Evaluation of Mean Time to System Failure of a Repairable 3-out-of-4 System with Online Preventive Maintenance

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Abstract Most of the literature assumed that systems undergo preventive maintenance. Little literature is found on whether the preventive maintenance is online or offline. It is known that most of the engineering systems undergo both online and offline preventive maintenance. In this paper, we studied the mean time to system failure of a repairable redundant 3-out-of-4 system with online preventive maintenance involving four types of failures. We develop the explicit expressions for mean time to system failure for the system using Chapman-Kolmogorov equations. Various cases are analyzed graphically to investigate the impact of system parameters on mean time to system failure. Results have shown that system with online preventive maintenance is better in terms of mean time to system failure of system than system without preventive maintenance.

Keywords: Mean time to system failure, online preventive maintenance

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1. Introduction

Preventive Maintenance schedules that minimize resource consumption or maximize availability can be determined through the use of quantitative decisionmodels, based on factual information such as time-tofailure distributions, cost of intervention (e.g. for inspection, repair or replacement) and consequences of failure. Most of engineering systems undergo either offline or online preventive. Under online Preventive maintenance is carried out when the system is operating and intends to slow down the wear process and reduce the frequency of occurrence of system or components failures.

There are systems of three/four units in which two/three units are sufficient to perform the entire function of the system. Example of such systems are 2-out-of-3,2-out-of-4, or 3-out-of-4 redundant systems which can be seen in modular communication amplifier system.

Many research results have been reported on reliability of redundant systems. For example, Chander and Bhardwaj[1], analyzed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Chander and Bhardwaj [2] present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik [3] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Wang et al [4] examined the cost benefit analysis of series systems with cold standby components and repairable service station. El-Said [5] and Haggag [6] examined the cost analysis of two unit cold standby system involving preventive maintenance respectively. Haggag [7] analyzed cost analysis of repairable k-out-of-n system with dependable failures and standby support. Wang and Kuo [8] studied the cost and probabilistic analysis of series system with mixed standby components. Wang et al [9] studied cost benefit analysis of series systems with warm standby components involving general repair time where the server is not subject to breakdowns. Yusuf [10] studied the availability and profit of 3-out-of-4 system.

Reference [10] studied the availability and profit of 3out-of-4 system with preventive maintenance. In his study, it is not certain whether the preventive maintenance is offline or online. In fact most of studies on system reliability evaluation, did not mention which category of preventive maintenance the system will undergo. In our study, we assume that the system undergo online preventive maintenance where the system is operation while the preventive maintenance taking place. Also availability and profit cannot be use to judge the effectiveness of the system. The time interval from the time system is new to the time when the system experience its first failure (Mean time to system failure) is equally important in determining the effectiveness which has not been studied in Reference [10].

This paper is continuation of the work of Yusuf [10], a repairable 3-out-of-4 system under online preventive

maintenance is considered and derived its corresponding mathematical model. The main contribution of this paper is three fold. First, is to develop the explicit expressions mean time to system failure. The second is to perform a parametric investigation of system parameters on mean time to system failure and capture their effect. The third to compare the mean time to system failure with and without online preventive maintenance to highlight the impact of online preventive maintenance on mean time to system failure.

The rest of the paper is organized as follows. Section 2 gives the notations and assumptions of the study. Section 3 is the description of the system. Section 4 deals with model formulation. The results of our numerical simulations are presented and discussed in Section 5. The paper is concluded in Section 6.

2. Notations and Assumptions

 α_i : Constant repair rates for type i = 1, 2, 3, 4

 β_i : Constant failure rates for types i = 1, 2, 3, 4

 μ : Constant rate end of preventive maintenance

 $\boldsymbol{\lambda}$: Constant rate of taking the unit into preventive maintenance

A : System transition rate matrix

 $MTSF_1$: Mean time to system failure with online preventive maintenance

 $MTSF_2$: Mean time to system failure without online preventive maintenance

- 1. The system is 3-out-of-4 system
- 2. The system can be in Operation, Fail state or online preventive maintenance
- 3. The system suffer four types of failures
- 4. The system is down when number of units failure goes beyond one
- 5. Failure and repair time follow exponential
- 6. Failure rates and repair rates are constant
- 7. The system is attended by one repairman

3. Description of the System

In this section, the 3-out-of-4 redundant system is considered. Through Markov assumption, the Chapman-Kolmogorov's equations are obtained for the analysis of state probabilities. The system comprise of four units in which at least three units most be in operational for the system to work. Malfunctioning of two units lead the system to go down. The units can work consecutively or randomly as can be seen in the states of the system given below. The system transit to preventive maintenance before failure at the rate λ with corresponding rate of preventive maintenance μ . Unit *i* fails with rate β_i and is under minimal repair with rate α_i and the standby unit is switch on. It is assumed that the switch from standby to operation is perfect. The system failed when two units failed. The states of the system according Markov chain is shown in Table 1 below.

	\mathbf{S}_0	\mathbf{S}_1	\mathbf{S}_2	S_3	S_4	S_5	S_6	S_7	S_8	S ₉	\mathbf{S}_{10}
\mathbf{S}_0	0	β ₃	β_2	β_1	0	0	0	0	0	0	λ
\mathbf{S}_1	α ₃	0	0	0	0	β_1	β_4	0	0	0	0
S_2	α_2	0	0	0	β_1	0	0	0	β3	β_4	0
S ₃	α_1	0	0	0	β_2	β_3	0	β_4	0	0	0
S_4	0	0	α_1	α_2	0	0	0	0	0	0	0
S ₅	0	α_1	0	α ₃	0	0	0	0	0	0	0
S_6	0	α_4	0	0	0	0	0	0	0	0	0
S ₇	0	0	0	α_4	0	0	0	0	0	0	0
S ₈	0	0	α ₃	0	0	0	0	0	0	0	0
S ₉	0	0	α_4	0	0	0	0	0	0	0	0
S ₁₀	μ	0	0	0	0	0	0	0	0	0	0

State of the System

State 0: Units 1,2 and 3 are working, unit 4 in standby, the system is working

State 1: units 1,2,and 4 are working; unit 3 is down and under repair, the system is working

State 2: units 1,3 and 4 are working, unit 2 is down and under repair, the system is working

State 3: units 2,3 and 4 are working, unit 1 is down and under repair, the system is working

State 4: units 1 is down, under repair, units 2 is down and waiting for repair, units 3 and 4 are idle, the system failed

State 5: unit 1 is down, under repair, unit 3 is down, waiting for repair, units 2 and 4 are idle, the system failed

State 6: unit 1 and 2 are idle, unit 3 is down, and waiting for repair, unit 4 is down, under repair, the system failed

State 7: unit 1 is down, waiting for repair, units 2 and 3 are idle, unit 4 is down, under repair, the system failed

State 8: units 1 and 4 are idle, unit 2 is down, and waiting for repair, unit 3 is down, under repair, the system failed

State 9: units 1 and 3 are idle, unit 2 is down, and waiting for repair, unit 4 is down, under repair, the system failed

State 10: all units are under online preventive maintenance, the system is working

Table 1. Transition rate table

4. Model Formulation

Let $P_i(t)$ be the probability that the system is in state *i* at time *t*. Let P(t) be the probability row vector at time *t*, then the initial conditions for this problem are as follows:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)]$$
$$= [1, 0, 0, 0, 0, 0, 0]$$

The corresponding set of kolmogorov's differential equations obtained from Table 1 is as follows:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2 + \beta_3 + \lambda)P_0(t) + \alpha_3P_1(t) + \alpha_2P_2(t) + \alpha_1P_3(t) + \mu P_{10}(t) \frac{dP_1(t)}{dt} = -(\alpha_3 + \beta_1 + \beta_4)P_1(t) + \beta_3P_0(t) + \alpha_1P_5(t) + \alpha_4P_6(t) dP_4(t)$$

$$\frac{dI_2(t)}{dt} = -(\alpha_2 + \beta_1 + \beta_3 + \beta_4)P_2(t) + \beta_2 P_0(t) + \alpha_1 P_4(t) + \alpha_3 P_8(t) + \alpha_4 P_9(t)$$

$$\frac{dP_3(t)}{dt} = -(\alpha_1 + \beta_2 + \beta_3 + \beta_4)P_3(t) + \beta_1 P_0(t) + \alpha_2 P_4(t) + \alpha_3 P_5(t) + \alpha_4 P_7(t)$$

$$\begin{aligned} \frac{dP_4(t)}{dt} &= -(\alpha_1 + \alpha_2)P_4(t) + \beta_1 P_2(t) + \beta_2 P_3(t) \\ \frac{dP_5(t)}{dt} &= -(\alpha_1 + \alpha_3)P_5(t) + \beta_1 P_1(t) + \beta_3 P_3(t) \\ \frac{dP_6(t)}{dt} &= -\alpha_4 P_6(t) + \beta_4 P_1(t) \\ \frac{dP_7(t)}{dt} &= -\alpha_4 P_7(t) + \beta_4 P_3(t) \\ \frac{dP_8(t)}{dt} &= -\alpha_3 P_8(t) + \beta_3 P_2(t) \\ \frac{dP_9(t)}{dt} &= -\alpha_4 P_9(t) + \beta_4 P_2(t) \\ \frac{dP_{10}(t)}{dt} &= -\mu P_{10}(t) + \lambda P_0(t) \end{aligned}$$
(1)

The differential equations in (1) above is transformed into matrix as

$$P' = TP \tag{2}$$

where

	$-y_1$	α_3	α_2	α_1	0	0	0	0	0	0	μ
	β_3	$-y_{2}$	0	0	0	α_1	$lpha_4$	0	0	0	0
	β_2	0	$-y_{3}$	0	α_1	0	0	0	α_3	$lpha_4$	0
	β_1	0	0	$-y_{4}$	α_2	α_3	0	$lpha_4$	0	0	0
	0	0	β_1	β_2	$-y_{5}$	0	0	0	0	0	0
T =	0	β_1	0	β_3	0	$-y_{6}$	0	0	0	0	0
	0	β_4	0	0	0	0	$-\alpha_4$	0	0	0	0
	0	0	0	β_4	0	0	0	$-\alpha_4$	0	0	0
	0	0	β_3	0	0	0	0	0	$-\alpha_3$	0	0
	0	0	β_4	0	0	0	0	0	0	$-\alpha_4$	0
	λ	0	0	0	0	0	0	0	0	0	$-\mu$

$$y_1 = (\lambda + \beta_1 + \beta_2 + \beta_3), y_2 = (\alpha_3 + \beta_1 + \beta_4),$$

$$y_3 = (\alpha_2 + \beta_1 + \beta_3 + \beta_4), y_4 = (\alpha_1 + \beta_2 + \beta_3 + \beta_4)$$

$$y_5 = (\alpha_1 + \alpha_2), y_6 = (\alpha_1 + \alpha_3)$$

It is difficult to evaluate the transient solutions, hence we follow El-said [5], Haggag [6] and Wang [9], the procedure to develop the explicit expression for MTSF is to delete the rows and columns of an absorbing state in matrix T and take the transpose to produce a new matrix, say Q . The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} = \frac{N}{D} (3)$$

Where

$$Q = \begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \beta_3 & \beta_2 & \beta_1 & \lambda \\ \alpha_3 & -(\alpha_3 + \beta_1 + \beta_4) & 0 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_2 + \beta_1 + \beta_3 + \beta_4) & 0 & 0 \\ \alpha_1 & 0 & 0 & -(\alpha_2 + \beta_2 + \beta_3 + \beta_4) & 0 \\ \mu & 0 & 0 & 0 & -\mu \end{bmatrix}$$

$$N = \mu \begin{pmatrix} a_{3}\beta_{2}\beta_{3} + \beta_{1}\beta_{2}\beta_{3} + \beta_{2}\beta_{3}\beta_{4} + a_{3}\beta_{2}\beta_{4} + 2\beta_{1}\beta_{2}\beta_{4} + \beta_{2}\beta_{4}^{2} + a_{2}a_{3}\beta_{2} + a_{2}\beta_{1}\beta_{2} + a_{2}\beta_{1}\beta_{4} + a_{3}\beta_{1}\beta_{4} + a_{2}\beta_{3}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{3}\beta_{1}\beta_{3}\beta_{4} + a_{2}a_{3}\beta_{3} + 2a_{3}\beta_{3}\beta_{4} + \beta_{1}^{2}\beta_{4} + 2\beta_{1}\beta_{4}^{2} + \beta_{3}^{2}\beta_{4} + a_{1}\beta_{1}\beta_{3} + a_{2}\beta_{3}^{2} + a_{1}\beta_{2}^{2} + a_{1}a_{2}\beta_{1} + a_{1}a_{2}\beta_{1} + a_{1}a_{2}\beta_{1} + a_{1}\beta_{1}\beta_{4} + a_{1}\beta_{1}\beta_{3} + \beta_{3}^{2} + a_{1}a_{2}\beta_{1}^{2} + a_{1}a_{2}\beta_{1} + a_{1}a_{3}\beta_{1} + a_{1}a_{3}\beta_{3} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{2}\beta_{3} + \beta_{1}\beta_{4} + a_{2}\beta_{3} + \beta_{1}\beta_{4}^{2} + a_{1}a_{2}a_{3} + a_{2}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{2}a_{3} + a_{2}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{2}a_{3} + a_{2}a_{3}\beta_{4} + a_{1}a_{3}\beta_{4} + a_{2}a_{3} + a_{2}\beta_{1}\beta_{4} + a_{2}a_{3} + a_{2}\beta_{1}\beta_{4} + a_{2}\beta_{3}\beta_{4} + a_{2}a_{3}\beta_{4} + a_{2}\beta_{1}\beta_{4} + a_{2}\beta_{3}\beta_{4} + a_{2}a_{3}\beta_{4} + a_{2}\beta_{1}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{2}\beta_{1}\beta_{3} + a_{2}\beta_{1}\beta_{3} + a_{1}\beta_{3}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{1}\beta_{3}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{1}\beta_{3}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{1}\beta_{2}\beta_{4} + a_{1}\beta_{3}\beta_{4} + a_{1}\beta_{3}\beta_{4} + a_{1}\beta_{3}\beta_{4} + a_{2}\beta_{1}\beta_{3} + a_{2}\beta_{1}\beta_{3}\beta_{4} + a_{2}\beta_{1$$

5. Results and Discussions

In this section, we numerically obtained the results for mean time to system failure. For the model analysis, the following set of parameters values are fixed throughout the simulations for consistency:

$$\beta_1 = 0.2, \beta_2 = 0.2, \beta_3 = 0.3, \beta_4 = 0.5, \alpha_1 = 0.7,$$

 $\alpha_2 = 0.28, \alpha_3 = 0.7, \alpha_4 = 0.4, \mu = 0.8, \lambda = 0.98$

The simulations in Figure 1 and Figure 3 have shown that the mean time to system failure for both systems with and without preventive maintenance increase with increase in repair rate α_1 . It is evident from Figure 3 that the mean time to system failure of system with preventive maintenance increases more with respect to repair rate α_1 than the mean time to system failure of system of system without preventive maintenance. In Figure 2 and Figure 4 it is clear that the mean time to systems with and without preventive maintenance. Here also the mean time to system failure of system failure of system of system of system with preventive maintenance decreases more than the mean time to system failure of system of system with preventive maintenance decreases more than the mean time to system failure of system of system without preventive maintenance.



Figure 1. Effect of α_1 on MTSF





α,

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9



6. Conclusion

In this paper, we developed the explicit expression for mean time to system failure of repairable 3-out-of-4 system in the presence of online preventive maintenance. In order to determine the effectiveness of the system under study, we performed numerical investigation to see the effect of failure and repair rates on mean time to system failure. It is evident from the results obtained that failure and repair rates decrease and increase the meant time to system failure of the system respectively. Through the analysis, we conclude that system with online preventive maintenance is more effective than system without online preventive maintenance.

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