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Some Fixed Point Theorems of Semi Compatible and Occasionally Weakly Compatible Mappings in Menger Space

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Abstract The notion of semi compatible mappings was introduced by Cho. Sharma and Sahu (Semicompatibility and fixed points, *Math. Japon,* 42(1), 1995, 91-98) and the notion of occationally weakly compatible mappings was introduced by Al-Thagafi M. A., Shahzad N. (Generalized I-non expansive selfmaps and invariant approximations, *Acta. Math. Sinica* (*English series*) 24(5), 2008, 867-876). In this paper, we prove a common fixed point theorem in Menger space using the concept of semi compatible and occasionally weakly compatible mappings. Some results are also given as corollaries. Our results generalise some similar results in the literature.

Keywords: *common fixed point, compatible maps, menger space, probabilistic metric space, semi-compatible, weakly compatible, occasionally weakly compatible*

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1. Introduction

Many researchers have been generalising the notion of metric space in different ways and Menger space is one of such generalisations introduced by the great mathematician Karl Menger [1] in the year 1942 who used distribution functions instead of non-negative real numbers as the value of metric. Schweizer and Sklar [4] studied this concept and gave some fundamental results on this space. In 1972, Sehgal and Bharucha-Reid [2] obtained a generalization of Banach Contractive Principle on a complete Menger space which is a milestone in developing fixed point theory in Menger space.

In 1982, Sessa [3] improved the definition of commutativity in fixed point theorems by introducing the notion of weakly commuting maps. Then in 1986, Jungck [5] introduced the concept of compatible maps and this notion of compatible mappings in Menger space was introduced by Mishra [6]. Further this condition has been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades [8]. Recently, Singh and Jain [10] introduced weakly compatible maps in Menger space to establish a common fixed point theorem.

Al. Thagafi and Shahzad [13] introduced the notion of occasionally weakly compatible mappings in metric space which is more general than weakly compatible mappings. Recently, Jungck and Rhoades [11] extensively studied the notion occasionally weakly compatible mappings in semi-metric space and Chauhan *et.al.* [13] extended the notion of occassionally weakly compatible mappings to PM-space.

Cho, Sharma and Sahu [7] introduced the concept of semi-compatibility in a d-complete topological space and using this concept of semi compatibility in Menger space, Singh *et.al.* [9] proved a fixed point theorem using implicit relation. Recently, Rohen and Chhatrajit [16] used the concept of semi compatible mappings in cone metric space to prove some common fixed point theorems.

In this paper, we prove a common fixed point theorem in Menger space using the concept of semi compatible and occasionally weakly compatible mappings. Some results are also given as corollaries. Our results generalise some similar results [12,15,16].

2. Preliminaries

Definition 2.1 A triangular norm * (shortly *t*-norm) is a binary operation on the unit interval [0, 1] such that for all $a, b, c, d \in [0, 1]$ the following conditions are satisfied:

i) $a * 1 = a$;
ii) $a * b = b * a$;
ii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;
iv) $a * (b * c) = (a * b) * c$.

Example: $a * b = \min\{a, b\}$.

Definition 2.2 A distribution function is a function $F: [-\infty,\infty] \to [0,1]$ which is left continuous on **R,** non-decreasing and $F(-\infty) = 0$, $F(\infty) = 1$.

We will denote the family of all distribution functions on $[-\infty,\infty]$ by Δ. H is a special element of Δ defined by

$$H(t) = \begin{cases} 0 \text{ if } t \le 0 \\ 1 \text{ if } t > 0 \end{cases}$$

If X is a non-empty set, $F : X \times X \to \Delta$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by F_{xy}.

Definition 2.3 (Schweizer and Sklar [4]): The ordered pair (X, F) is called a probabilistic metric space (shortly PM-space) if X is a nonempty set and F is a probabilistic distance satisfying the following conditions:

(i) $F_{xy}(t) = 1 \Leftrightarrow x = y$;

(ii) $F_{xy}(0) = 0$;

(iii) $F_{xy} = F_{yx}$;

(iv) $F_{xz}(t) = 1$, $F_{zy}(s) = 1 \Rightarrow F_{xy}(t+s) = 1$.

The ordered triplet $(X, F, *)$ is called Menger space if (X, F) is a PM-space, * is a t-norm and the following condition is also satisfied i.e.

(v) $F_{xy}(t+s) \ge F_{xz}(t) * F_{zy}(s)$.

Proposition 2.4 (Sehgal and Bharucha-Reid [2]) Let (X, d) be a metric space. Then the metric d induces a distribution function F defined by $F_{xy}(t) = H(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. If t-norm * is $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$ then $(X, F, *)$ is a Menger space. Further, $(X, F, *)$ is a complete Menger space if (X, d) is complete.

Definition 2.5 (Mishra [6]) Let $(X, F, *)$ be a Menger space and * be a continuous t – norm.

i) A sequence $\{x_n\}$ in X is said to converge to a point x in X (written as $x_n \to x$) iff for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists an integer $n_0 = n_0(\varepsilon, \lambda)$ such that $Fx_nx(\varepsilon) > 1 - \lambda$ for all $n \ge n_0$.

ii) A sequence $\{x_n\}$ in X is said to be Cauchy if for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists an integer $n_0 = n_0(\varepsilon, \lambda)$ such that $Fx_nx_{n+p}(\varepsilon) > 1 - \lambda$ for all $n \ge n_0$ and $p > 0$.

iii) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.6 If * is a continuous t- norm, it follows from definition 2.3 (v) that the limit of sequence in Menger space is uniquely determined.

Definition 2.7 (Mishra[6]) Two self-maps S and T of a Menger space $(X, F, *)$ are said to be compatible if $F_{STx_nTSx_n}(t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that Sx_n, $Tx_n \to x$ for some x in X as $n \to \infty$.

Definition 2.8 (Singh and Jain [10]) Two self-maps S and T of a Menger space $(X, F, *)$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincident points i.e. if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

Definition 2.9(Al Thagafi and Shahzad [12]) Two self-maps S and T of a Menger space $(X, F, *)$ are said to be occasionally weakly compatible (owc) if and only if S and T commute at their coincidence point.

Definition 2.10 (Singh B. and Jain S.[10]) Two self-maps S and T of a Menger space $(X, F, *)$ are said to be semi compatible if $F_{STx_nTt}(x) \to 1$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that Sx_n, $Tx_n \to t$, for some t in X, as $n \to \infty$.

Lemma 2.11(Singh B. and Jain S. [10]) Let $\{x_n\}$ be a sequence in a Menger space $(X, F, *)$ with continuous t-norm * and $t * t \ge t$. If there exists a constant $k \in (0,1)$ such that $Fx_nx_{n+1}(kt) \ge Fx_{n-1}x_n(t)$ for all $t > 0$ and $n = 1, 2, 3, \ldots$ then $\{x_n\}$ is a Cauchy sequence in X.

3. Main Results

Theorem 3.1 Let A, B, S, T, L and M be self-maps on a complete Menger space $(X, F, *)$ with $t*t \ge t$ for all $t \in [0,1]$, satisfying:

i) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

ii) There exists a constant $k \in (0,1)$ such that

$$F^2_{LxMy}(kt) * [F_{ABxLx}(kt).F_{STyMy}(kt)]$$
$$\ge [pF_{ABxLx}(t) + qF_{ABxSTy}(t)].F_{ABxMy}(2kt)$$

for all $x, y \in X$ and $t > 0$ where $0 < p, q < 1$ such that $p + q = 1$;

iii) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

iv) Either AB or L is continuous;

v) The pair (L, AB) is semi compatible and (M, ST) is occasionally weakly compatible.

Then A, B, S, T, L and M have a unique common fixed point.

Proof: Let us choose an arbitrary point x_0 in X then by (i), there exist x_1, $x_2 \in X$ such that $Lx_0 = STx_1 = y_0$ and $Mx_1 = ABx_2 = y_1$. By induction we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Lx_{2n} = STx_{2n+1} = y_{2n}$ and $Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for $n = 0, 1, 2, 3\ldots$

By (ii), we have

$$F^2_{Lx_{2n}Mx_{2n+1}}(kt) * [F_{ABx_{2n}Lx_{2n}}(kt) F_{STx_{2n+1}Mx_{2n+1}}(kt)]$$
$$\ge \left[pF_{ABx_{2n}Lx_{2n}}(t) + qF_{ABx_{2n}STx_{2n+1}}(t) \right] F_{ABx_{2n}Mx_{2n+1}}(2kt)$$
$$F^2_{y_{2n}y_{2n+1}}(kt) * [F_{y_{2n-1}y_{2n}}(kt) F_{y_{2n}y_{2n+1}}(kt)]$$
$$\ge \left[pF_{y_{2n}y_{2n-1}}(t) + qF_{y_{2n-1}y_{2n}}(t) \right] F_{y_{2n-1}y_{2n+1}}(2kt),$$
$$F_{y_{2n}y_{2n+1}}(kt) \left[F_{y_{2n-1}y_{2n}}(kt) * F_{y_{2n}y_{2n+1}}(kt) \right]$$
$$\ge (p+q) F_{y_{2n}y_{2n-1}}(t) F_{y_{2n-1}y_{2n+1}}(2kt),$$
$$F_{y_{2n}y_{2n+1}}(kt) F_{y_{2n-1}y_{2n+1}}(2kt)$$
$$\ge F_{y_{2n-1}y_{2n}}(t) F_{y_{2n-1}y_{2n+1}}(2kt)$$

Hence, we have

$$F_{y_{2n}y_{2n+1}}(kt) \ge F_{y_{2n-1}y_{2n}}(t)$$

Similarly, we also have

$$F_{y_{2n+1}y_{2n+2}}(kt) \ge F_{y_{2n}y_{2n+1}}(t)$$

In general, for all n even or odd, we have

$$F_{y_n y_{n+1}}(kt) \geq F_{y_{n-1} y_n}(t)$$

for $k \in (0, 1)$ and $t > 0$. Thus, by Lemma 2.11, $\{ y_n \}$ is a Cauchy sequence in X. Since $(X, F, *)$ is complete, it converges to a point z in X. Also $\{ Lx_{2n} \} \to z$, $\{ ABx_{2n} \} \to z$, $\{ Mx_{2n+1} \} \to z$ and $\{ STx_{2n+1} \} \to z$.

First, let AB be continuous then we have, $AB(AB)\, x_{2n} \to ABz$ and $(AB)\, Lx_{2n} \to ABz$. Since (L, AB) is semi compatible, we have $L(AB)\, x_{2n} \to ABz$.

Again, by (*ii*), we have

$$F^2_{L(AB)x_{2n}Mx_{2n+1}}(kt) * \begin{bmatrix} F_{AB(AB)x_{2n}L(AB)x_{2n}}(kt) \\ F_{STx_{2n+1}Mx_{2n+1}}(kt) \end{bmatrix}$$
$$\geq \left[pF_{AB(AB)x_{2n}L(AB)x_{2n}}(t) + qF_{AB(AB)x_{2n}STx_{2n+1}}(t) \right]$$
$$\times F_{AB(AB)x_{2n}Mx_{2n+1}}(2kt)$$

Letting $n \to \infty$ we have

$$F^2_{zABz}(kt) * \left[F_{ABzABz}(kt) F_{zz}(kt \right]$$
$$\geq \left[pF_{ABzABz}(t) + qF_{zABz}(t) \right] F_{zABz}(2kt)$$
$$\geq \left[p + qF_{zABz}(t) \right] F_{zABz}(kt)$$
$$F_{zABz}(kt) \geq p + qF_{zABz}(t) \geq p + qF_{zABz}(kt)$$
$$F_{zABz}(kt) \geq \frac{p}{1-q} = 1.$$

For $k \in (0, 1)$ and all $t > 0$. Thus, we have $z = ABz$.

Now by (ii), we have

$$F^2_{LzMx_{2n+1}}(kt) * \left[F_{ABzLz}(kt) F_{STx_{2n+1}Mx_{2n+1}}(kt) \right]$$
$$\geq \left[pF_{ABzLz}(t) + qF_{ABzSTx_{2n+1}}(t) \right] F_{ABzMx_{2n+1}}(2kt)$$

Letting $n \to \infty$ we have

$$F^2_{zLz}(kt) * \left[F_{zLz}(kt) F_{zz}(kt) \right]$$
$$\geq \left[pF_{zLz}(t) + qF_{zz}(t) \right] F_{zz}(2kt)$$
$$F^2_{zLz}(kt) F_{zLz}(kt) \geq pF_{zLz}(t) + q$$

Noting that $F^2_{zLz}(kt) \leq 1$ and using (iii) in definition 2.1, we have

$$F_{zLz}(kt) \geq pF_{zLz}(t) + q$$
$$\geq pF_{zLz}(kt) + q$$
$$F_{zLz}(kt) \geq \frac{q}{1-p} = 1 \text{ for } k\ (0,1) \text{ and all } t > 0.$$

Thus, we have $z = Lz = ABz$.

By (ii), we have

$$F^2_{L(Bz)Mx_{2n+1}}(kt) * [F_{AB(Bz)L(Bz)}(kt) F_{STx_{2n+1}Mx_{2n+1}}(kt)]$$
$$\geq \begin{bmatrix} pF_{AB(Bz)L(Bz)}(t) \\ +qF_{AB(Bz)STx_{2n+1}}(t) \end{bmatrix} F_{AB(Bz)Mx_{2n+1}}(2kt)$$

Since $AB=BA$ and $BL=LB$, we have $L(Bz)=B(Lz)=Bz$ and $AB(Bz)=B(ABz)=Bz$. Letting $n \to \infty$, we have

$$F^2_{zBz}(kt) * [F_{BzBz}(kt) F_{zz}(kt)]$$
$$\geq [pF_{BzBz}(t) + qF_{zBz}(t)] F_{zBz}(2kt)$$
$$F^2_{zBz}(kt) \geq [p + qF_{zBz}(t)] F_{zBz}(2kt)$$
$$\geq [p + qF_{zBz}(t)] F_{zBz}(kt)$$
$$F_{zBz}(kt) \geq p + qF_{zBz}(t) \geq p + qF_{zBz}(kt)$$
$$F_{zBz}(kt) \geq \frac{p}{1-q} = 1$$

For $k \in (0, 1)$ and all $t > 0$. Thus, we have $z = Bz$. Since $z = ABz$, we also have $z = Az$. Therefore, $z = Az = Bz = Lz$.

Since $L(X) \subseteq ST(X)$, there exists $v \in X$ such that $z = Lz = STv$. By (ii), we have

$$F^2_{Lx_{2n}Mv}(kt) * [F_{ABx_{2n}Lx_{2n}}(kt) F_{STvMv}(kt)]$$
$$\geq [pF_{ABx_{2n}Lx_{2n}}(t) + qF_{ABx_{2n}STv}(t)] F_{ABx_{2n}Mv}(2kt)$$

Letting $n \to \infty$ we have

$$F^2_{zMv}(kt) * [F_{zz}(kt) F_{zMv}(kt)]$$
$$\geq [pF_{zz}(t) + qF_{zz}(t)] F_{zMv}(2kt)$$
$$F^2_{zMv}(kt) * F_{zMv}(kt)$$
$$\geq (p+q) F_{zMv}(2kt)$$

Noting that $F^2_{zMv}(kt) \leq 1$ and using (iii) in definition 2.1, we have

$$F_{zMv}(kt) \geq F_{zMv}(2kt) \geq F_{zMv}(t)$$

Thus, by Lemma 2.11, we have $z = Mv$ and so $z = Mv = STv$. Since (M, ST) is occasionally weakly compatible, we have $STMv = MSTv$. Thus, $STz = Mz$.

By (ii), we have

$$F^2_{Lx_{2n}Mz}(kt) * \left[F_{ABx_{2n}Lx_{2n}}(kt) F_{STzMz}(kt) \right]$$
$$\geq \left[pF_{ABx_{2n}Lx_{2n}}(t) + qF_{ABx_{2n}STz}(t) \right] F_{ABx_{2n}Mz}(2kt)$$

Letting as $n \to \infty$ we have

$$F^2_{zMz}(kt) * \left[F_{zz}(kt) F_{MzMz}(kt) \right]$$
$$\geq \left[pF_{zz}(t) + qF_{zMz}(t) \right] F_{zMz}(2kt)$$
$$F^2_{zMz}(kt) \geq \left[p + qF_{zMz}(t) \right] F_{zMz}(2kt)$$
$$\geq \left[p + qF_{zMz}(t) \right] F_{zMz}(kt)$$
$$F_{zMz}(kt) \geq p + qF_{zMz}(t) \geq p + qF_{zMz}(kt)$$
$$F_{zMz}(kt) \geq \frac{p}{1-q} = 1$$

Thus, we have $z = Mz$ and therefore $z = Az = Bz = Lz = Mz = STz$.

By (ii), we have

$$F^2_{Lx_{2n}M(Tz)}(kt) * \left[F_{ABx_{2n}Lx_{2n}}(kt) F_{ST(Tz)M(Tz)}(kt) \right]$$
$$\geq \begin{bmatrix} pF_{ABx_{2n}Lx_{2n}}(t) \\ +qF_{ABx_{2n}ST(Tz)}(t) \end{bmatrix} F_{ABx_{2n}M(Tz)}(2kt)$$

Since $MT = TM$ and $ST = TS$, we have $MTz = TMz = Tz$ and $ST(Tz) = T(STz) = Tz$. Letting $n \to \infty$, we have

$$F^2_{zTz}(kt)*[F_{zz}(kt)F_{TzTz}(kt)]$$
$$\geq [pF_{zz}(t)+qF_{zTz}(kt)]F_{zTz}(2kt)$$
$$F^2_{zTz}(kt)\geq [p+qF_{zTz}(t)]F_{zTz}(kt)$$
$$F_{zTz}(kt)\geq p+qF_{zTz}(t)\geq p+qF_{zTz}(kt)$$
$$F_{zTz}(kt)\geq \frac{p}{1-q}=1$$

Thus, we have $z = Tz$. Since $Tz = STz$, we also have $z = Sz$. Therefore, $z = Az = Bz = Lz = Mz = Sz = Tz$, and hence z is the common fixed point of A, B, L, M, S and T.

Secondly, let L be continuous then we have, $LL\,x_{2n} \to Lz$ and $L(AB)\ x_{2n} \to Lz$.

Since (L, AB) is semi compatible, we have $L\,(AB)\ x_{2n} \to ABz$ and $ABz = Lz$.

By (ii), we have

$$F^2_{LLx_{2n}Mx_{2n+1}}(kt)*\begin{bmatrix}F_{ABLx_{2n}LLx_{2n}}(kt)\\ F_{STx_{2n+1}Mx_{2n+1}}(kt)\end{bmatrix}$$
$$\geq \begin{bmatrix}pF_{ABLx_{2n}LLx_{2n}}(t)\\ +qF_{ABLx_{2n}STx_{2n+1}}(t)\end{bmatrix}F_{ABLx_{2n}Mx_{2n+1}}(2kt)$$

Letting $n\to\infty$ we have

$$F^2_{zLz}(kt)*[F_{LzLz}(kt)F_{zz}(kt)]$$
$$\geq [pF_{LzLz}(t)+qF_{zLz}(t)]F_{zLz}(2kt)$$
$$F^2_{zLz}(kt)\geq [p+qF_{zLz}(t)]F_{zLz}(2kt)$$
$$\geq [p+qF_{zLz}(t)]F_{zLz}(kt)$$
$$F_{zLz}(kt)\geq p+qF_{zLz}(t)\geq p+qF_{zLz}(kt)$$
$$F_{zLz}(kt)\geq \frac{p}{1-q}=1$$

Thus, we have $z = Lz$. Hence $z = Lz = Mz = S\,z = Tz$.

Since $M(X) \subseteq AB(X)$, there exists $v \in X$ such that $z = Mz = ABv$. By (ii), we have

$$F^2_{LvMx_{2n+1}}(kt)*[F_{ABvLv}(kt)F_{STx_{2n+1}Mx_{2n+1}}(kt)]$$
$$\geq \begin{bmatrix}pF_{ABvLv}(t)\\ +qF_{ABvSTx_{2n+1}}(t)\end{bmatrix}F_{ABvMx_{2n+1}}(2kt)$$

Letting $n\to\infty$

$$F^2_{zLv}(kt)*[F_{zLv}(kt)F_{zz}(kt)]$$
$$\geq [pF_{zLv}(t)+qF_{zz}(t)]F_{zz}(2kt)$$
$$F^2_{zLv}(kt)*F_{zLv}(kt)\geq pF_{zLv}(t)+q$$
$$\geq pF_{zLv}(kt)+q$$

Noting that $F^2_{zLv}(kt)\leq 1$ and using (iii) in definition 2.1, we have

$$F_{zMv}(kt)\geq pF_{zLv}(kt)+q$$
$$F_{zMv}(kt)\geq \frac{q}{1-p}=1$$

Thus, we have $z = Lv = ABv$. Since (L, AB) is occasionally weakly compatible, we have $Lz = ABz$ and using $z = Bz$ as shown above. Hence $z = Az = Bz = Sz = Tz = Lz = Mz$, that is, z is the common fixed point of the six mappings in this case also.

In order to prove the uniqueness of fixed point let w be another common fixed point of A, B, S, T, L and M. Then by (ii), we have

$$F^2_{LzMw}(kt)*[F_{ABzLz}(kt)F_{STwMw}(kt)]$$
$$\geq [pF_{ABzLz}(t)+qF_{ABzSTw}(t)]F_{ABzMw}(2kt)$$

which implies that

$$F^2_{zw}(kt)\geq [p+qF_{zw}(t)]F_{zw}(2kt)$$
$$\geq [p+qF_{zw}(t)]F_{zw}(kt)$$
$$F_{zw}(kt)\geq p+qF_{zw}(t)$$
$$\geq p+qF_{zw}(kt)$$
$$F_{zw}(kt)\geq \frac{p}{1-q}=1$$

Thus, we have $z = w$. This completes the proof of the theorem.

If we take $B = T = I_X$ (the identity map on X) in the main theorem, we have the following:

Corollary 3.2: Let A, S, L and M be self-maps on a complete Menger space (X, F, *) with $t*t \geq t$ for all $t \in [0, 1]$, satisfying:

(i) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(ii) There exists a constant $k \in (0, 1)$ such that

$$F^2_{LxMy}(kt)*[F_{AxLx}(kt).F_{SyMy}(kt)]$$
$$\geq [pF_{AxLx}(t)+qF_{AxSy}(t)]F_{AxMy}(2kt)$$

for all $x, y \in$ X and $t > 0$ where $0 < p, q < 1$ such that $p + q=1$;

(iii) either A or L is continuous;

(iv) the pair (L, A) is semicompatible and (M, S) is occassionally weakly compatible.

Then A, S, L, and M have a unique common fixed point.

If we take $A = S$, $L = M$ and $B = T = I_X$ in the main Theorem, we have the following:

Corollary 3.3: Let (X, F, *) be a complete Menger space with $t*t \geq t$ for all $t \in [0, 1]$ and let A and L be compatible maps on X such that $L(X) \subseteq A(X)$. If A is continuous and there exists a constant $k \in (0, 1)$ such that

$$F^2_{LxLy}(kt)*[F_{AxLx}(kt).F_{AyLy}(kt)]$$
$$\geq [pF_{AxLx}(t)+qF_{AxAy}(t)]F_{AxLy}(2kt)$$

for all $x, y \in X$ and $t > 0$ where $0 < p, q < 1$ such that $p + q = 1$, then A and L have a unique fixed point.

Example 3.4: Let $X = [0, 1]$ with the metric d defined by $d(x, y) = |x-y|$ and defined $F_{xy}(t) = H(t - d(x, y))$ for all $x, y \in X$, $t > 0$. Clearly (X, F, *) is a complete Menger space where t-norm * is defined by $a*b = \min\{a, b\}$ for all $a, b \in [0, 1]$. Let A, B, S, T, L and M be maps from X into itself defined as

$$Ax=\frac{x}{4}, Bx=\frac{x}{3}, Sx=x, Tx=\frac{x}{2}, Lx=\frac{x}{6}, Mx=\frac{x}{15}$$

for all x $\in$ X. Then

$$L(X) = \left[0, \frac{1}{6}\right] \subset \left[0, \frac{1}{2}\right] = ST(X)$$

$$and \;\; M(X) = \left[0, \frac{1}{15}\right] \subset \left[0, \frac{1}{12}\right] = AB(X).$$

Clearly $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$ and AB, L are continuous. If we take $k = \frac{1}{3}$ and t=1, we see that the condition (ii) of the main Theorem is also satisfied. Moreover, the maps L and AB are semi compatible if $\lim_{n\to\infty} x_n = 0$, where $\{ x_n \}$ is a sequence in X such that $\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} ABx_n = 0$ for $0 \in X$. The maps M and ST are occasionally weakly compatible at 0. Thus, all conditions of the main Theorem are satisfied and 0 is the unique common fixed point of A, B, S, T, L, and M.

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