1}{\Phi_1(1,z) - S} \Phi_1(r_D, z) \tag{27}$$

$$\overline{p}_{2fD}(r_D, z) = \overline{q}_D \frac{1}{C_D z - \frac{1}{\Phi_1(1,z) - S}} \frac{1}{\Phi_1(1,z) - S} \cdot \frac{\beta^{\frac{1-\gamma_1+\theta_{1f}}{2}} \psi_{v_1, v_1-1}(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2})}{\left[\begin{array}{l} \gamma_f \beta^{\beta_{22}-\beta_{21}} \psi_{v_1-1, v_1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \\ -\Phi_2(\beta, z)\sqrt{X_1}\beta^{\frac{\theta_{1f}}{2}} \\ \cdot \psi_{v_1-1, v_1-1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \end{array}\right]} \cdot \Phi_2(r_D, z) \tag{28}$$

From (19) and (21), we can obtain

$$\overline{p}_{1mD}(r_D, z) = \frac{\lambda_1}{(1-\omega) z r_D^{\theta_{1m}} + \lambda_1} \overline{q}_D \cdot \frac{1}{C_D z - \frac{1}{\Phi_1(1,z) - S}} \frac{1}{\Phi_1(1,z) - S} \Phi_1(r_D, z) \tag{29}$$

$$\overline{p}_{2mD}(r_D, z) = \frac{\lambda_2}{\sigma_m z r_D^{\theta_{2m}} + \lambda_2} \cdot \overline{q}_D \frac{1}{C_D z - \frac{1}{\Phi_1(1,z) - S}} \frac{1}{\Phi_1(1,z) - S} \cdot \frac{\beta^{\frac{1-\gamma_1+\theta_{1f}}{2}} \psi_{v_1, v_1-1}(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2})}{\left[\begin{array}{l} \gamma_f \beta^{\beta_{22}-\beta_{21}} \psi_{v_1-1, v_1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \\ -\Phi_2(\beta, z)\sqrt{X_1}\beta^{\frac{\theta_{1f}}{2}} \\ \cdot \psi_{v_1-1, v_1-1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \end{array}\right]} \cdot \Phi_2(r_D, z) \tag{30}$$

Taking the Laplace transform about t_D for (A.4), and using (27), the dimensionless well-bore's pressure can be obtained in Laplace space, as follow:

$$\overline{p}_{wD}(z) = \overline{q}_D \frac{1}{C_D z - \frac{1}{\Phi_1(1,z) - S}} \tag{31}$$

Where $\Phi_2(r_D, z)$ is outer region similar kernel function, and $\Phi_1(r_D, z)$ is inner region similar kernel function. They are the following.

For $\overline{p}_{2fD}(\infty, z) = 0$, using the properties of modified Bessel function $\lim\limits_{x\to\infty} I_v(x) = \infty$, $\lim\limits_{x\to\infty} K_v(x) = 0$,(14) and (15), we can obtain the similar kernel function as follows:

$$\Phi_2(r_D, z) = \frac{r_D^{\frac{1-\gamma_2}{2}} K_{v_2}(\frac{2\sqrt{X_2}}{\theta_{2f}+2} r_D^{\frac{\theta_{2f}+2}{2}})}{-\sqrt{X_2}\beta^{\frac{\theta_{2f}+1-\gamma_2}{2}} K_{v_2-1}(\frac{2\sqrt{X_2}}{\theta_{2f}+2}\beta^{\frac{\theta_{2f}+2}{2}})} \tag{32}$$

$$\Phi_1(r_D, z) = -\frac{r_D^{\frac{1-\gamma_1}{2}}}{\sqrt{X_1}} \cdot \frac{\left[\begin{array}{l} \Phi_{21}(\beta)\sqrt{X_1} r_D^{\frac{\theta_{1f}}{2}} \psi_{v_1, v_1-1}(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \\ -\gamma_f \beta^{\beta_{22}-\beta_{21}} \psi_{v_1, v_1}(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \end{array}\right]}{\left[\begin{array}{l} \Phi_{21}(\beta)\sqrt{X_1}\beta^{\frac{\theta_{1f}}{2}} \psi_{v_1-1, v_1-1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \\ -\gamma_f \beta^{\beta_{22}-\beta_{21}} \psi_{v_1-1, v_1}(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}) \end{array}\right]} \tag{33}$$

For $\left.\frac{d\overline{p}_{2fD}(r_D, z)}{dr_D}\right|_{r_D = R_D} = 0$,

$$\Phi_2(r_D,z) = \frac{r_D^{\frac{1-\gamma_2}{2}}\psi_{\nu_2,\nu_2}\left(r_D^{\frac{\theta_{2f}+2}{2}}, R_D^{\frac{\theta_{2f}+2}{2}}, \frac{2\sqrt{X_2}}{\theta_{2f}+2}\right)}{-\sqrt{X_2}\beta^{\frac{\theta_{2f}+1-\gamma_2}{2}}\psi_{\nu_2-1,\nu_2}\left(\beta^{\frac{\theta_{2f}+2}{2}}, R_D^{\frac{\theta_{2f}+2}{2}}, \frac{2\sqrt{X_2}}{\theta_{2f}+2}\right)} \tag{34}$$

$$\Phi_1(r_D,z) = -\frac{r_D^{\frac{1-\gamma_1}{2}}}{\sqrt{X_1}} \cdot \frac{\left[\begin{array}{l}\Phi_2(\beta)\sqrt{X_1}r_D^{\frac{\theta_{1f}}{2}}\psi_{\nu_1,\nu_1-1}\left(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right) \\ -\gamma_f\beta^{\beta_{22}-\beta_{21}}\psi_{\nu_1,\nu_1}\left(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right)\end{array}\right]}{\left[\begin{array}{l}\Phi_2(\beta)\sqrt{X_1}\beta^{\frac{\theta_{1f}}{2}}\psi_{\nu_1-1,\nu_1-1}\left(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right) \\ -\gamma_f\beta^{\beta_{22}-\beta_{21}}\psi_{\nu_1-1,\nu_1}\left(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right)\end{array}\right]} \tag{35}$$

For $\overline{p}_{2fD}(R_D,z)=0$,

$$\Phi_2(r_D,z) = \frac{r_D^{\frac{1-\gamma_2}{2}}\psi_{\nu_2,\nu_2-1}\left(r_D^{\frac{\theta_{2f}+2}{2}}, R_D^{\frac{\theta_{2f}+2}{2}}, \frac{2\sqrt{X_2}}{\theta_{2f}+2}\right)}{-\sqrt{X_2}\beta^{\frac{\theta_{2f}+1-\gamma_2}{2}}\psi_{\nu_2-1,\nu_2-1}\left(\beta^{\frac{\theta_{2f}+2}{2}}, R_D^{\frac{\theta_{2f}+2}{2}}, \frac{2\sqrt{X_2}}{\theta_{2f}+2}\right)} \tag{36}$$

$$\Phi_1(r_D,z) = -\frac{r_D^{\frac{1-\gamma_1}{2}}}{\sqrt{X_1}} \cdot \frac{\left[\begin{array}{l}\Phi_2(\beta)\sqrt{X_1}r_D^{\frac{\theta_{1f}}{2}}\psi_{\nu_1,\nu_1-1}\left(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right) \\ -\gamma_f\beta^{\beta_{22}-\beta_{21}}\psi_{\nu_1,\nu_1}\left(r_D^{\frac{\theta_{1f}+2}{2}}, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right)\end{array}\right]}{\left[\begin{array}{l}\Phi_2(\beta)\sqrt{X_1}\beta^{\frac{\theta_{1f}}{2}}\psi_{\nu_1-1,\nu_1-1}\left(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right) \\ -\gamma_f\beta^{\beta_{22}-\beta_{21}}\psi_{\nu_1-1,\nu_1}\left(1, \beta^{\frac{\theta_{1f}+2}{2}}, \frac{2\sqrt{X_1}}{\theta_{1f}+2}\right)\end{array}\right]} \tag{37}$$

Where

$$\nu_i = (1-\gamma_i)/(\theta_{if}+2), \quad i=1,2.$$

4. Conclusions

For the radial seepage model of fractal composite reservoir with double-porosity, introducing dimensionless variables, using the Laplace transform, structuring similarity kernel functions (32)~(37). Then we can obtain the model's solutions by using SSM, and know the solutions of dimensionless reservoir's pressure (27)~(30), dimensionless well-bore's pressure (31) under three outer boundary conditions (infinite, closed, constant pressure) have similar formula in Laplace space. The formulas can clearly shows that well-bore storage coefficient and skin factor generated influence for reservoir pressure and bottom hole pressure.

Appendices

A. Mathematical Model of Fractal Composite Reservoir with Double-porosity with Radial Seepage.

In order to study the radial seepage model of fractal composite reservoir with double-porosity, the main assumptions for fluid flow in porous media as follows: Ignoring the influence of quadratic pressure gradient term, gravity and capillary pressure; Fluids are of low compressibility and single-phase; The flow follows Darcy's law; Radial seepage which has one production; Both consider the well-bore storage and the skin factor.

The distributions of the porosity and permeability of inner and outer region of reservoir are defined as follows [1]:

$$\varphi_{ij} = \varphi_{wij}\left(\frac{r}{r_w}\right)^{D^*_{ij}-d},$$

$$K_{ij} = K_{wij}\left(\frac{r}{r_w}\right)^{D^*_{ij}-\theta_{ij}-d} \quad (i=1,2;\ j=f,m)$$

where D^*, d, θ, r and r_w are fractal dimension, Euclidean dimension, fractal index, radial distance, and well radius respectively.

The radial seepage model of fractal composite reservoir with double-porosity is obtained:

The basic seepage equation:

$$\begin{cases} \dfrac{\partial^2 p_{1f}}{\partial r^2} + \dfrac{D^*_{1f}-\theta_{1f}-d+1}{r}\dfrac{\partial p_{1f}}{\partial r} \\ +\alpha_1\dfrac{K_{w1m}}{K_{w1f}}\left(\dfrac{r}{r_w}\right)^{(D^*_{1m}-D^*_{1f})-(\theta_{1m}-\theta_{1f})}(p_{1m}-p_{1f}) \\ =\dfrac{\mu_1\varphi_{w1f}C_{t1f}}{K_{w1f}}\left(\dfrac{r}{r_w}\right)^{\theta_{1f}}\dfrac{\partial p_{1f}}{\partial t}, \quad r_w<r<\beta r_w, t>0; \\ \dfrac{\varphi_{w1f}C_{t1m}}{K_{w1f}}\left(\dfrac{r}{r_w}\right)^{\theta_{1m}}\dfrac{\partial p_{1f}}{\partial t}+\dfrac{\alpha_1 K_{w1m}}{\mu_1}(p_{1m}-p_{1f})=0, \\ \qquad r_w<r<\beta r_w, t>0; \\ \dfrac{\partial^2 p_{2f}}{\partial r^2} + \dfrac{D^*_{2f}-\theta_{2f}-d+1}{r}\dfrac{\partial p_{2f}}{\partial r} \\ +\alpha_1\dfrac{K_{w1m}}{K_{w1f}}\left(\dfrac{r}{r_w}\right)^{(D^*_{2m}-D^*_{2f})-(\theta_{2m}-\theta_{2f})}(p_{2m}-p_{2f}) \\ =\dfrac{\mu_2\varphi_{w2f}C_{t2f}}{K_{w2f}}\left(\dfrac{r}{r_w}\right)^{\theta_{2f}}\dfrac{\partial p_{2f}}{\partial t}, \quad R>r>\beta r_w, t>0; \\ \dfrac{\varphi_{w2f}C_{t2m}}{K_{w2f}}\left(\dfrac{r}{r_w}\right)^{\theta_{2m}}\dfrac{\partial p_{2f}}{\partial t}+\dfrac{\alpha_2 K_{w2m}}{\mu_2}(p_{2m}-p_{2f})=0, \\ \qquad R>r>\beta r_w, t>0. \end{cases}$$

Initial condition:

$$p_{1f}(r,0) = p_{1m}(r,0) = p_{2f}(r,0) = p_{2m}(r,0) = p_0;$$

Inner boundary condition:

$$p_w = [p_{1f} - Sr\frac{\partial p_{1f}}{\partial r}]\Big|_{r=r_w},$$

$$(r^{D^*_{1f}-\theta_{1f}-d+1}\frac{\partial p_{1f}}{\partial r})\Big|_{r=r_w} = \frac{\mu_1}{2\pi K_{1f}h}(Bq + C\frac{dp_w}{dt});$$

Convergence condition:

$$p_{1f}(\beta r_w,t) = p_{2f}(\beta r_w,t),$$

$$\frac{K_{w1f}}{\mu_1}(\frac{r}{r_w})^{D^*_{1f}-\theta_{1f}-d}\frac{\partial p_{1f}}{\partial r}\Big|_{r=\beta r_w} = \frac{K_{w2f}}{\mu_2}(\frac{r}{r_w})^{D^*_{2f}-\theta_{2f}-d}\frac{\partial p_{2f}}{\partial r}\Big|_{r=\beta r_w};$$

Outer boundary condition:

1). Infinite pressure outer boundary condition:

$$p_{2f}(\infty,t) = p_{2m}(\infty,t) = p_0;$$

2). Closed pressure outer boundary condition:

$$\frac{\partial p_{2f}}{\partial r}\Big|_{r=R} = \frac{\partial p_{2m}}{\partial r}\Big|_{r=R} = 0;$$

3). Constant pressure outer boundary condition:

$$p_{2f}(R,t) = p_{2m}(R,t) = p_0.$$

The following dimensionless parameters can be defined to simplify the formulation:

$$p_{ijD} = \frac{2\pi K_{w1f}h}{Bq_e\mu_1} = [p_0 - p_{ij}(r,t)];$$

$$p_{wD} = \frac{2\pi K_{w1f}h}{Bq_e\mu_1}[p_0 - p_w(t)];$$

$$q_D = \frac{q(t)}{q_e}; r_D = \frac{r}{r_w}; R_D = \frac{R}{r_w};$$

$$\gamma_f = [\frac{K_{w2f}}{\mu_2}]/[\frac{K_{w1f}}{\mu_1}];$$

$$t_D = \frac{K_{w1f}t}{[\varphi_{w1f}C_{t2f} + \varphi_{w1m}C_{t2m}]\mu_1 r_w^2};$$

$$C_D = \frac{C}{\delta\pi[\varphi_{w1f}C_{t2f} + \varphi_{w1m}C_{t2m}]hr_w^2};$$

$$\lambda_i = \alpha_i r_w^2\frac{K_{wim}}{K_{wif}}; \omega = \frac{\varphi_{w1f}C_{t2f}}{\varphi_{w1f}C_{t1f} + \varphi_{w1m}C_{t1m}};$$

$$\sigma_f = \frac{K_{w1f}}{K_{w2f}}\frac{\mu_2\varphi_{w2f}C_{t2f}}{\mu_1[\varphi_{w1f}C_{t1f} + \varphi_{w1m}C_{t1m}]r_w^2};$$

$$\sigma_m = \frac{K_{w1f}}{K_{w2f}}\frac{\mu_2\varphi_{w2m}C_{t2m}}{\mu_1[\varphi_{w1f}C_{t1f} + \varphi_{w1m}C_{t1m}]r_w^2};$$

$$\beta_{i1} = D^*_{if} - \theta_{if}; \beta_{i2} = D^*_{im} - \theta_{im};$$

$$\gamma_i = D^*_{if} - \theta_{if} - d + 1; (i = 1,2; j = f,m).$$

The dimensionless radial seepage model of fractal composite reservoir with double-porosity is obtained:

The basic seepage equations:

$$\begin{cases} \frac{\partial^2 p_{1fD}}{\partial r_D^2} + \frac{\gamma_1}{r_D}\frac{\partial p_{1fD}}{\partial r_D} + \lambda_1 r_D^{\beta_{12}-\beta_{11}}(p_{1mD} - p_{1fD}) \\ = \omega r_D^{\theta_{1f}}\frac{\partial p_{1fD}}{\partial t_D}, \\ \quad 1 < r_D < \beta,\ t_D > 0\ ; \\ (1-\omega)r_D^{\theta_{1m}}\frac{\partial p_{1mD}}{\partial t_D} + \lambda_1(p_{1mD} - p_{1fD}) = 0, \\ \quad 1 < r_D < \beta,\ t_D > 0\ ; \\ \frac{\partial^2 p_{2fD}}{\partial r_D^2} + \frac{\gamma_2}{r_D}\frac{\partial p_{1fD}}{\partial r_D} + \lambda_1 r_D^{\beta_{22}-\beta_{21}}(p_{2mD} - p_{2fD}) \\ = \sigma_f r_D^{\theta_{2f}}\frac{\partial p_{2fD}}{\partial t_D}, \\ \quad R_D > r_D > \beta,\ t_D > 0\ ; \\ \sigma_m r_D^{\theta_{2m}}\frac{\partial p_{2f}}{\partial t} + \lambda_2(p_{2m} - p_{2f}) = 0, \\ \quad R_D > r_D > \beta,\ t_D > 0. \end{cases} \quad (A.1)$$

Initial conditions:

$$p_{1fD}(r_D,0) = p_{1mD}(r_D,0) = p_{2fD}(r_D,0) = p_{2mD}(r_D,0) = 0; \quad (A.2)$$

Inner boundary conditions:

$$p_{wD}(t_D) = [p_{1fD} - Sr_D\frac{\partial p_{1fD}}{\partial r_D}]\Big|_{r_D=1}, \quad (A.3)$$

$$(r_D^{\gamma_1}\frac{\partial p_{1fD}}{\partial r_D})\Big|_{r_D=1} = -[q_D(t_D) - C_D\frac{dp_{wD}}{dt_D}]; \quad (A.4)$$

Convergence conditions:

$$p_{1fD}(\beta,t_D) = p_{2fD}(\beta,t_D),$$

$$\frac{\partial p_{1fD}}{\partial r_D}\Big|_{r_D=\beta} = \gamma_f\beta^{\beta_{22}-\beta_{21}}\frac{\partial p_{2fD}}{\partial r_D}\Big|_{r_D=\beta}; \quad (A.5)$$

Outer boundary conditions:

1). Infinite outer boundary conditions:

$$P_{2fD}(\infty,t_D) = P_{2mD}(\infty,t_D) = 0; \quad (A.6)$$

2). Closed outer boundary conditions:

$$\frac{\partial p_{2fD}}{\partial r_D}\Big|_{r_D=R_D} = \frac{\partial p_{2mD}}{\partial r_D}\Big|_{r_D=R_D} = 0; \quad (A.7)$$

3). Constant pressure outer boundary conditions:

$$P_{2fD}(R_D,t_D) = P_{2mD}(R_D,t_D) = 0. \quad (A.8)$$

Nomenclature

p :Reservoir pressure, MPa ; p_0 :Initial reservoir pressure, MPa ; C :Well-bore storage coefficient, $\mathrm{m}^3/\mathrm{MPa}$; S :Skin factor; C_t :Total compressibility, 1/MPa ; B :Formation volume factor, $\mathrm{m}^3/\mathrm{m}^3$; K : Permeability, $\mu\mathrm{m}^2$; φ :Porosity, % ; μ : Viscosity, $\mathrm{mPa \cdot s}$; r :Radial distance, m ; R :Radial distance of the outer boundary, m ; q :Production rate or injection rate, m^3/d ; t :Time, h ; β :The internal and external interface radius; $D*$:Fractal dimension; d :Euclid dimension; θ :Fractal index; z : Laplace space variable.

Superscript

$\bar{\ }$: Laplace domain.

Subscript

$i = 1$:Inner region; $i = 2$:Outer region; $j = f$:Fissure media; $j = m$:Matrix block; w :Well-bore; we :Well-bore effective; D :Dimensionless.

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