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Unique Fixed Points on OWC for Self-Maps with Generalized Contractive Type Conditions in CMS

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Abstract In this paper, we obtain fixed point theorem for OWC (Occasionally Weakly Compatible) self-mappings satisfying a generalized contractive type condition in CMS (Cone Metric Space). Our results are generalizing and improving some of the well known comparable results existing in this literature.

Keywords: *fixed point, occasionally weakly compatible, cone metric space, normal cone*

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1. Introduction

The fixed point theory is an important area of non-linear analysis. Recently Huang and Zhang [1] generalized the concept of metric space into cone metric space. And the concept of cone metric space initially introduced by the Huang and Zhang [1] and they replacing the real numbers by an ordered Banach space and they also proved some fixed point theorems in cone metric space. Later on many authors inspired with this results extended these results in many way (see for e.g., [2-11]) in different types of contractive type conditions in cone metric space. Recently Bhatt and Chandra [6]. In this paper, we proved a unique common fixed point theorem for OWC satisfying a generalized contractive condition in CMS.

2. Preliminaries

The following important preliminaries are useful in our main results.

Definition 2.1. Let L be a real Banach space. A subset M of L is called a cone iff

(i) M is a non-empty and closed also $M \neq \{0\}$,

(ii) $u, v \in \mathbb{R}$, and $\alpha, \beta \geq 0$, $u, v \in M \Rightarrow u\alpha + v\beta \in M$,

(iii) $M \cap (-M) = \{0\}$.

Given a cone $M \subset L$, we define a partial ordering $\leq$ with respect to M by $\alpha \leq \beta$ iff $\beta - \alpha \in M$. A cone M is called normal if $\exists$ a number $K > 0 \ni \forall\ \alpha, \beta \in M$, $0 \leq \alpha \leq \beta$ implies $||\alpha|| \leq K\,||\beta||$.

The least positive number satisfying the above inequality is called the normal constant of M, while $\alpha << \beta$ stands for $\beta - \alpha \in$ interior of M.

Definition 2.2. Let X be a nonempty set. And suppose that the mapping $\rho : X \times X \rightarrow L$ satisfying the following conditions:

(1). $0 \leq \rho(\alpha, \beta)$ for all $\alpha, \beta \in X$ and $\rho(\alpha,\beta) = 0$ if and only if $\alpha = \beta$,

(2). $\rho(\alpha, \beta) = \rho(\beta, \alpha)$ for all $\alpha, \beta \in X$;

(3). $\rho(\alpha, \beta) \leq \rho(\alpha, \gamma) + \rho(\gamma, \beta)$ for all $\alpha, \beta, \gamma \in X$.

Then ρ is called a cone metric on X and (X, ρ) is called a CMS.

Definition 2.3. Let (X, ρ) be CMS. We say that $\{x_n\}$ is

(i) a convergent sequence if for any b>>0, there exists a natural number N such that for all n>N, $\rho(x_n, x) << b$, for some fixed x in X .We denote this $x_n \rightarrow x$ (as $n \rightarrow \infty$) .

(ii) a Cauchy sequence if for every b in M with b>>0, there exists a natural number N such that for all n, m>N, $\rho(x_n, x_m) << b$.

Definition 2.4. A cone metric space (X, ρ) is said to be complete if every Cauchy sequence is convergent.

Definition 2.5 [9]. Let A and B be self-mappings of a set X. If $q = A\alpha = B\alpha$ for some α in X, then α is called a coincidence point of A and B, and q is called a point of coincidence of A and B.

Proposition 2.1. Let A and B be OWC self-mappings of a set X if and only if there is a point α in X which is coincidence point of A and B at which A and B are commute.

Lemma 2.1. Let X be a set, A and B are OWC self-mappings of X. If A and B have a unique point of coincidence $q = A\alpha = B\alpha$, then q is the unique.

3. Main Results

In this section, we prove a unique common fixed point theorem for OWC self-mappings in CMS (Cone Metric

Space). Ours result is improvement and generalization of the results of [6].

Let Φ: $\mathbb{R}^+ \to \mathbb{R}^+$ be a function satisfying the condition $\Phi(t) < t$ for each $t > 0$.

Now we prove the our main theorem

Theorem 3.1. Let (X, ρ) be a CMS and M be a normal cone. And suppose that l, m are two self- mappings of X and they satisfy the following conditions:

(i)

$$\rho(lx,\ ly) \le \Phi\left(Max \begin{Bmatrix} [\rho(mx,\ my) + \rho(mx,\ ly)/2], \\ [\rho(my,\ lx) + \rho(my,\ ly)]/2 \end{Bmatrix} \right)$$

for each x, y $\in$ X.

(ii) l and m are OWC.

Then l and m have a unique common fixed point.

Proof. Given (by (ii)) l and m are occasionally weakly compatible, then there exists point p $\in$ X such that lp = mp, lmp = mlp. We claim that, lp is the unique common fixed point of l and m. First we ascertain that lp is a fixed point of p. For if, llp $\neq$ lp, then by (i) we get that

$$\rho(lp,llp) \le \Phi\left(Max \begin{Bmatrix} \left[\rho(mp,mlp) + \rho(mp,llp)/2\right], \\ [\rho(mlp,lp) + \rho(mlp,llp)]/2 \end{Bmatrix} \right)$$

$$= \Phi\left(Max \begin{Bmatrix} [\rho(lp,\ lmp) + \rho(lp,\ llp)/2], \\ \rho[(lmp,\ lp) + \rho(lmp,\ llp)]/2 \end{Bmatrix} \right)$$

$$= \Phi\left(Max \begin{Bmatrix} [\rho(lp,\ llp) + \rho(lp,\ llp)/2], \\ [\rho(llp,\ lp) + \rho(llp,\ llp)]/2\} \end{Bmatrix} \right)$$

$$= \Phi(Max\{\rho(lp,\ llp), \rho[(llp,\ lp)]/2\})$$

$$= \Phi(\rho(lp,\ llp))$$

$$< \rho(lp,\ llp), \textit{ which is a contradiction.}$$

Therefore, llp = lp and llp = mlp = lp. Thus lp is a common fixed point of l and m.

Uniqueness: suppose that p, q $\in$ X such that lp = mp = p and lq = mq = q and p $\neq$ q. Then by (i) we get that

$$\rho(p,q) = \rho(lp,\ lq)$$

$$\le \Phi\left(Max \begin{Bmatrix} [\rho(mp,mq) + \rho(mp,lq)/2], \\ [\rho(mq,lp) + \rho(mq,lq)]/2 \end{Bmatrix} \right)$$

$$= \Phi\left(Max \begin{Bmatrix} [\rho(p,q) + \rho(p,q)/2], \\ [\rho(q,p) + \rho(q,q)]/2 \end{Bmatrix} \right)$$

$$= \Phi(Max\{\rho(p,q), (q,p)/2\})$$

$$= \Phi(\rho(p,q))$$

$$< \rho(p,q), \textit{ which is a contradiction.}$$

Therefore, p = q. Therefore, l and m have a unique common fixed point. This completes the proof of the theorem.

4. Conclusion

Our results are more general then the results of [6].

Conflict of Interest

Author has declared there is no conflict of interest.

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