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The Fourier, Laplace Transformations and the Newton Potential

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Abstract With help of a methods of Fourier`s analysis we consider the analytic continuation of Laplace transformation (with help a values on the complex axis) to the left part of plane, when the continuation is odd ore even.

Keywords: *format, microsoft laplace transform, fourier transform, even laplace transform, regularity of the laplace transform*

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1. Introduction

A communication between potentials of Newton and evenness of functions of type of Laplace transformations from Fourier transformations (the theorems 1, 3) is of principle for the article. The main result of the article are the theorems 1 and 2.

The second purpose of the article is a clarification of results of the works [1,2] (the remarks 1, 2, 3 and the theorem 3).

The results look like were formulated in works of author ([1,2,3,4]). In the works of author [1,2] the specifying investigation (the remark 2) was absent, and in the works ([3,4]) other methods were used.

We will mark, that unforeseeable results of theorem 2 and the theorems 3 are proved by different methods fully. A primary purpose of the article is a proof of these theorems.

A discussion of an consequences in the direction of possible compound of axiomatic notions of theory of functions of complex variable and the classic mathematical analysis is beyond the article.

2. The Laplace Transformation on the Complex Axis

Theorem 1.

The equalities

1.

$$-\int_{-\infty}^{\infty} S_0(x)e^{-(iv+ix)b} / (iv+xi)]dx = \pi S_0(-v), v \in (-\infty,\infty),$$

$$\int_{-\infty}^{\infty} S_0(-x)e^{(iv+ix)b} / (iv+xi)]dx = \pi S_0(v), v \in (-\infty,\infty),$$

2. for all $b > B > 0$

$$\int_{-\infty}^{\infty} S_0(x)\cos(v+x)b / (v+x)]dx = 0, v \in (-\infty,\infty),$$

$$\int_{-\infty}^{\infty} S_0(x)\sin(v+x)b / (v+x)]dx = \pi S_0(-v), v \in (-\infty,\infty),$$

take place, if $S(-B) = S(B) = 0$, if $d^2S(x)/dx^2$ is continuous on $[-B, B]$, and

$$S_0(x) = \int_{-B}^{B} S(x)e^{ixu}du, B \in (0,+\infty).$$

Proof.

We use (it is well-known)

$$L(t) = \int_{-\infty}^{\infty} [S_0(x)/(t+xi)] =$$

$$\int_{-\infty}^{\infty} [S_0(x) - S_0(-v)]/(t+xi)]dx +$$

$$S_0(-v)\int_{-\infty}^{\infty} 1/(t+xi)]dx =$$

$$= \int_{-\infty}^{\infty} [S_0(x) - S_0(-v)]/(t+xi)]dx + S_0(-v)\pi,$$

$$t \in (0,+\infty), v \in (-\infty,+\infty),$$

$$L(p) \to L(iv) = \int_{-\infty}^{\infty} S_0(x)/(iv+xi)]dx + S_0(-v)\pi,$$

$$t = p \to iv, v \in (-\infty,+\infty),$$

where from remark 1

$$L(iv) = \lim_{p \to iv} \int_{-\infty}^{\infty} [S_0(x)/(p+xi)] =$$

$$\int_{-\infty}^{\infty} [S_0(x)[1-e^{-(iv+ix)b}]/(iv+xi)]dx = L_b(iv), Re\, p > 0,$$

$$L(iv) = \int_{-\infty}^{\infty} [S_0(x)/(iv+xi)]dx + J(iv), v \in (-\infty, \infty).$$

(After replacement of variable $x = -x_1$ and after the substitution in place of $-v$ value v we obtain the second part of the equality 1 in the theorem 1).

The complex and the real parts of the first equality are the second equalities of the theorem 1.

Remark 1.

In the conditions of the theorem 1 the equalities

$$L(p) = \int_{-\infty}^{\infty} [S_0(x)/(p+xi)] =$$

$$\int_{-\infty}^{\infty} [S_0(x)[1-e^{-(p+ix)b}]/(p+xi)]dx = L_b(p), Re\, p > 0,$$

take place.

Proof.

From the inversion formula of the Fourier transformation we get

$$Z(t) = \int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx =$$

$$\int_{-\infty}^{\infty} e^{-ixt}dx \int_{-B}^{B} S(x)e^{ixu}du = 2\pi S(x)I(t),$$

$$I(t) = 1, t \in [-B, B], , I(t) = 0, |t| > B > 0.$$

We obtain

$$\int_0^{\infty} e^{-pt}dt \int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx = \int_0^{b} e^{-pt}dt \int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx,$$

$$Re\, p \geq 0, b > B > 0.$$

From the condition $S(-B) = S(B) = 0$ we obtain all the conditions of the proposition 1 (taking into account the formula of integration on parts).

Proposition 1.

$$L(p) = \int_{-\infty}^{\infty} S_0(x)dx \int_0^{\infty} e^{-(p+ix)t}dt =$$

$$\int_0^{\infty} e^{-pt}dt \int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx, Re\, p > 0,$$

if $S_0(x)$ is continuous for all $x \in (-\infty, \infty)$. By definition,

$$L(p) = \int_{-\infty}^{\infty} S_0(x)dx/(p+xi), Re\, p > 0 .$$

(The proposition 1 takes place, if $\int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx \equiv 0,$ for all $|t| > N > 0$; the functions are considered in the theorem 1).

Proof.

For $u = Re\, p > 0$ from inequality

$$|e^{-(p+ix)t}| \leq e^{-ut}, u > 0.$$

we can change the limits of integration in the initial expression $L(p)$ ([5]).

The proposition 1 is proved.

After change of order of integration ([5]) from the proposition 1 we get the last part of the remark 1 :

$$\int_0^{b} e^{-pt}dt \int_{-\infty}^{\infty} e^{-ixt} S_0(x)dx =$$

$$\int_{-\infty}^{\infty} [S_0(x)[1-e^{-(p+ix)b}]/(p+xi)]dx = L_b(p),$$

$$Re\, p \geq 0, b > B > 0,$$

and $L(p) = L_b(p), Re\, p \geq 0$.

We will prove the main theorem 2.

Theorem 2.

In the conditions of the theorem 1

$$\int_0^{b} \cos tudt \int_{-\infty}^{\infty} \sin tx\, S_0(x)dx \equiv 0, u \in (0, +\infty),$$

Proof.

From the theorem 1 we have

$$0 = \int_{-\infty}^{\infty} S_0(x)[\cos(v+x)b/(v+x) - \cos(v-x)b/(v-x)]dx$$

$$= \int_{-\infty}^{\infty} S_0(x)dx[\int_0^{b} \sin t(v+x) - \sin t(v-x)]dt$$

$$= 2\int_0^{b} \cos tvdt \int_0^{\infty} (\sin tx)S_0(x)dx, v \in (0, \infty),$$

for $S(-x) = S(x), x \in (0, +\infty)$.

The main theorem 2 is proved.

Now, we will prove the remark 2. The remark is absent in works of author ([1,2]).

Remark 2.

In conditions of the remark 1

$$J(p) = -\int_{-\infty}^{\infty} [S_0(x)e^{-(p+ix)b}/(p+xi)]dx \equiv 0,$$

$$Re\, p < 0.$$

Proof.

After the change of limits of integration ([5]) $x = -x_1$, we obtain

$$J_{left}(p) = \int_{-\infty}^{\infty} [S_0(x)e^{-(-p+ix)b}/(-p+xi)]dx =$$

$$-\int_{-\infty}^{\infty} [S_0(-x)e^{(p+ix)b}/(p+xi)]dx.$$

Obviously, for the $S(-x) = S(x)$ function there is a limit

$$-\int_{-\infty}^{\infty}[S_0(-x)e^{(p+ix)b}/(p+xi)]dx -$$

$$(-\int_{-\infty}^{\infty}[S_0(x)e^{-(p+ix)b}/(p+xi)]dx) \to$$

$$\to \int_{-\infty}^{\infty}[S_0(-x)e^{(iv+ix)b}/(iv+xi)]dx -$$

$$\int_{-\infty}^{\infty}[S_0(x)e^{-(iv+ix)b}/(iv+xi)]dx = 0$$

where for the last equality we use the first part of the remark 3. We obtain $J(iv) = J_{left}(iv), v \in (-\infty,\infty)$, and $J(p) = J_{left}(p), Re\, p > 0$ too ([6]), (the regularity of the $J(p)$ function in an open area of imaginary axis ensues from the obvious regularity of $L_b(p) = L(p), \mathrm{Re}\, p > 0,$ in all complex plane).

We get. $J_{left}(p) = 0, Re\, p > 0$ from $J(p) = 0, Re\, p > 0$. For the odd the $S(x)$ function the remark 2 is proved.

For the $S(-x) = -S(x)$ function we repeat the same reasoning of the remark 3 but for the sum.

Consequence.

From the remark 2 as analytical continuation of the $J(p)$ function (not as in the theorem 1, see the theorem 2 too) we get

$$J(p) = -e^{-pb}\int_{-\infty}^{\infty}[S_0(x)e^{-ixb}]/(p+xi)dx \equiv 0,$$

$$Re\, p > 0, b = const.$$

Theorem 3

In the conditions of remark 1 to the lemma 1 we get, that the analytic continuation of function $L_b(p) = L(p), Re\, p > 0,$ from the right part of plane to left is equal to the analytic expression $L(p), Re\, p < 0$.

Proof.

From the remark 2 we get $L(p) = L_p(p)$ not only for $Re\, p > 0$, but for all $Re\, p < 0$. The existence of the $dL(p)/dp, \mathrm{Re}\, p \neq 0,$ is obvious (in the form $L(p)$). We can use ([6])

$$\lim_{p\to iv, \mathrm{Re}\, p>0} L_b(p) = \lim_{p\to iv, \mathrm{Re}\, p<0} L_b(p) =$$

$$L_b(iv) = L(iv), v \in (-\infty,\infty),$$

and as in ([1,2]) from the theorem about the analytic continuation of the functions continuous on border we obtain, that $L(p) = L_p(p)$ is the same analytic function as for $Re\, p > 0$ so as $Re\, p < 0$.

Remark 3. ([1,2]).

The analytic expression $L(p)$ is even or odd expression, if the $S_0(-v)$ function is odd or even accordingly. It is obviously, if $p \in (-\infty,\infty)$. From the theorem 3 we obtain, that the analytic function $L(p) = L_p(p)$ is even or odd in the all complex plain too.

Remark 4.

We will notice, that from the investigation of evenness of $L(p)$ in the area of imaginary axis, we obtain

$$\int_{-\infty}^{\infty}[S_0(x)e^{-ixb}/(iv+xi)]dx = 0, v \in (-\infty,\infty),$$

which it follows from taking into account the remark 2 and the theorem 3, but from point of the convergence of $L(p)$ to $L(iv)$, the $\mathrm{Re}\, L(iv) \neq 0$, and from point of analytic continuations (physically from point of smooth transition-flow through the imaginary axis) the such actual values are absent.

The first given fact was marked by other methods in a few more general situation, but for other functions, in the works of author ([3,4]).

3. Conclusion

The first unforeseeable result is the theorem 2 (see so the formula (5) on the 209 page in the work [6]).

The second unforeseeable result we obtain from the remark 2 and the theorem 3: from point of the convergence of $L(p)$ to $L(iv)$, the $\mathrm{Re}\, L(iv) \neq 0$ (the $L(iv)$ value in the theorem 1), but from point of analytic continuations (physically from point of smooth transition-flow through the complex axis) the such values are absent (the fact was considered by other methods in the similar situation in the works of author ([6,7])); we get a similar situation from the given reasoning: if $S_0(-x) = S_0(x), x \in (-\infty,\infty),$ we get $L(-iv) = -L(iv), v \in (-\infty,\infty),$ where as in remark 3

$$L(iv) = \mathrm{Re}\, L(iv) - iQ_1(v), S_0(-x) = S_0(x), v \in (-\infty,\infty),$$

and

$$-iQ_1(v) = \int_{-\infty}^{\infty}[S_0(x)/(iv+xi)]dx,$$

$$Q_1(v) = -Q_1(-v) \in (-\infty,\infty);$$

we obtain

$$\mathrm{Re}\, L(-iv) = -\mathrm{Re}\, L(iv), v \in (-\infty,\infty),$$

But ([7])

$$\mathrm{Re}\, L(iv) = 2\int_0^{\infty}(\cos vx)dx \int_{-\infty}^{\infty} S_0(x)(\cos tu)dx,$$

$$\mathrm{Re}\, L(-iv) = \mathrm{Re}\, L(iv), v \in (-\infty,\infty), if$$

$$S_0(-x) = S_0(x), v \in (-\infty,\infty),$$

and the only $J(p) = 0$ function is odd and even simultaneously, we get

$$-J(iv) = \int_{-\infty}^{\infty}[S_0(x)e^{-ixb}/(iv+xi)]dx = 0,$$

$$v \in (-\infty,\infty).$$

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