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Common Fixed Point Theorem of Comatible Mappings in Metric Space

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Abstract In this paper we prove a common fixed point theorem of compatible mappings of type (R). Our result modify the result of M. Koireng Meitei [4].

Keywords: *fixed point, complete metric space, compatible mappings*

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1. Introduction

The first important result in the theory of fixed point of compatible mappings was obtained by Gerald Jugck in 1986 [2] as a generalization of commuting mappings. Pathak, Chang and Cho [3] in 1994 introduced the concept of compatible mappings of type(P). In 2004 Rohen, Singh and shambhu [5] introduced the concept of compatible mappings of type(R) by combining the definitions of compatible mappings and compatible mappings of type(P). The aim of this paper is to prove a common fixed point theorem of compatible mappings of type(R) in metric space by considering eight self mappings.

2. Preliminaries

Definition 2.1: [2] A metric space is given by a set X and a distance function $d: X\times X \to \mathbb{R}$ such that

(i) (Positivity) For all $x, y \in X, 0 \le d(x, y)$.

(ii) (Non-degenerated) For all $x, y \in X$,

$$0 = d(x, y) \Leftrightarrow x = y.$$

(iii) (Symmetry) For all $x, y \in X$,

$$d(x, y) = d(y, x).$$

(iv) (Triangle inequality) For all $x, y, z \in X$,

$$d(x, y) \le d(x, z) + d(z, y).$$

Definition 2.2: [4] Let S and T be mappings from a complete metric space X into itself. The mappings S and T are said to be compatible if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

Definition 2.3: [4] Let S and T be mappings from a complete metric space X into itself. The mappings S and T are said to be compatible of type (P) if $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that for $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 2.4: [4] Let S and T be mappings from a complete metric space X into itself. The mappings S and T are said to be compatible of type (R) if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$ and $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that for $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Proposition 2.5. [4] Let S and T be mappings from a complete metric space (X, d) into itself. If a pair {S, T} is compatible of type (R) on X and Sz = Tz for z ∈ X,Then STz = TSz = SSz = TTz.

Proposition 2.6. [4] Let S and T be mappings from a complete metric space (X, d) into itself. If a pair {S, T} is compatible of type (R) on X and $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$, then we have

(i) $d(TSx_n, Sz) \to 0$ as $n \to \infty$ if S is continuous,

(ii) $d(STx_n, Tz) \to 0$ as $n \to \infty$ if T is continuous and

(iii) STz = TSz and Sz = Tz if S and T are continuous at z.

Lemma 2.7. [4] Let A, B, S and T be mapping from a metric space(X, d) into itself satisfying the following conditions:

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(2)
$$[d(Ax, By)]^2 \le k_1 \begin{bmatrix} d(Ax, Sx)d(By, Ty) \\ +d(By, Sx)d(Ax, Ty) \end{bmatrix} + k_2 \begin{bmatrix} d(Ax, Sx)d(Ax, Ty) \\ +d(By, Ty)d(By, Sx) \end{bmatrix}$$

Where $0 \le k_1 + 2k_2 < 1; k_1, k_2 \ge 0$.

(3) Let $x_0 \in X$ then by (1) there exists $x_1 \in X$ such that $Tx_1 = Ax_0$ and for x_1 there exists $x_2 \in X$ such that $Sx_2 = Bx_1$ and so on. Continuing this process we can define a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = Tx_{2n+1} = Ax_{2n}$$
$$\text{and } y_{2n} = Sx_{2n} = Bx_{2n-1}.$$

Then the sequence $\{y_n\}$ is Cauchy sequence in X.

Theorem: [4] Let A, B, S and T be mapping from a metric space (X, d) into itself satisfying the following conditions:

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(2) $$\left[d(Ax,By)\right]^2 \le k_1\begin{bmatrix} d(Ax,Sx)d(By,Ty) \\ +d(By,Sx)d(Ax,Ty)\end{bmatrix} + k_2\begin{bmatrix} d(Ax,Sx)d(Ax,Ty) \\ +d(By,Ty)d(By,Sx)\end{bmatrix}$$

Where $0 \le k_1 + 2k_2 < 1;\ k_1, k_2 \ge 0.$

(3) Let $x_0 \in X$ then by (1) there exists $x_1 \in X$ such that $Tx_1 = Ax_0$ and for x_1 there exists $x_2 \in X$ such that $Sx_2 = Bx_1$ and so on. Continuing this process we can define a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = Tx_{2n+1} = Ax_{2n}$$
$$\text{and } y_{2n} = Sx_{2n} = Bx_{2n-1}.$$

Then the sequence $\{y_n\}$ is Cauchy sequence in X.

(4) One of A, B, S or T is continuous.

(5) [A, S] and [B, T] are compatible of type (R) on X.

Then A, B, S and T have a unique common fixed point in X.

3. Main Result

Lemma 3.1: Let C, D, E, F, K, M, P and V be self maps of a complete metric space (X, d) satisfying the following conditions:

(1) $C(X) \subseteq DPV(X)$ and $E(X) \subseteq FKM(X)$

(2) $$\left[d(Cx,Ey)\right]^2 \le \alpha_1\begin{bmatrix} d(Cx,FKMx)d(Ey,DPVy) \\ +d(Ey,FKMx)d(Cx,DPVy)\end{bmatrix} + \alpha_2\begin{bmatrix} d(Cx,FKMx)d(Cx,DPVy) \\ +d(Ey,DPVy)d(Ey,FKMx)\end{bmatrix}$$

Where $0 \le \alpha_1 + 2\alpha_2 < 1;\ \propto_1, \propto_2 \ge 0.$

(3) Let $x_0 \in X$ then by (1) there exists $x_1 \in X$ such that $DPVx_1 = Cx_0$ and for x_1 there exists $x_2 \in X$ such that $FKMx_2 = Ex_1$ and so on.continuing this process we candefine a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = DPVx_{2n+1} = Cx_{2n}$$
$$\textit{and } y_{2n} = FKMx_{2n} = Ex_{2n-1}$$

Then the sequence $\{y_n\}$ is a Cauchy sequence in X.

Proof: By condition (2) and (3), we have

$$[d(y_{2n+1}, y_{2n})]^2 = [d(Cx_{2n}, Ex_{2n-1})]^2$$
$$\le \alpha_1\begin{bmatrix} d(Cx_{2n}, FKMx_{2n})d(Ex_{2n-1}, DPVx_{2n-1}) \\ ++d(Ex_{2n-1}, FKMx_{2n})d(Cx_{2n}, DPVx_{2n-1})\end{bmatrix} + \alpha_2\begin{bmatrix} d(Cx_{2n}, FKMx_{2n})d(Cx_{2n}, DPVx_{2n-1}) \\ +d(Ex_{2n-1}, DPVx_{2n-1})d(Ex_{2n-1}, FKMx_{2n})\end{bmatrix}$$
$$= \alpha_1\left[d(y_{2n+1}, y_{2n})d(y_{2n}, y_{2n-1}) + 0\right] + \alpha_2[d(y_{2n+1}, y_{2n})d(y_{2n+1}, y_{2n-1}) + 0]$$

$$[d(y_{2n+1}, y_{2n})] \le \alpha_1 d(y_{2n}, y_{2n-1}) + \alpha_2\begin{bmatrix} d(y_{2n+1}, y_{2n}) \\ +d(y_{2n}, y_{2n-1})\end{bmatrix}$$

$$\left[d(y_{2n+1}, y_{2n})\right] \le p\,d(y_{2n}, y_{2n-1})$$

where $p = \dfrac{\alpha_1 + \alpha_2}{1 - \alpha_2} < 1.$

Hence $\{y_n\}$ is Cauchy sequence.

Theorem 3.2: Let C, D, E, F, K, M, P and V be self maps of a complete metric space (X, d) satisfying the following conditions:

(1) $C(X) \subseteq DPV(X)$ and $E(X) \subseteq FKM(X)$

(2) $$\left[d(Cx,Ey)\right]^2 \le \alpha_1\begin{bmatrix} d(Cx,FKMx)d(Ey,DPVy) \\ +d(Ey,FKMx)d(Cx,DPVy)\end{bmatrix} + \alpha_2\begin{bmatrix} d(Cx,FKMx)d(Cx,DPVy) \\ +d(Ey,DPVy)d(Ey,FKMx)\end{bmatrix}$$

Where $0 \le \alpha_1 + 2\alpha_2 < 1;\ \propto_1, \propto_2 \ge 0.$

(3) Let $x_0 \in X$ then by (1) there exists $x_1 \in X$ such that $DPVx_1 = Cx_0$ and for x_1 there exists $x_2 \in X$ such that $FKMx_2 = Ex_1$ and so on.continuing this process we candefine a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = \mathit{DPV}x_{2n+1} = Cx_{2n} \text{ and } y_{2n} = FKMx_{2n} = Ex_{2n-1}.$$

Then the sequence $\{y_n\}$ is a Cauchy sequence in X.

(4) One of C, E, FKM, DPV is continuous.

(5) [C, FKM] and [E, DPV] are compatible of type (R) on X.

Then C, D, E, F, K, M, P and V have a unique common fixed point in X.

Proof: By lemma 3.1, $\{y_n\}$ is Cauchy sequence. and since X is complete so there exists a point $z \in X$ such that $\lim y_n = z$ as $n \to \infty$.

Consequently subsequences $Cx_{2n}, FKMx_{2n}, Ex_{2n-1}$ and $DPVx_{2n+1}$ converges to z. Let FKM be continuous. Since C and FKM are compatible of type (R) on X. Then by proposition 2.6, We have $(FKM)^2 x_{2n} \to FKMz$ and $(C)(FKM)x_{2n} \to FKMz$ as $n \to \infty$.

Now by condition (2), we have

$$\left[d(CFKMx_{2n}, Ex_{2n-1})\right]$$
$$\le \alpha_1\begin{bmatrix} d\left(CFKMx_{2n}, (FKM)^2 x_{2n}\right)d(Ex_{2n-1}, DPVx_{2n-1}) \\ +d\left(Ex_{2n-1}, (FKM)^2 x_{2n}\right)d(CFKMx_{2n}, DPVx_{2n-1})\end{bmatrix}$$
$$+\alpha_2\begin{bmatrix} d\left(CFKMx_{2n}, (FKM)^2 x_{2n}\right)d(CFKMx_{2n}, DPVx_{2n-1}) \\ +d(Ex_{2n-1}, DPVx_{2n-1})d\left(Ex_{2n-1}, (FKM)^2 x_{2n}\right)\end{bmatrix}$$

$$\left[d\left(CFKMx_{2n}, y_{2n-1}\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(CFKMx_{2n}, (FKM)^2 x_{2n}\right) d\left(y_{2n}, y_{2n-1}\right) \\ +d\left(y_{2n}, (FKM)^2 x_{2n}\right) d\left(CFKMx_{2n-1}, y_{2n-1}\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(CFKMx_{2n}, (FKM)^2 x_{2n}\right) d\left(CFKMx_{2n}, y_{2n-1}\right) \\ +d\left(y_{2n}, y_{2n-1}\right) d\left(y_{2n}, (FKM)^2 x_{2n}\right) \end{array}\right]$$

Letting $n \to \infty$, we have

$$\left[D\left(FKMz, z\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(FKMz, FKMz\right) d\left(z, z\right) \\ +d\left(z, FKMz\right) d\left(FKMz, z\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(FKMz, FKMz\right) d\left(FKMz, z\right) \\ +d\left(z, z\right) d\left(z, FKMz\right) \end{array}\right]$$

$$\left[d\left(FKMz, z\right)\right]^2 \le \alpha_1\left[d\left(z, FKMz\right) d\left(FKMz, z\right)\right]$$

$$\left[d\left(FKMz, z\right)\right]^2 \le \alpha_1\left[d\left(FKMz, z\right)\right]^2$$

Which is a contradiction. Hence

$$FKMz = z \tag{3.1}$$

Now by putting x = z and $y = x_{2n-1}$ in condition (2), then we have

$$\left[d\left(Cz, Ex_{2n-1}\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Ex_{2n-1}, DPVx_{2n-1}\right) \\ +d\left(Ex_{2n-1}, FKMz\right) d\left(Cz, DPVx_{2n-1}\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Cz, DPVx_{2n-1}\right) \\ +d\left(Ex_{2n-1}, DPVx_{2n-1}\right) d\left(Ex_{2n-1}, FKMz\right) \end{array}\right] \tag{3.2}$$

$$\left[d\left(Cz, y_{2n}\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(y_{2n}, y_{2n-1}\right) \\ +d\left(y_{2n}, FKMz\right) d\left(Cz, y_{2n-1}\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Cz, y_{2n-1}\right) \\ +d\left(y_{2n}, y_{2n-1}\right) d\left(y_{2n}, FKMz\right) \end{array}\right]$$

Letting $n \to \infty$, we have

$$\left[d\left(Cz, z\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(Cz, z\right) d\left(z, z\right) \\ +d\left(z, z\right) d\left(Cz, z\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(Cz, z\right) d\left(CDz, z\right) \\ +d\left(z, z\right) d\left(z, z\right) \end{array}\right]$$

$$\left[d\left(Cz, z\right)\right]^2 \le \alpha_2\left[d\left(Cz, z\right) d\left(Cz, z\right)\right]$$

$$\left[d\left(Cz, z\right)\right]^2 \le \alpha_2\left[d\left(Cz, z\right)\right]^2$$

Which is a contradiction. Hence

$$Cz = z. \tag{3.3}$$

Now since $Cz = z$, by condition (1) $z \in DPV(X)$. Also DPV is self map of X, so there exists a point $u \in X$ such that

$$z = Cz = DPVu. \tag{3.4}$$

Moreover by putting Cz = z and $x_{2n-1} = u$ in condition (3.2), we obtain

$$\left[d\left(z, Eu\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(z, FKMz\right) d\left(Eu, DPVu\right) \\ +d\left(Eu, FKMz\right) d\left(z, DPVu\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(z, FKMz\right) d\left(z, DPVu\right) \\ +d\left(Eu, DPVu\right) d\left(Eu, FKMz\right) \end{array}\right]$$

$$\left[d\left(z, Eu\right)\right]^2 \le \alpha_1\left[d\left(z, z\right) d\left(Eu, z\right) + d\left(Eu, z\right) d\left(z, z\right)\right] + \alpha_2\left[d\left(z, z\right) d\left(z, z\right) + d\left(Eu, z\right) d\left(Eu, z\right)\right]$$

$$\left[d\left(z, Eu\right)\right]^2 \le \alpha_2\left[d\left(z, Eu\right)\right]^2$$

Which is a contradiction.
Hence Eu = z, i.e., z = DPVu = Eu.
By condition (5), we have

$$[d(DPV(Eu), E(DPVu))] = 0.$$

Hence d(DPVz, Ez) = 0 i.e., DPVz = Ez.
Now

$$\left[d\left(z, DPVz\right)\right]^2 = \left[d\left(Cz, Ez\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Ez, DPVz\right) \\ +d\left(Ez, FKMz\right) d\left(Cz, DPVz\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Cz, DPVz\right) \\ +d\left(Ez, DPVz\right) d\left(Ez, FKMz\right) \end{array}\right]$$

$$\left[d\left(z, DPVz\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(z, z\right) d\left(DPVz, DPVz\right) \\ +d\left(DPVz, z\right) d\left(z, DPVz\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(z, z\right) d\left(z, DPVz\right) \\ +d\left(DPVz, DPVz\right) d\left(DPVz, z\right) \end{array}\right]$$

$$\left[d\left(z, DPVz\right)\right]^2 \le \alpha_1\left[d\left(DPVz, z\right) d\left(z, DPVz\right)\right]$$

$$\left[d\left(z, DPVz\right)\right]^2 \le \alpha_1\left[d\left(z, DPVz\right)\right]^2$$

Which is a contradiction.

$$\text{Hence } z = DPVz, \text{ i.e., } z = DPVz = Ez. \tag{3.5}$$

Now to prove Vz = z, put x = z and y = Vz in (1) and using (3.1), (3.3) and (3.5), we have

$$\left[d\left(Cz, E\left(Vz\right)\right)\right]^2 \le \alpha_1\left[\begin{array}{l} d\left(Cz, FKM\left(Vz\right)\right) d\left(E\left(Vz\right), DPV\left(Vz\right)\right) \\ +d\left(E\left(Vz\right), FKMz\right) d\left(Cz, DPV\left(Vz\right)\right) \end{array}\right] + \alpha_2\left[\begin{array}{l} d\left(Cz, FKMz\right) d\left(Cz, DPV\left(Vz\right)\right) \\ +d\left(E\left(Vz\right), DPV\left(Vz\right)\right) d\left(E\left(Vz\right), FKMz\right) \end{array}\right]$$

$$\left[d\left(z, Vz\right)\right]^2 \le \alpha_1\left[d\left(z, Vz\right) d\left(Vz, Vz\right) + d\left(Vz, z\right) d\left(z, Vz\right)\right] + \alpha_2\left[d\left(z, z\right) d\left(z, Vz\right) + d\left(Vz, Vz\right) d\left(Vz, z\right)\right]$$

$$\left[d\left(z, Vz\right)\right]^2 \le \alpha_1\left[d\left(z, Vz\right)\right]^2$$

Which is a contradiction.

Hence z = Vz. Since DPVz = z, implies that DPz = z.

Now to prove Pz = z, put x = z and y = Pz in (1) and using (3.1), (3.3) and (3.5), we have

$$\left[d(Cz,E(Pz))\right]^2$$
$$\leq \alpha_1\begin{bmatrix} d(Cz,FKM(Pz))d(E(Pz),DPV(Pz)) \\ +d(E(Pz),FKMz)d(Cz,DPV(Pz)) \end{bmatrix}$$
$$+\alpha_2\begin{bmatrix} d(Cz,FKMz)d(Cz,DPV(Pz)) \\ +d(E(Pz),DPV(Pz))d(E(Pz),FKMz) \end{bmatrix}$$
$$\left[d(z,Pz)\right]^2$$
$$\leq \alpha_1\left[d(z,Pz)d(Pz,Pz)+d(Pz,z)d(z,Pz)\right]$$
$$+\alpha_2\left[d(z,z)d(z,Pz)+d(Pz,Pz)d(Pz,z)\right]$$
$$\left[d(z,Pz)\right]^2 \leq \alpha_1\left[d(z,Pz)\right]^2$$

Which is a contradiction.

Hence Pz = z. Since DPz = z, implies that Dz = z.

Now to prove Mz = z, put x = Mz and y = z in (1) and using (3.1), (3.3) and (3.5), we have

$$[d(C(Mz),Ez)]^2$$
$$\leq \alpha_1\begin{bmatrix} d(C(Mz),FKM(Mz))d(Ez,DPVz) \\ +d(Ez,FKM(Mz))d(C(Mz),DPVz) \end{bmatrix}$$
$$+\alpha_2\begin{bmatrix} d(C(Mz),FKM(Mz))d(C(Mz),DPVz) \\ +d(Ez,DPVz)d(Ez,FKM(Mz)) \end{bmatrix}$$
$$\left[d(Mz,z)\right]^2$$
$$\leq \alpha_1\left[d(Mz,Mz)d(z,z)+d(z,Mz)d(Mz,z)\right]$$
$$+\alpha_2\left[d(Mz,Mz)d(Mz,z)+d(z,z)d(z,Mz)\right]$$
$$\left[d(Mz,z)\right]^2 \leq \alpha_1\left[d(Mz,z)\right]^2$$

Which is a contradiction.

Hence Mz = z. Since FKMz = z, implies that FKz = z.

Now to prove Kz = z, put x = Kz and y = z in (1) and using (3.1), (3.3) and (3.5), we have

$$[d(C(Kz),Ez)]^2$$
$$\leq \alpha_1\begin{bmatrix} d(C(Kz),FKM(Kz))d(Ez,DPVz) \\ +d(Ez,FKM(Kz))d(C(Kz),DPVz) \end{bmatrix}$$
$$+\alpha_2\begin{bmatrix} d(C(Kz),FKM(Kz))d(C(Kz),DPVz) \\ +d(Ez,DPVz)d(Ez,FKM(Kz)) \end{bmatrix}$$
$$[d(Kz,z)]^2$$
$$\leq \alpha_1\left[d(Kz,Kz)d(z,z)+d(z,Kz)d(Kz,z)\right]$$
$$+\alpha_2\left[d(Kz,Kz)d(Kz,z)+d(z,z)d(z,Kz)\right]$$
$$\left[d(Mz,z)\right]^2 \leq \alpha_1\left[d(Kz,z)d(z,Kz)\right]$$
$$\left[d(Mz,z)\right]^2 \leq \alpha_1\left[d(Kz,z)\right]^2$$

Which is a contradiction.

Hence Kz = z. Since FKz =z, implies that Fz = z.Thus Cz = Dz = Ez = Fz = Kz = Mz = Pz = Vz = z. Therefore z is common fixed point of C, D, E, F, K, M, P and V. Similarly we can prove this any one of C, D, E, F, P and V is continuous.

4. Uniqueness

Suppose w be another common fixed point of C, D, E, F, K. M, P and V. Then we have

$$\left[d(z,w)\right]^2 = \left[d(Cz,Ew)\right]^2$$
$$\leq \alpha_1\begin{bmatrix} d(Cz,FKMz)d(Ew,DPVw) \\ +d(Ew,FKMz)d(Cz,DPVw) \end{bmatrix}$$
$$+\alpha_2\begin{bmatrix} d(Cz,FKMz)d(Cz,DPVw) \\ +d(Ew,DPVw)d(Ew,FKMz) \end{bmatrix}$$
$$[d(z,w)]^2$$
$$\leq \alpha_1\left[d(z,z)d(w,w)+d(w,z)d(z,w)\right]$$
$$+\alpha_2\left[d(z,z)d(z,w)+d(w,w)d(w,z)\right]$$
$$\left[d(z,w)\right]^2 \leq \alpha_1\left[d(z,w)d(z,w)\right]$$
$$\left[d(z,w)\right]^2 \leq \alpha_1\left[d(z,w)\right]^2$$

Which is a contradiction.

Hence z = w. Therefore z is a unique common fixed point of C, D, E, F, K, M, P and V.

Corollary: Let C, D, E, K, M and V be self maps of a complete metric space (X, d) satisfying the following conditions:

(1) $C(X) \subseteq DV(X)$ *and* $E(X) \subseteq KM(X)$

$$\left[d(Cx,Ey)\right]^2$$

(2) $\leq \alpha_1\begin{bmatrix} d(Cx,KMx)d(Ey,DPVy) \\ +d(Ey,KMx)d(Cx,DVy) \end{bmatrix}$

$$+\alpha_2\begin{bmatrix} d(Cx,KMx)d(Cx,DVy) \\ +d(Ey,Vy)d(Ey,KMx) \end{bmatrix}$$

Where $0 \leq \alpha_1 + 2\alpha_2 < 1; \propto_1, \propto_2 \geq 0$.

(3) Let $x_0 \in X$ then by (1) there exists $x_1 \in X$ such that $DVx_1 = Cx_0$ and for x_1 there exists $x_2 \in X$ such that $KMx_2 = Ex_1$ and so on.continuing this process we candefine a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = DVx_{2n+1} = Cx_{2n}$$
$$\textit{and}\quad y_{2n} = KMx_{2n} = Ex_{2n-1}.$$

Then the sequence $\{y_n\}$ is a Cauchy sequence in X.

(4) One of C, E, KM, DV is continuous.

(5) [C, KM] and [E, DV] are compatible of type (R) on X.

Then C, D, E, K, M and V have a unique common fixed point in X.

5. Conclusion

In this paper, we have presented common fixed point theorem for eight mappings in metric spaces through concept of compatibility.

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