American Journal of Applied Mathematics and Statistics, 2014, Vol. 2, No. 4, 239-243
Available online at http://pubs.sciepub.com/ajams/2/4/12

DOI:10.12691/ajams-2-4-12

Common Fixed Points of a Countable Family of I-Nonexpansive Multivalued Mappings in Banach Spaces

Poonam Lata Sagar[1,*], S.K. Malhotra[2]

[1]Samrat Ashok Technological Institute, Vidisha (M.P.) India
[2]M.P. Professional Board of Examination, Bhopal (M.P.) India
*Corresponding author: poonamlata.sagar@gmail.com

Received April 24, 2014; Revised August 01, 2014; Accepted August 12, 2014

Abstract In this paper, we introduce a modified Ishikawa iteration for a countable family of multi-valued mappings. We use the best approximation operator to obtain weak and strong convergence theorems in a Banach space. We apply the main results to the problem of finding a common fixed point of a countable family of I-Nonexpansive multi-valued mappings.

Keywords: *I-Nonexpansive multi-valued mapping, fixed point, weak convergence, strong convergence, Banach space, Ishikawa iteration*

Cite This Article: Poonam Lata Sagar, and S.K. Malhotra, "Common Fixed Points of a Countable Family of I-Nonexpansive Multivalued Mappings in Banach Spaces." *American Journal of Applied Mathematics and Statistics*, vol. 2, no. 4 (2014): 239-243. doi: 10.12691/ajams-2-4-12.

1. Introduction

Let D be a nonempty and convex subset of a Banach spaces E. The set D is called proximinal if for each $x \in E$, there exists an element $y \in D$ such that $\|x-y\| = d(x,D)$, where $d(x,D) = \inf\{\|x-z\|: z \in D\}$. Let CB(D), CCB(D), K(D) and P(D) denote the families of nonempty closed bounded subsets, nonempty closed convex bounded subsets, nonempty compact subsets, and nonempty proximinal bounded subsets of D, respectively. The Hausdorff metric on CB(D) is defined by

$$H(A,B) = \max\left\{\sup_{x \in A} d(x,B) \quad \sup_{x \in B} d(y,A)\right\}$$

for $A,B \in CB(D)$. A single-valued map T: D→D is called nonexpansive if $\|T_x - T_y\| \le \|x-y\|$ for all $x,y \in D$. A multi-valued mapping T: D→CB(D) is called nonexpansive if $H(T_x,T_y) \le \|x-y\|$ for all $x,y \in D$. An element $p \in D$ is called a fixed point of T: D→D (respectively, T: D→CB(D)) if $p = T_p$ (respectively, $p \in T_p$). The set of fixed points of T is denoted by F(T). The mapping T: D→CB(D) is called quasi-nonexpansive [1] if $F(T) \neq \emptyset$ and $H(T_x,T_p) \le \|x-p\|$ for all $x \in D$ and all $p \in F(T)$. It is clear that every nonexpansive multi-valued mapping T with $F(T) \neq \emptyset$; is quasi-nonexpansive. But there exist quasi-nonexpansive mappings that are not nonexpansive (see [2]). It is known that if T is a quasi-nonexpansive multi-valued mapping, then F(T) is closed.

Throughout this paper, we denote the weak convergence and the strong convergence by – and→, respectively. The mapping T: D→CB(D) is called hemicompact if, for any sequence $\{x_n\}$ in D such that $d(x_n, Tx_n) \to 0$ as $n \to 1$, there exists a subsequence $\{x_{nk}\}$ of $\{x_n\}$ such that $x_{nk} \to p \in D$. We note that if D is compact, then every multi-valued mapping T: D→CB(D) is hemicompact.

A Banach space E is said to satisfy Opial's condition [3] if for each $x \in E$ and a sequence $\{x_n\}$ in E such that $x_n \to x$, the following condition holds for all $x \neq y$:

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|$$

The mapping T: D→CB(D) is called demi-closed if for every sequence $\{x_n\} \subset D$ and any $y_n \in Tx_n$ such that $x_n - x$ and $y_n \to y$, we have $x \in D$ and $y \in Tx$.

Remark 1.1 ([4]). If the space E satisfies Opial's condition, then I-T is demi-closed at 0, where T: D→K(D) is a nonexpansive multi-valued mapping.

For a single-valued case, in 1953, Mann [5] introduced the following iterative procedure to approximate a fixed point of a nonexpansive mapping T in a real Hilbert space H:

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n) Tx_n, \forall n \in \mathbb{N}, \tag{1.1}$$

where the initial point x_1 is taken in D arbitrarily and $\{\alpha_n\}$ is a sequence in (0,1).

However, we note that Mann's iteration process (1.1) has only weak convergence, in general; for instance, see [6,7,8].

Since 1953, Mann's iteration has extensively been studied by many authors (see, for examples, [9-18]). However, the studying of multivalued nonexpansive mappings is harder than that of single-valued nonexpansive mappings in both Hilbert spaces and Banach spaces.

The result of fixed points for multi-valued contractions and nonexpansive mappings by using the Hausdorff metric was initiated by Markin [19]. Later, different iterative processes have been used to approximate fixed points of multi-valued nonexpansive mappings (see also [1,20-26]).

In 2009, Song and Wang [26] proved strong and weak convergence theorems for Mann's iteration of a multi-valued nonexpansive mapping T in a Banach space. They studied strong convergence of the modified Mann iteration which is independent of the implicit anchor-like continuous path $z_t \in tu+(1-t)Tz_t$.

Let D be a nonempty and closed subset of a Banach space E, $\{\beta_n\}\subset[0,1]$, $\{\alpha_n\}\subset[0,1]$ and $\{\gamma_n\}\subset(0,+\infty)$ such that $\lim_{n\to\infty}\gamma_n=0$.

(A) Choose $x_0\in D$,

$$x_{n+1}=(1-\alpha_n)x_n+\alpha_n y_n, \forall n\geq 0,$$

where $y_n\in Tx_n$ such that $\|y_{n+1}-y_n\|\leq H(Tx_{n+1},Tx_n)+\gamma_n$.

(B) For fixed u 2 D, the sequence of modified Mann iteration is defined by $x_0\in D$,

$$x_{n+1}=\beta_n u+\alpha_n x_{n+}(1-\alpha_n-\beta_n)\gamma_n+, \forall n\geq 0,$$

where $y_n\in Tx_n$ such that $\|y_{n+1}-y_n\|\leq H(Tx_{n+1},Tx_n)+\gamma_n$.

Very recently, Shahzad and Zegeye [2] obtained the strong convergence theorems for a quasi-nonexpansive multi-valued mapping. They relaxed the compactness of domain of T and constructed an iterative scheme which removes the restriction of T namely $T_p=\{p\}$ for any $p\in F(T)$. The results provided an affirmative answer to some questions raised in [21]. In fact, they introduced iterations as follows:

Let D be a nonempty and convex subset of a Banach space E, let T: D→CB(D) and let $\{\alpha_n\}$, $\{\alpha'_n\}\subset[0,1]$.

(C) The sequence of Ishikawa's iteration is defined by $x_0\in D$,

$$y_n=\alpha'_n z'_n+(1-\alpha'_n)x_n,$$
$$x_{n+1}=\alpha_n z_n+(1-\alpha_n)x_n, \forall n\geq 0.$$

where $z'_n\in Tx_n$ and $z_n\in Ty_n$.

(D) Let T: D→P(D) and $P_Tx=\{y\in Tx:\|x-y\|=d(x,Tx)\}$, where PT is the best approximation operator. The sequence of Ishikawa's iteration [30] is defined by $x_0\in D$,

$$y_n=\alpha'_n z'_n+(1-\alpha'_n)x_n,$$
$$x_{n+1}=\alpha_n z_n+(1-\alpha_n)x_n, \forall n\geq 0.$$

where $z'_n\in \Pr x_n$ and $z_n\in \Pr y_n$.

It is remarked that Hussain and Khan [27], in 2003, employed the best approximation operator PT to study fixed points of *-nonexpansive multi-valued mapping T and strong convergence of its iterates to a fixed point of T defined on a closed and convex subset of a real Hilbert space.

Let D be a nonempty, closed and convex subset of a Banach space E. Let $\{T_n\}_{n=1}^{\infty}$ be a family of multi-valued mappings from D into 2^D and let $P_{T_n}x=\{y_n\in T_nx \ \|x-y_n\|=d(x,T_nx)\}$, $n\geq1$. Let $\{\alpha_n\}$ be a sequence in (0,1).

(E) The sequence of the modified Ishikawa's iteration is defined by $x_1\in D$ and

$$x_{n+1}\in\alpha_n x_n+(1-\alpha_n)\Pr y_n, \forall n\geq 1, \quad (1.2)$$

In this paper, we modify Mann's iteration by using the best approximation operator P_{T_n}, $n\geq1$ to find common fixed points of a countable family of nonexpansive multi-valued mappings $\{T_n\}_{n=1}^{\infty}, n\geq 1$. Then we prove weak and strong convergence theorems for a countable family of multi-valued mappings in Banach spaces. Finally, we apply our main result to the problem of finding a common fixed point of a family of nonexpansive multi-valued mappings.

2. Preliminaries

In this section, we give some characterizations and properties of the metric projection in a real Hilbert space.

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. Let D be a closed and convex subset of H. If, for any point x 2 H, there exists a unique nearest point in D, denoted by P_Dx, such that

$$\|x-P_Dx\|\leq\|x-y\|, \forall y\in D$$

then PD is called the metric projection of H onto D. We know that PD is a nonexpansive mapping of H onto D.

Lemma 2.1 ([28]). Let D be a closed and convex subset of a real Hilbert space H and PD be the metric projection from H onto D. Then, for any $x\in H$ and $z\in D$, z = PDx if and only if the following holds:

$$\langle x-z, y-z\rangle\leq 0, \forall y\in D$$

Using the proof line in Lemma 3.1.3 of [28], we obtain the following result.

Proposition 2.2. Let D be a closed and convex subset of a real Hilbert space H. Let T: D→CCB(D) be a multi-valued mapping and PT the best approximation operator. Then, for any $x\in D$, $z\in P_Tx$ if and only if the following holds:

$$\langle x-z, y-z\rangle\leq 0, \forall y\in Tx$$

Lemma 2.3 ([28]). Let H be a real Hilbert space. Then the following equations hold:

(1) $\|x-y\|^2=\|x\|^2-\|y\|^2-2\langle x-y,y\rangle$ *for all* $x,y\in H$;

(2) $\|tx+(1-t)y\|^2=t\|x\|^2+(1-t)\|y\|^2-t(1-t)\|x-y\|^2$

for all $t\in[0,1]$ and $x,y\in H$.

We next show that P_T is nonexpansive under some suitable conditions imposed on T.

Remark 2.4. Let D be a closed and convex subset of a real Hilbert space H. Let T: D→CCB(D) be a multi-valued mapping. If Tx = Ty, ∀ $x,y\in D$, then P_T is a nonexpansive multi-valued mapping.

In fact, let $x,y\in D$. For each $a\in P_Tx$, we have

$$d(a,P_T y)\leq\|a-b\|, \forall b\in P_T y \quad (2.1)$$

From Proposition 2.2, we have

$$\langle x-y-(a-b), a-b\rangle=\langle x-a,a-b\rangle+\langle y-b,b-a\rangle\geq 0$$

It follows that

$$\|a-b\|^2 = \langle x-a, a-b\rangle + \langle a-b-(x-y), a-b\rangle \le (x-y, a-b) \le \|x-y\|\|a-b\| \quad (2.2)$$

This implies that

$$\|a-b\| \le \|x-y\| \quad (2.3)$$

From (2.1) and (2.3), we obtain

$$d(a, P_T y) \le \|x-y\|$$

for every $a \in P_T x$. Hence $\sup_{a \in P_T x} d(a, P_T y) \le \|x-y\|$. Similarly, we can show that $\sup_{b \in P_T x} d(P_T x, b) \le \|x-y\|$. Therefore $H(P_T x, P_T y) \le \|x-y\|$.

It is clear that if a nonexpansive multi-valued mapping T satisfies the condition that Tx = Ty, ∀ x,y∈D, then PT is nonexpansive. The following example shows that if T is a nonexpansive multi-valued mapping satisfying the property that Tx = Ty, ∀ x,y∈D, then Tx is not a singleton for all x∈D.

3. Strong and Weak Convergence of the Modified Ishikawa Iteration in Banach Spaces

In this section, we first prove a strong convergence theorem for a countable family of multi-valued mappings under the SC-condition and Condition (A) and then prove a weak convergence theorem under the SC-condition in Banach spaces.

Theorem 3.1. Let D be a closed and convex subset of a uniformly convex Banach space E which satisfies Opial's condition. Let $\{T_n\}$ and τ be two families of multivalued mappings from D into P(D) with $F(\tau) = \bigcap_{n=1}^{\infty} F(T_n) \neq \varnothing$. Let $\{\alpha_n\}$ be a sequence in (0,1) such that 0 $0 < \liminf_{n\to\infty} \alpha_n \le \limsup_{n\to\infty} \alpha_n < 1$. Let $\{x_n\}$ be generated by (1.2). Assume that

(A1) for each $n \in \mathbb{N}, H\left(P_{T_n} x, P_{T_n} p\right) \le \|x-p\|, \forall x \in D, p \in F(\tau)$;

(A2) I-T is demi-closed at 0 for all T∈τ.

If $\{T_n\}$ satisfies the SC-condition, then $\{x_n\}$ converges weakly to an element in F(τ).

Proof. Since

$$x_{n+1} \in \alpha_n x_n + (1-\alpha_n) PT_n y_n$$
$$y_n \in \beta_n x_n + (1-\beta_n) PT_n x_n$$

there exists $z_n \in PT_n x_n$, $p \in F(\tau)$ and $n \in \mathbb{N}$. We note that $PT_n p = \{p\}$ for all. It follows from (A1) that

$$\begin{aligned}\|y_n - p\| &= \|\beta_n x_n + (1-\beta_n) PT_n x_n - P\| \\ &\le \beta_n \|x_n - p\| + (1-\beta_n)\|PT_n x_n - P\| \\ &\le \beta_n \|x_n - p\| + (1-\beta_n)\|z_n - P\| \\ &= \beta_n \|x_n - p\| + (1-\beta_n) d(z_n, PT_n p) \\ &\le \beta_n \|x_n - p\| + (1-\beta_n) H(PT_n x_n, PT_n p) \\ &\le \beta_n \|x_n - p\| + (1-\beta_n)\|x_n - P\| \le \|x_n - P\|\end{aligned} \quad (3.1)$$

and Also

$$\begin{aligned}\|x_{n+1} - P\| &= \|a_n x_n + (1-\alpha_n) PT_n y_n - P\| \\ &= \|a_n x_n + (1-\alpha_n) PT_n y_n - (1-a_n+a_n)P\| \\ &\le a_n \|x_n - P\| + (1-a_n)\|PT_n y_n - P\| \\ &\le a_n \|x_n - P\| + (1-a_n) PT_n \|y_n - P\| \\ &\le a_n \|x_n - P\| + (1-a_n) PT_n \|x_n - P\| \\ &\le a_n \|x_n - P\| + (1-a_n)\|PT_n x_n - P\| \\ &\le a_n \|x_n - P\| + (1-a_n)\|z_n - P\| \\ &= a_n \|x_n - P\| + (1-a_n) d(z_n, PT_n p) \\ &\le a_n \|x_n - P\| + (1-a_n) H(PT_n x_n, PT_n p) \\ &\le a_n \|x_n - P\| + (1-a_n)\|x_n - P\| \\ &\le \|x_n - P\|\end{aligned} \quad (3.2)$$

for every p∈F(T). Then $<\|x_n-p\|>$ is a decreasing sequence and hence $\lim_{n\to\infty}\|x_n - p\|$ exists for every p∈F(T). For p∈F(T), since $<x_n>$ and $<z_n>$ are bounded by Lemma 2.9, there exists a continuous, strictly increasing and convex function g: [0,1)→[0,1) with g(0) = 0 such that

$$\begin{aligned}\|y_n - P\|^2 &= \|\beta_n (x_n - p) + (1-\beta_n)(z_n - P)\|^2 \\ &\le \beta_n \|x_n - p\|^2 + (1-\beta_n)\|z_n - P\|^2 \\ &\quad -\beta_n (1-\beta_n) g(\|x_n - z_n\|) \\ &\le \beta_n \|x_n - p\|^2 + (1-\beta_n) H(PT_n x_n, PT_n p) \\ &\quad -\beta_n (1-\beta_n) g(\|x_n - z_n\|) \\ &\le \|x_n - p\|^2 - \beta_n (1-\beta_n) g(\|x_n - z_n\|)\end{aligned}$$

It follows that

$$\beta_n (1-\beta_n) g(\|x_n - z_n\|) \le \|x_n - p\|^2 - \|y_n - p\|^2 = 0$$

Since $\lim_{n\to\infty}\|x_n - p\|$ exists and $0 < \liminf_{n\to\infty} \alpha_n \le \limsup_{n\to\infty} \alpha_n < 1$,

$$\lim_{n\to\infty} g(\|x_n - z_n\|) = 0$$

By the properties of g, we can conclude that

$$\lim_{n\to\infty} \|x_n - z_n\| = 0$$

Since $<T_n>$ satisfies the SC-condition, there exists $C_n \in T_{x_n}$ such that

$$\lim_{n\to\infty} \|x_n - z_n\| = 0 \quad (3.3)$$

for every T∈τ. Since $<x_n>$ is bounded, there exists a subsequence $<x_{nk}>$ of $<x_n>$ converges weakly to some $q_1 \in D$. It follows from (A2) and (3.3) that $q_1 \in Tq_1$ for every T∈τ. Next, we show that $<x_n>$ converges weakly to q1, take another subsequence $<x_{mk}>$ of $<x_n>$ converging weakly to some $q_2 \in D$. Again, as above we can conclude that $q_2 \in Tq_2$ for every T∈τ. Finally, we show that $q_1 = q_2$. Assume $q_1 \neq q_2$. Then by Opial's condition of E, we have

$$\begin{aligned}\lim_{n\to\infty} \|x_n - q_1\| &= \lim_{k\to\infty} \|x_{nk} - q_1\| < \lim_{k\to\infty} \|x_{nk} - q_2\| \\ &= \lim_{n\to\infty} \|x_n - q_2\| = \lim_{k\to\infty} \|x_{mk} - q_2\| \\ &< \lim_{k\to\infty} \|x_{mk} - q_1\| = \lim_{n\to\infty} \|x_n - q_1\|\end{aligned}$$

which is a contradiction. Therefore $q_1 = q_2$. This shows that <x_n> converges weakly to a fixed point of τ for every $T \in \tau$. This completes the proof.

Corollary 3.2. Let D be a closed and convex subset of a uniformly convex Banach space E which satisfies Opial's condition. Let $\{T_n\}$ and τ be two families of nonexpansive multivalued mappings from D into K(D) with $F(\tau) = \bigcap_{n=1}^{\infty} F(T_n) \neq \varnothing$. Let $\{\alpha_n\}$ be a sequence in (0, 1) such that $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$. Let $\{x_n\}$ be generated by (1.2). Assume that for each $n \in \mathbb{N}$,

$$H\left(P_{T_n} x, P_{T_n} p\right) \leq \|x - p\|$$

$\forall\, x \in D$, $p \in F(\tau)$. If $\{T_n\}$ satisfies the SC-condition, then $\{x_n\}$ converges weakly to an element in $F(\tau)$.

Theorem 3.3. Let D be a closed and convex subset of a uniformly convex Banach space E which satisfies Opial's condition. Let $\{T_n\}$ and τ be two families of multivalued mappings from D into P(D) with $F(\tau) = \bigcap_{n=1}^{\infty} F(T_n) \neq \varnothing$. Let $\{\alpha_n\}$ be a sequence in (0,1) such that 0 $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$. Let $\{x_n\}$ be generated by (1.2). Assume that

(B1) for each $n \in \mathbb{N}, H\left(P_{T_n} x, P_{T_n} p\right) \leq \|x - p\|, \forall x \in D, p \in F(\tau)$;

(B2) the best approximation operator P_T is nonexpansive for every $T \in \tau$;

(B3) $F(\tau)$ is closed.

If $\{T_n\}$ satisfies the SC-condition and Condition (A), then fxng converges strongly to an element in $F(\tau)$.

Proof. It follows from the proof of Theorem 3.1 that $\lim_{n\to\infty} \|x_n - p\|$ exists for every $p \in F(\tau)$ and $\lim_{n\to\infty} \|x_n - z_n\|$ where $z_n \in P_{T_n} x_n$. Since fTng satisfies the SC- condition, there exists $c_n \in Tx_n$ such that

$$\lim_{n\to\infty} \|x_n - c_n\| = 0$$

for every $T \in \tau$. This implies that

$$\lim_{n\to\infty} d\left(x_n, Tx_n\right) \leq \lim_{n\to\infty} d\left(x_n, P_T x_n\right) < \lim_{n\to\infty} \|x_n - c_n\| = 0$$

for every $T \in \tau$. Since that $\{T_n\}$ satisfies Condition (A), we have $\lim_{n\to\infty} d\left(x_n, F(\tau)\right) = 0$.

It follows from (B3), there is subsequence $\left\{x_{n_k}\right\}$ of $\{x_n\}$ and a sequence $\{p_k\} \subset F(\tau)$ such that

$$\left\|x_{n_k} - p_k\right\| < \frac{1}{2^k} \tag{3.4}$$

for all k. From (3.1), we obtain

$$\begin{aligned} \left\|x_{n_{k+1}} - p\right\| &\leq \left\|x_{n_{k+1}-1} - p\right\| \\ &\leq \left\|x_{n_{k+1}-2} - p\right\| \\ &\vdots \\ &\leq \left\|x_{n_k} - p\right\| \end{aligned}$$

for all $p \in F(\tau)$. This implies that

$$\left\|x_{n_{k+1}-1} - p_k\right\| \leq \left\|x_{n_k} - p_k\right\| < \frac{1}{2^k} \tag{3.5}$$

Next, we show that $\{p_k\}$ is a Cauchy sequence in D. From (3.4) and (3.5), we have

$$\begin{aligned} \|p_{k+1} - p_k\| &\leq \|p_k\| \\ &\leq \left\|p_{k+1} - x_{n_{k+1}}\right\| + \left\|x_{n_{k+1}} - p_k\right\| \leq \frac{1}{2^{k-1}} \end{aligned} \tag{3.6}$$

This implies that $\{p_k\}$ is a Cauchy sequence in D and thus converges to $q \in D$. Since PT is nonexpansive for every $T \in \tau$,

$$d\left(p_k, Tq\right) \leq d\left(p_k, P_T q\right) \leq H\left(P_T p_k, P_T q\right) \leq \|p_k - q\| \tag{3.7}$$

for every $T \in \tau$. It follows that $d(q,Tq) = 0$ for every $T \in \tau$ and thus $q \in F(\tau)$. It implies by (3.4) that $\{x_{nk}\}$ converges strongly to q. Since $\lim_{n\to\infty} \|x_n - q\|$ exists, it follows that $\{x_n\}$ converges strongly to q. This completes the proof.

We know that if T is a quasi nonexpansive multivalued mapping, then F(T) is closed. So we have the following result:

Corollary 3.4. Let D be a closed and convex subset of a uniformly convex Banach space E. Let $\{T_n\}$ and τ be two families of nonexpansive multivalued mappings from D into P(D) with $F(\tau) = \bigcap_{n=1}^{\infty} F(T_n) \neq \varnothing$. Let $\{\alpha_n\}$ be a sequence in (0,1) such that $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$. Let $\{x_n\}$ be generated by (1.2). Assume that for each $n \in \mathbb{N}, H\left(P_{T_n} x, P_{T_n} p\right) \leq \|x - p\|, \forall x \in D, p \in F(\tau)$ and the best approximation operator P_T is nonexpansive for every $T \in \tau$.

If $\{T_n\}$ satisfies the SC-condition and Condition (A), then $\{x_n\}$ converges strongly to an element in $F(\tau)$.

References

[1] C. Shiau, K.K. Tan, C.S. Wong, Quasi-nonexpansive multi-valued maps and selection, Fund. Math. 87 (1975) 109-119.

[2] N. Shahzad, H. Zegeye, On Mann and Ishikawa iteration schemes for multivalued maps in Banach spaces, Nonlinear Anal. 71 (2009) 838-844.

[3] Z. Opial, Weak convergence of the sequence of successive approximations for nonexpansive mappings, Bull. Am. Math. Soc. 73 (1967) 591-597.

[4] E.L. Dozo, Multivalued nonexpansive mappings and Opials condition, Proc. Am. Math. Soc. 38 (1973) 286-292.

[5] W.R. Mann, Mean value methods in iteration, Proc. Am. Math. Soc. 4 (1953) 506-510.

[6] H.H. Bauschke, E. Matouskova, S. Reich, Projection and proximal point methods: convergence results and counterexamples, Nonlinear Anal. 56 (2004) 715-738.

[7] A. Genal, J. Lindenstrass, An example concerning fixed points, Israel J.Math. 22 (1975) 81-86.

[8] S. Reich, Weak convergence theorems for nonexpansive mappings in Banach spaces, J. Math. Anal. Appl. 67 (1979) 274-276.

[9] D. Boonchari, S. Saejung, Construction of common fixed points of a countable family of -demicontractive mapping in arbitrary Banach spaces, Appl. Math. Comput. 216 (2010) 173-178.

[10] P. Cholamjiak, S. Suantai, Weak convergence theorems for a countable family of strict pseudocontractions in Banach spaces, Fixed Point Theory Appl., Vol. 2010, Article ID 632137, 16 pages.

[11] Y.J. Cho, S.M. Kang, H. Zhou, Some conditions on iterative methods, Commun. Appl. Nonlinear Anal. 12 (2005) 27-34.

[12] S. Itoh, W. Takahashi, Singlevalued mappings, multivalued mappings and fixed point theorems, J. Math. Anal. Appl. 59 (1977) 514-521.

[13] W.A. Kirk, Transfinite methods in metric fixed point theory, Abstr. Appl. Anal. 5 (2003) 311-324.

[14] T.C. Lim, Remarks on some fixed point theorems, Proc. Am. Math. Soc. 60 (1976) 179-182.

[15] N. Shioji, W. Takahashi, Strong convergence of approximated sequences for nonexpansive mappings in Banach spaces, Proc. Am. Math. Soc. 125 (1997) 3641-3645.

[16] Y. Song, On a Mann type implicit iteration processes for continuous pseudo-contractive mappings, Nonlinear Anal. 67 (2007) 3058-3063.

[17] Y. Song, S. Hu, Strong convergence theorems for nonexpansive semigroup in Banach spaces, J. Math. Anal. Appl. 338 (2008) 152-161.

[18] T. Suzuki, Strong convergence theorems for infinite families of nonexpansive mappings in general Banach space, Fixed Point Theory Appl. 1 (2005) 103-123.

[19] J.T. Markin, Continuous dependence of fixed point sets, Proc. Am. Math. Soc. 38 (1973) 545-547.

[20] M. Abbas, S.H. Khan, A.R. Khan, R.P. Agarwal, Common fixed points of two multivalued nonexpansive mappings by one-step iterative scheme, Appl. Math. Lett. 24 (2011) 97-102.

[21] B. Panyanak, Mann and Ishikawa iterative processes for multivalued mappings in Banach spaces, Comput. Math. Appl. 54 (2007) 872-877.

[22] D.R. Sahu, Strong convergence theorems for nonexpansive type and non-self multi- valued mappings, Nonlinear Anal. 37 (1999) 401-407.

[23] K.P.R. Sastry, G.V.R. Babu, Convergence of Ishikawa iterates for a multivalued map-pings with a fixed point, Czechoslovak Math. J. 55 (2005) 817-826.

[24] Y. Song, H. Wang, Erratum to Mann and Ishikawa iterative processes for multivalued mappings in Banach spaces [Comput. Math. Appl. 54 (2007) 872-877]. Comput. Math. Appl. 55 (2008) 2999-3002.

[25] Y. Song, Y.J. Cho, Iterative approximations for multivalued nonexpansive mappings in reexive Banach spaces, Math. Ineq. Appl. 12 (3) (2009) 611-624.

[26] Y. Song, H. Wang, Convergence of iterative algorithms for multivalued mappings in Banach spaces, Nonlinear Anal. 70 (2009) 1547-1556.

[27] N. Hussain, A.R. Khan, Applications of the best approximation operator to *- nonexpansive maps in Hilbert spaces, Numer. Funct. Anal. Optim. 24 (2003) 327-338.

[28] W. Takahashi, Nonlinear Functional Analysis: Fixed Point Theory and its Applications, Yokohama Publishers, Yokohama, 2000. 10 Poonam Lata Sagar and S.K. Malhotra

[29] K. Nakajo, K. Shimoji, W. Takahashi, Strong convergence to common fixed points of families of nonexpansive mappings in Banach spaces, J. Nonlinear and Convex Anal. 8 (2007) 11-34.

[30] S. Ishikawa, Fixed points by a new iteration method, Proc. Am. Math. Soc. 44 (1974) 147-150.

[31] H.K. Xu, Inequalities in Banach spaces with applications, Nonlinear Anal. 16 (1991) 1127-1138.

[32] W. Cholamjiak, S. Suantai,Weak and strong convergence theorems for a _nite family of generalized asymptotically quasi-nonexpansive mappings, Comput. Math. Appl. 60 (2010) 1917-1923.