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Application of Linear ODE as Auxiliary Equation to the Nonlinear Evolution Equation

Hasibun Naher*, Fardousi Ara Begum

Department of Mathematics and Natural Sciences, BRAC University, Mohakhali, Dhaka, Bangladesh
*Corresponding author: hasibun06tasauf@gmail.com

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Abstract In this article, simplified Modified Camassa-Holm (SMCH) equation is investigated to construct some new analytical solutions via the improved (G'/G)-expansion method. Second order linear ordinary differential equation is used with constant coefficients in the method. As a result, some new travelling wave solutions are obtained through the hyperbolic function, the trigonometric function and the rational forms. If parameters take specific values, the solitary waves are derives from the travelling waves. Furthermore, some of the solutions are presented in the figures with the aid of commercial software Maple.

Keywords: *The SMCH equation, analytical solutions, nonlinear partial differential equation, ordinary differential equation, auxiliary equation, the improved (G'/G)-expansion method*

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1. Introduction

In recent years, various methods have been introduced to generate analytical solutions of the nonlinear evolution equations (NLEEs) in mathematical physics, engineering sciences and other real world problems. For example, the inverse scattering method [1], the homogeneous balance method [2], the generalized Riccati equation method [3,4,5,6], the F-expansion method [7], the tanh-coth method [8], the Exp-function method [9-11] and others [12,13,14].

Firstly, the (G'/G)-expansion method is proposed by Wang *et al.* [15] in 2008. They applied this method to some nonlinear partial differential equations and $u(\xi)=\sum_{i=0}^{m} a_i (G'/G)^i$ is used as the travelling wave solution where $a_m \neq 0$. Subsequently, many researchers applied this method to study various nonlinear partial differential equations [16,17,18].

Recently, (G'/G)-expansion method is extended by Zhang *et al.* [19] and they presented the solution form as $u(\xi)=\sum_{j=-m}^{m} a_j (G'/G)^j$, where either a_{-m} or a_m may be zero, but both a_{-m} or a_m cannot be zero at a time. Consequently, a diverse group of scientists investigated a different class of nonlinear PDEs by using this effective and straightforward method for establishing many new travelling wave solutions. For example, Hamad *et al.* [20] studied higher dimensional potential YTSF equation to construct analytical solutions via this method. Nofel *et al.* [21] implemented same method to the fifth-order KdV equation for obtaining travelling wave solutions whereas Naher *et al.* [22] investigated compound KdV-Burgers equation.

Naher and Abdullah [23] executed the same method to construct travelling wave solutions of the nonlinear reaction diffusion equation whilst they [24] studied the (2+1)-dimensional modified Zakharov-Kuznetsov equation for constructing abundant new travelling wave solutions by applying this method. Again, Naher and Abdullah [25] constructed analytical solutions of the (2+1)-dimensional breaking soliton equation and Naher *et al.* [26] investigated the higher dimensional Jimbo-Miwa equation via the same method for obtaining some new solutions. Furthermore, in Ref. [27] Naher and Abdullah applied this method to the higher dimensional Kadomstev-Petviashvili equation to establish some new analytical solutions and so on.

In this article, auxiliary equation intends to be used to generate analytical solutions of the SMCH equation. In addition, some new solutions coincide with previous results which are available in the open literature. Also, some new solutions are displayed in the figures.

2. Description of the Method

Consider the general nonlinear partial differential equation:

$$P\left(u,u_t,u_x,u_{xt},u_{tt},u_{xx},\ldots\right)=0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and the subscripts stand for the partial derivatives.

The main steps of the method [19] are as follows:

Step 1. Suppose, combining the real variables x and t by a new variable ξ:

$$u(x,t) = v(\xi), \quad \xi = x - Vt, \tag{2}$$

where V is the speed of the travelling wave. Now using transformation of (2) in equation (1) obtain to the following ordinary differential equation for $v(\xi)$:

$$A(v, v', v'', \ldots) = 0, \tag{3}$$

where the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2. According to possibility, equation (3) can be integrated term by term one or more times, then we can obtain constant(s) of integration. For simplicity, we can consider zero for integral constant.

Step 3. Suppose that the travelling wave solution of (3) can be expressed in the following form:

$$v(\xi) = \sum_{j=-m}^{m} a_j \left(G'/G\right)^j \tag{4}$$

with $G = G(\xi)$ satisfies the following second order linear ODE:

$$G'' + \lambda G' + \mu G = 0, \tag{5}$$

where $a_j\ (j = 0, \pm 1, \pm 2, \ldots, \pm m), \lambda$ and μ are constants to be determined later.

Step 4. To determine the positive integer m, for this reason need to take the homogeneous balance between highest order nonlinear terms and highest order derivatives of equation (3).

Step 5. Substituting equation (4) and equation (5) into equation (3) with the value of m which obtained in Step 4, then obtained polynomials in $(G'/G)^r$, $(r = 0, \pm 1, \pm 2, \ldots)$, and setting each coefficient of the resulted polynomials to zero, it follows a set of algebraic equations for $a_j\ (j = 0, \pm 1, \pm 2, \ldots, \pm m), V, \lambda$ and μ.

Step 6. Solving the system of algebraic equations which are obtained in step 5 with the aid of algebraic software Maple and then, we can obtain values for $a_j\ (j = 0, \pm 1, \pm 2, \ldots, \pm m)$ and V. Furthermore, substituting the values of $a_j\ (j = 0, \pm 1, \pm 2, \ldots, \pm m)$ and V in equation (4) along with the general solution of (5) which is well known, then obtaining some new travelling wave solutions of equation (1).

3. Application of the Method

In this section, we have investigated the SMCH equation by construction some new analytical solutions which including solitons solutions and periodic solutions via the improved (G'/G)-expansion method.

3.1. The SMCH Equation

In this work, we choose the SMCH equation:

$$u_t + 2k\, u_x - u_{xxt} + \beta u^2 u_x = 0, \tag{6}$$

where $k \in \Re$ and $\beta > 0$.

Details of CH and MCH equation can be found in references [28,29,30,31,32].

Now, we use the wave transformation equation (2) into equation (6), which yields:

$$-Vv' + 2kv' + Vv''' + \beta v^2 v' = 0, \tag{7}$$

where the superscripts stand for the derivatives with respect to ξ.

Equation (7) is integrable, therefore, integrating with respect to once yields:

$$(2k - V)v + Vkv'' + \frac{\beta}{3} v^3 + C = 0, \tag{8}$$

where C is an integral constant which is to be determined later.

Balancing v'' with v^3 in equation (8), we obtain $m = 1$. So, the solution of (8) is the form:

$$v(\xi) = a_{-1}\left(G'/G\right)^{-1} + a_0 + a_1\left(G'/G\right), \tag{9}$$

where a_{-1}, a_0 and a_1 are constants to be determined.

Substituting equation (9) together with equation(5) into (8), the left-hand side of (8) is converted into a polynomial of $(G'/G)^r, (r = 0, \pm 1, \pm 2, \ldots)$. According to Step 5, collecting all terms with the same power of (G'/G). Then, setting each coefficient of the resulted polynomial to zero, yields a set of algebraic equations (for simplicity, which are not presented) for $a_{-1}, a_0, a_1, V, C, \lambda$ and μ.

Solving the system of obtained algebraic equations with the help of algebraic software Maple, we obtain three different values.

Case 1:

$$\begin{gathered} a_{-1} = 0,\ a_0 = \pm\lambda\sqrt{\frac{6k}{4\beta\mu - 2\beta - \beta\lambda^2}}, \\ a_1 = \pm 2\sqrt{\frac{6k}{4\beta\mu - 2\beta - \beta\lambda^2}}, \\ V = \frac{-4k}{4\mu - 2 - \lambda^2},\ C = 0, \end{gathered} \tag{10}$$

where k, λ, μ are free parameters and $\beta > 0$.

Case 2:

$$\begin{gathered} a_{-1} = \pm 2\sqrt{\frac{6k}{4\beta\mu - 2\beta - \beta\lambda^2}}, \\ a_0 = \pm\lambda\sqrt{\frac{6k}{4\beta\mu - 2\beta - \beta\lambda^2}}, \\ a_1 = 0,\ V = \frac{-4k}{4\mu - 2 - \lambda^2},\ C = 0, \end{gathered} \tag{11}$$

where k, λ, μ are free parameters and $\beta > 0$.

Case 3:

$$a_{-1}=\pm 2\mu i\sqrt{\frac{6k}{\beta\left(8\mu+2+\lambda^2\right)}},$$

$$a_0=\pm\lambda i\sqrt{\frac{6k}{\beta\left(8\mu+2+\lambda^2\right)}},a_1=\pm 2i\sqrt{\frac{6k}{\beta\left(8\mu+2+\lambda^2\right)}},\quad (12)$$

$$V=\frac{4k}{8\mu+2+\lambda^2},\ C=\pm\frac{16i\lambda\mu k^{3/2}\sqrt{6}}{\left(8\mu+2+\lambda^2\right)^{3/2}\sqrt{\beta}},$$

where k,λ,μ are free parameters and $\beta>0$.

Substituting the general solution equation (5) into (9), we obtain three different families of travelling wave solutions of (8):

Family 1: Hyperbolic function solutions:

When $\lambda^2-4\mu>0$, we obtain

$$\begin{aligned}v(\xi)&=a_{-1}\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\frac{\left[\begin{array}{l}C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\\+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\end{array}\right]}{\left[\begin{array}{l}C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\\+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\end{array}\right]}\right)\\&+a_0+a_1\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\frac{\left[\begin{array}{l}C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\\+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\phi\end{array}\right]}{\left[\begin{array}{l}C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\\+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\end{array}\right]}\right),\end{aligned}\quad (13)$$

If C_1 and C_2 are taken particular values, various known solutions can be rediscovered.

Family 2: Trigonometric function solutions:

When $\lambda^2-4\mu<0$, we obtain

$$\begin{aligned}v(\xi)&=a_{-1}\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\frac{\left[\begin{array}{l}-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\\+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\end{array}\right]}{\left[\begin{array}{l}C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\\+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\end{array}\right]}\right)^{-1}\\&+a_0+a_1\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\frac{\left[\begin{array}{l}-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\\+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\end{array}\right]}{\left[\begin{array}{l}C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\\+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\end{array}\right]}\right)\end{aligned}\quad (14)$$

If C_1 and C_2 are taken particular values, various known solutions can be rediscovered.

Family 3: Rational function solution:

When $\lambda^2-4\mu=0$, we obtain

$$\begin{aligned}v(\xi)&=a_{-1}\left(\frac{-\lambda}{2}+\frac{C_2}{C_1+C_2\xi}\right)^{-1}\\&+a_0+a_1\left(\frac{-\lambda}{2}+\frac{C_2}{C_1+C_2\xi}\right),\end{aligned}\quad (15)$$

Substituting expression of (10), (11) and (12) in (9), account into the general solution (5), then yields the hyperbolic function solution for equation (13), with corresponds to the conditions $C_1=0$, $C_2\neq 0$:

$$v_1(x,t)=\pm\sqrt{\frac{6k\left(\lambda^2-4\mu\right)}{\beta\left(4\mu-\lambda^2-2\right)}}\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi,$$

where $\xi=x-\dfrac{4k}{\lambda^2-4\mu+2}t$.

$$v_2(x,t)=\pm\sqrt{\frac{6k}{\beta\left(4\mu-\lambda^2-2\right)}}\times\left(2\left(\frac{-\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)^{-1}+\lambda\right),$$

where $\xi=x-\dfrac{4k}{\lambda^2-4\mu+2}t$.

$$v_3(x,t)=\pm i\sqrt{\frac{6k}{\beta\left(8\mu+2+\lambda^2\right)}}\times\left(\begin{array}{l}2\mu\left(\frac{-\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\right)^{-1}\\+\sqrt{\lambda^2-4\mu}\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi\end{array}\right),$$

where $\xi=x-\dfrac{4k}{8\mu+2+\lambda^2}t$.

Substituting expression of (10), (11) and (12) in (9), account into the general solution (5), then yields the trigonometric function solution for equation (14), which are traveling wave solutions corresponds to the conditions $C_2=0$, $C_1\neq 0$:

$$v_4(x.t)=\mp\sqrt{\frac{6k\left(4\mu-\lambda^2\right)}{\beta\left(4\mu-\lambda^2-2\right)}}\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi,$$

$$v_5(x,t)=\pm\sqrt{\frac{6k}{\beta\left(4\mu-\lambda^2-2\right)}}\times\left(2\left(\frac{-\lambda}{2}-\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi\right)^{-1}+\lambda\right),$$

$$v_6(x,t) = \pm i\sqrt{\frac{6k}{\beta\left(8\mu+2+\lambda^2\right)}} \times \left(2\mu\left(\frac{-\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \xi\right)^{-1} - \sqrt{4\mu-\lambda^2}\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \xi \right),$$

Substituting expression of (10), (11) and (12) in (9), account into the general solution of (5) then yields the rational function solution for equation (15), we obtain following traveling wave solutions corresponds to the condition $\lambda^2 - 4\mu = 0$:

$$v_7(x,t) = \pm\sqrt{\frac{6k}{\beta\left(4\mu-\lambda^2-2\right)}}\frac{2C_2}{\left(C_1+C_2\xi\right)},$$

where $\xi = x - \dfrac{4k}{\lambda^2 - 4\mu + 2}t.$

$$v_8(x,t) = \pm\sqrt{\frac{6k}{\beta\left(4\mu-\lambda^2-2\right)}}\left(2\left(\frac{-\lambda}{2} + \frac{C_2}{C_1+C_2\xi}\right)^{-1} + \lambda\right),$$

where $\xi = x - \dfrac{4k}{\lambda^2 - 4\mu + 2}t.$

$$v_9(x,t) = \pm i\sqrt{\frac{6k}{\beta\left(8\mu+\lambda^2+2\right)}} \times \left(2\mu\left(\frac{-\lambda}{2} + \frac{C_2}{C_1+C_2\xi}\right)^{-1} + \frac{C_2}{C_1+C_2\xi}\right),$$

where $\xi = x - \dfrac{4k}{8\mu + 2 + \lambda^2}t.$

4. Results and Discussion

It is worth declaring that some of our obtained solutions are in good agreement with already published results which are presented in the following table. Moreover, some of obtained traveling wave solutions are described from the Figure 1 to Figure 8.

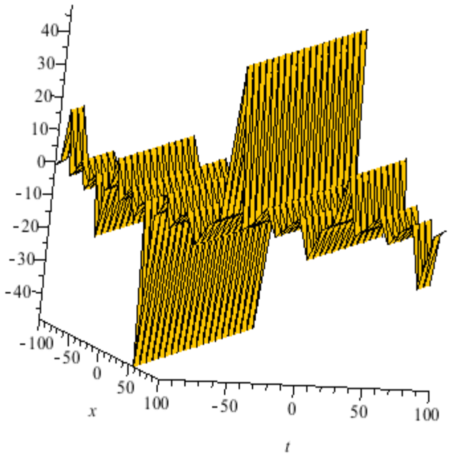

Figure 1. Periodic solution for $\lambda = 1, \mu = 1, \beta = 1, k = 0.25$

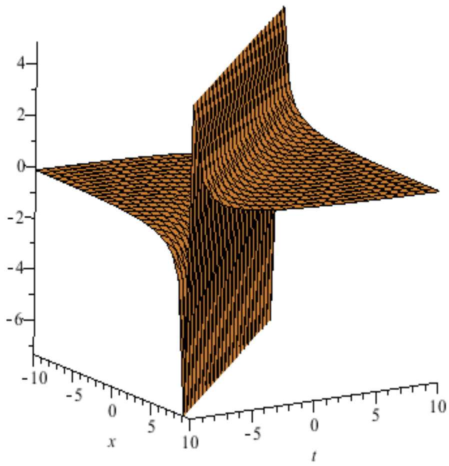

Figure 2. Periodic solution for $\lambda = 2, \mu = 1, \beta = 1, k = -0.5, C_1 = 1, C_2 = 2$

4.1. Table: Comparison between Liu *et al.* [32] Solutions and New Solutions

Liu *et al.* [32] solutions	New solutions
i. If $C_1 = 1, C_2 = 0, \lambda^2 - 4\mu = -4, k = a = 1$ solution (from example 1 of section 3) becomes: $u_{(3,4)}(x,t) = \pm 2\sqrt{3}\tan(x+2t)$.	i. If $k = 1, \beta = 1, 4\mu - \lambda^2 = 4$ and $v_4(x,t) = u_{(3,4)}$, solution $v_4(x,t)$ becomes: $u_{(3,4)}(x,t) = \pm 2\sqrt{3}\tan(x+2t)$.
ii. If $C_1 = 1, C_2 = 1, \mu = 1, \lambda = 2, a = 1$ and $k = -1$, solution (from example 1 of section 3) becomes: $u_{(3,4)}(x,t) = \pm 2\sqrt{3}\dfrac{1}{x+2t}$.	ii. If $C_1 = 0, C_2 = 1, \mu = 1, \lambda = 2, \beta = 1$ $k = -1$ and $v_7(x,t) = u_{(3,4)}$, solution $v_7(x,t)$ becomes: $u_{(3,4)}(x,t) = \pm 2\sqrt{3}\dfrac{1}{x+2t}$.
iii. If $C_1 = 1, C_2 = 0, \mu = \dfrac{1}{4}, \lambda = 0, a = 1$ and $k = 1$, solution (from example 2 of section 3) becomes: $u_{(3,4)}(x,t) = \pm i\sqrt{6}\tan\dfrac{1}{2}(x-4t)$.	iii. If $\lambda = 0, k = 1, \mu = \dfrac{1}{4}, \beta = 1$ and $v_4(x,t) = u_{(3,4)}$, solution $v_4(x,t)$ becomes: $u_{(3,4)}(x,t) = \pm i\sqrt{6}\tan\dfrac{1}{2}(x-4t)$.
iv. If $C_1 = 1, C_2 = 1, \mu = 1, \lambda = 2, a = 1$ and $k = 1$, solution (from example 2 of section 3) becomes: $u_{(3,4)}(x,t) = \pm i\,2\sqrt{3}\dfrac{1}{x-2t}$.	iv. If $C_1 = 0, C_2 = 1, \mu = 1, \lambda = 2, \beta = 1$ $k = 1$ and $v_7(x,t) = u_{(3,4)}$, solution $v_7(x,t)$ becomes: $u_{(3,4)}(x,t) = \pm i\,2\sqrt{3}\dfrac{1}{x-2t}$.

Beyond this table, we obtain new exact travelling wave solutions $v_1, v_2, v_3, v_5, v_6, v_8$ and v_9 which are not being established in the previous literature.

4.2. Graphical Representations of the Solutions

The graphical presentations of some new solutions are depicted in the figures with the aid of commercial software Maple:

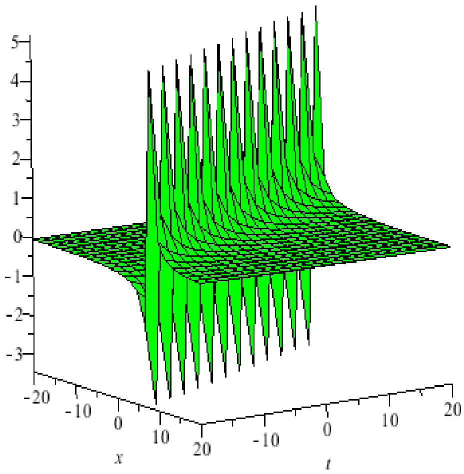

Figure 3. Solitons solution for $\lambda = 2, \mu = 1, \beta = 1, k = -0.25, C_1 = 2, C_2 = 1$

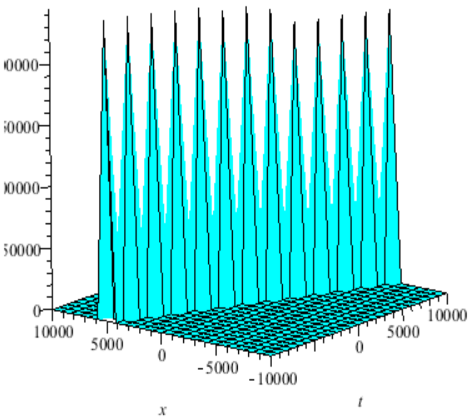

Figure 4. Solitons solution for $\lambda = 2, \mu = 1, \beta = 2, k = -0.25, C_1 = 0, C_2 = 5$

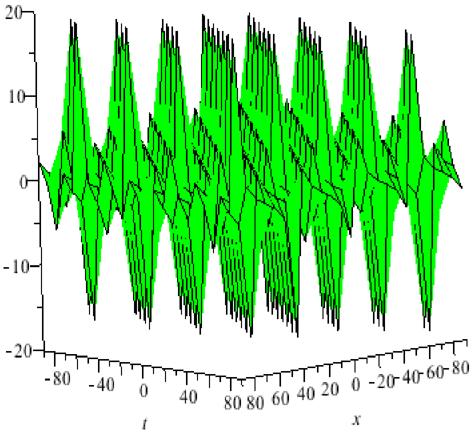

Figure 5. Solitons solution for $\lambda = 3, \mu = 4, \beta = 1, k = 0.25$

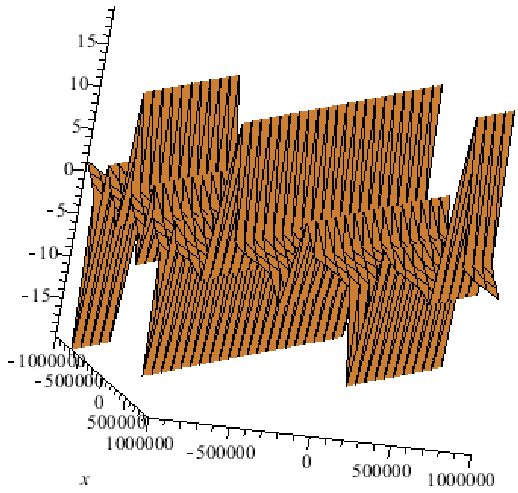

Figure 6. Periodic solution for $\lambda = 2, \mu = 2, \beta = 3, k = 0.5$

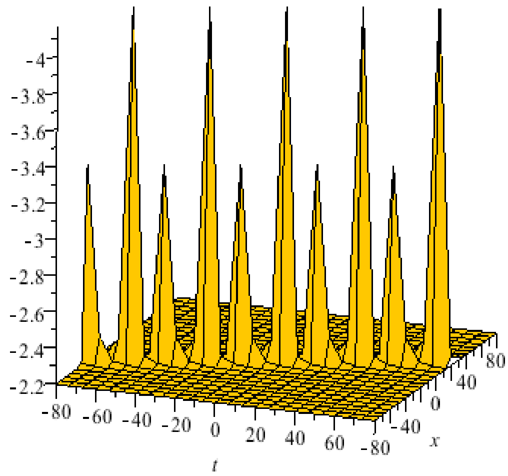

Figure 7. Solitons solution for $\lambda = 4, \mu = 4, \beta = 2, k = 5, C_1 = 9, C_2 = 1$

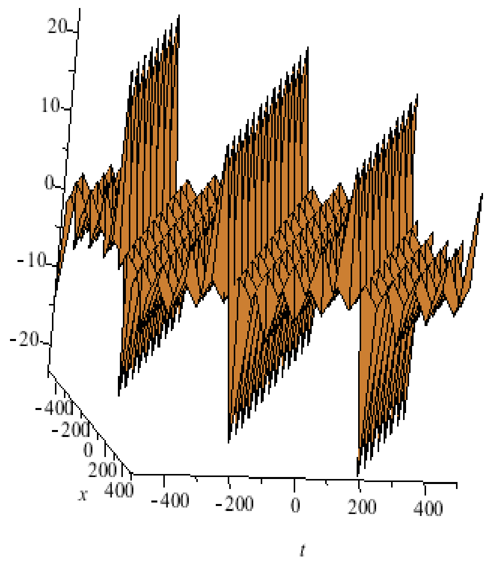

Figure 8. Periodic solution for $\lambda = 4, \mu = 5, \beta = 5, k = 1$

5. Conclusions

In this article, the linear ODE is used with the improved (G'/G) -expansion method to construct some new travelling wave solutions of the SMCH equation. The new obtained solutions are presented through the hyperbolic function, the trigonometric function and the rational functions form. If parameters take particular values some of the obtained solutions coincide with published results which describes in result and discussion section and verify that our obtained all solutions are correct. In addition, some figures are illustrated which examined new obtained solutions.

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