American Journal of Applied Mathematics and Statistics, 2023, Vol. 11, No. 2, 50-60
Available online at http://pubs.sciepub.com/ajams/11/2/2
Published by Science and Education Publishing
DOI:10.12691/ajams-11-2-2

The Function Number Method: Basis and Applications

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Received February 25, 2023; Revised April 02, 2023; Accepted April 10, 2023

Abstract In this paper, we present a new method to solve some mathematics problems such as integral calculus, derivative calculus and differential equations. The method consists to transform an analytic problem or function to a real number. This real number obtained represents the Function Number. After finding the Function Number solution, it is also possible to transform it to a semi-analytic function which represents the definitive solution of the problem. We qualify the solution as semi-analytic solution because to solve the problem, we make some approximations. So, the semi-analytic function obtained is an approximate analytic solution. This method is simple and concise. It gives strong approximate solutions near to the real solutions.

Keywords: *function, number, method, differential equation, approximation*

Cite This Article: Marcel Julmard Ongoumaka Yandza, "The Function Number Method: Basis and Applications." *American Journal of Applied Mathematics and Statistics*, vol. 11, no. 2 (2023): 50-60. doi: 10.12691/ajams-11-2-2.

1. Introduction

Nowadays, there are many mathematical functions used to solve complex problems in Physics and Engineering. Among them, we have numerical function, logarithm function, exponential function, rational function, irrational function and other [1]. To formulate any of them, mathematicians have introduced the variables. A variable is an abstract value which can change in an ensemble by applying a numerical application. This method represents the foundation of mathematics. Many mathematical problems have been solved by this way. But till now they are some equations which cannot be solved by this old technics. As example, the analytic primitive of $\exp(x^2)$ is not found till now.

So, to cross this limit, we have conceived the Function Number Method. As its name specifies, a Function Number is a real number associated to a variable function in an ensemble. That means, if we have a variable function *f*, we can transform it to a real number. The real number obtained by this transformation represents the Function Number $Q_{n,f}$ of the function f. In this article, we will present the methodology to make the transformation from a variable function to a Function Number. And also, from a Function Number to a variable function. This new method is helpful. It permits to find the semi-analytic solution of some unsolved equations like the primitive of $\exp(x^2)$. We will solve this integral in the section 3 associated to the Applications. Before the resolution of some exercises, we will first present the methodology in section 2. So, we will define the way to derive and to integrate a function or a differential equation.

2. Methods

2.1. Proposition

We consider a variable function f defined in the ensemble $\Omega = [x_0, x_{N-1}]$. N is a natural number which represents the number of nodes in Ω. The Main Function Number $Q_{n,f}$ associated to f is the sum of the images $f(x_i)$ of f in the ensemble Ω like shown by the equation (1).

$$Q_{n,f} = \sum_{\Omega} f(x_i) \tag{1}$$

To calculate $Q_{n,f}$, it is compulsory to define a pace h. The smaller the pace is, the more the result is precise. The reader is free to choose his pace in accordance with his application. If we are confronted to Physics problems, like the resolution of heat equation [2,3], Navier-Stokes equation [4,5] or Schrödinger equation [6,7], it is better to define the pace by using the results of the reference [8]. That means, the pace must be associated to the smallest density particle in the Universe: the photon [1]. The volume of a photon $V_{ph} = 9\pi h^2 c^5$ is equal to

[1] In the expression of the volume of a photon V_{ph}, h represents the Max Planck constant and c represents the speed of light

$3,00613084.10^{-23}m^3$. As we found that the Universe is a conical shape [9], we can consider this shape and deduct the side of the volume of a photon V_{ph}. In this case, the value of the side represents the pace. The numerical application gives as value $5,42012883.10^{-8}$ meters. Or, we can also use the cubic volume of V_{ph} by referring to the atomic structure in materials. For the cubic shape, the side is $3,10934772.10^{-8}$ meters. The best choice is the minimal value between the two. It means the value of the cubic structure is suitable.

To study a Function Number $Q_{n,f}$, we must consider it as a volume of water in a recipient. If we pour the liquid of this recipient from the origin x_0 to the last node x_{N-1}, the quantity in the recipient will decrease progressively. If we also consider that at any point x_i a certain quantity of water is stocked in the nodes; we will have two currents. The first current I is the main current it means the remaining volume in the recipient; and the second one is the residual current I_p which is stocked in the nodes (Figure 1). The residual current I_p can be variable from one node to another. It can also be *homogeneous*. In this last case, we note it I_{ph}.

On this basis, if we consider the nodes i and $i+1$. The relation between the main currents I_i, I_{i+1} and the residual current I_{p_i} at the node i, is given by the equation (3). The main current I_i is equal to $f(x_i)$ (2). In the continuation of this article, we will use I_i to define the main current.

$$Q_{n,f} = \sum_{\Omega} f(x_i) = \sum_{\Omega} I_i \tag{2}$$

$$I_{i+1} = I_i - I_{p_i} \tag{3}$$

The relation (3) is crucial. We will use it frequently. As precision, the study of a Function Number is done only for monotonous functions. If the function varies (increases and decreases) from an interval to another in Ω, we will study it fragment by fragment as a piecewise function. Each fragment will be associated to an interval in which the function is monotonous.

The Function Number can also be defined by the Residual Function Number $F_{n,f}$ such as $F_{n,f}$ is equal to the sum of residual currents I_{p_i} (4).

$$F_{n,f} = \sum_{\Omega} I_{p_i} \tag{4}$$

To define the Function Number, we must provide the following details: $Q_{n,f}$ or $F_{n,f}$, the ensemble of the function Ω, the pace h and the initial condition I_0 or/and boundary condition I_N.

2.2. Derivatives

To derive any function, we will use the Finite Difference Method (FDM) [10]. And, we will incorporate in it the equation (3). The combination of (3) and the FDM gives the Function Number Method (FNM). The derivative of the main current I at the node i is given by the equation (5).

$$\left(\frac{\partial I}{\partial i}\right)_i = \frac{I_{i+1} - I_i}{h} \tag{5}$$

By introducing the relation (3) in (5), we find the equation (6).

$$\left(\frac{\partial I}{\partial i}\right)_i = -\frac{I_{p_i}}{h} \tag{6}$$

By the same way:

$$\left(\frac{\partial^2 I}{\partial i^2}\right)_i = \frac{\partial}{\partial i}\left(\frac{\partial I}{\partial i}\right)_i = \frac{\partial}{\partial i}\left(-\frac{I_{p_i}}{h}\right)_i$$
$$= -\frac{1}{h}\left(\frac{\partial I_{p_i}}{\partial i}\right)_i = -\frac{1}{h}\left(\frac{I_{p_i} - I_{p_{i-1}}}{h}\right)_i$$

Finally, we find:

$$\left(\frac{\partial^2 I}{\partial i^2}\right)_i = \frac{I_{p_{i-1}} - I_{p_i}}{h^2} \tag{7}$$

The equation (6) shows that *the derivative of the main current is proportional to the residual current*. That also means that we can reduce the degree of derivative by using the residual current I_{p_i} like presented by (6).

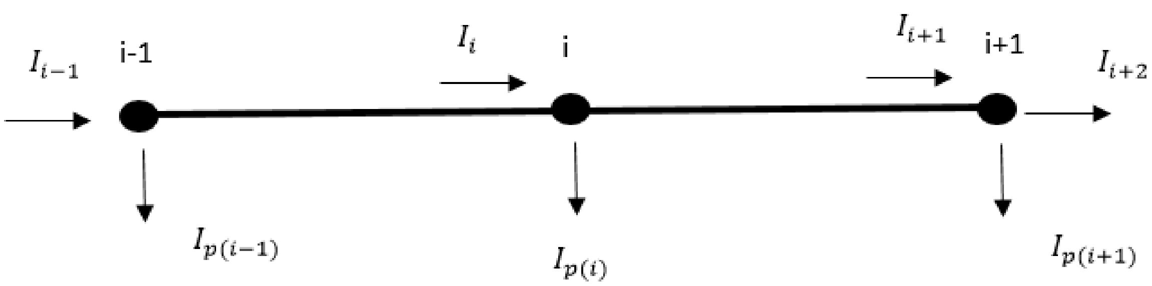

Figure 1. The Currents and the nodes

2.3. Primitives

For the integral calculus, we consider a known function f and an unknown function g such as (8):

$$\frac{dg}{dx} = f(x) \text{ on } \Omega \tag{8}$$

That means g is a primitive of f. To determine the primitive g, we apply the equation (6) in (8). That leads us to the following equation:

$$-\left(\frac{I_{p_g}}{h}\right)_i = f_i \tag{9}$$

By the equation (9), we find (10):

$$\left(I_{p_g}\right)_i = -hf_i \tag{10}$$

By applying the sum to (10), we find:

$$F_{n,g} = \sum_{\Omega}\left(I_{p_g}\right)_i = -h\sum_{\Omega} f_i = -hQ_{n,f} \tag{11}$$

So, the Residual Function Number $F_{n,g}$ of the primitive g is proportional to the Main Function Number $Q_{n,f}$ of the derivative f like we precise by the equation (12).

$$F_{n,g} = -hQ_{n,f} \tag{12}$$

As the Main Function Number of f $Q_{n,f}$ is known; by (12) we find the Residual Function Number of g $F_{n,g}$.

2.4. Transformation of a Function Number to an Analytic Function

For transforming $F_{n,g}$ to an semi-analytic function $g(x)$, the first step is to calculate the homogeneous residual current $I_{ph,g}$ of the function g. $I_{ph,g}$ represents the average value of the residual current I_{p_g}. It is determined by dividing the Function Number $F_{n,g}$ by the number of nodes N in the ensemble Ω like presented by (13).

$$I_{ph,g} = \frac{F_{n,g}}{N} \tag{13}$$

By summing the expression (3) $I_{i+1} = I_i - I_{p_i}$ from $i = 0$ till $i = N-1$, we find (14).

$$I_N = I_0 - F_n \tag{14}$$

By applying (14) in the case of our function g, we find (15).

$$I_{N,g} = I_{0,g} - F_{n,g} \tag{15}$$

We also know that:

$$I_{N,g} = I_{N-1,g} - I_{ph,g} \tag{16}$$

The relation (16) shows that I is an arithmetic sequence of ratio $-I_{ph,g}$. So, the sum of its terms $\sum_{\Omega} I_{i,g} = \sum_{\Omega} g_i = Q_{n,g}$ is presented by (17).

$$Q_{n,g} = \left(\frac{2I_{0,g} + I_{ph,g} - F_{n,g}}{2}\right)N \tag{17}$$

As we found $Q_{n,g} = \sum_{\Omega} g_i$, we can deduct the variable function $g(x) = ax + b$ by determining the constants a and b. The form of $g(x)$ can vary. The person who makes the calculus is free to use another type of function. We advise people that the type of the function $g(x)$ must be the same than its derivative $f(x)$. If $f(x)$ is an exponential function, $g(x)$ is also. In general, the type of the primitive $g(x)$ is a monotonous polynomial. Therefore, we choose $g(x) = ax + b$. The only condition to respect is that the derivative of $g(x)$, $f(x)$, must be monotonous in the ensemble Ω [2]. In addition, the form of $g(x)$ must also depend on the conditions given in the exercise. If we have, for example, three conditions to apply, we must choose $g(x)$ on the same basis, for example: $g(x) = ax^2 + bx + c$.

In our case, to find a and b we must have two conditions. The first one is a compulsory condition that any person must use when he uses this method. This condition is given by (18).

$$Q_{n,g} = a(x_0 + x_1 + ... + x_{N-1}) + bN \tag{18}$$

The second condition is associated to the initial or the boundary condition (19).

$$I_{0,g} = g(x_0) = ax_0 + b \tag{19}$$

After determining a and b, we find the semi-analytic expression of the primitive $g(x)$.

2.5. The Derivative of Function

The function t is known. We consider a function u derivative of t define on the ensemble Ω such as:

$$u(x) = \frac{dt}{dx} \text{ on } \Omega \tag{20}$$

At present, we want to find the Main Function Number of u $Q_{n,u}$.

To begin, we apply the equation (6) by using the functions u and t. The result is presented by (21).

$$u_i = \left(\frac{\partial t}{\partial i}\right)_i = -\left(\frac{I_{p_t}}{h}\right)_i \tag{21}$$

By applying the sum to (21), we get (22):

[2] If the derivative function $f(x)$ is not monotonous in Ω, its primitive $g(x)$ will be studied as a piecewise function in the ensemble Ω

$$\sum_{\Omega} u_i = \sum_{\Omega} \left(\frac{\partial t}{\partial i}\right)_i = -\frac{1}{h}\sum_{\Omega}\left(I_{p_t}\right)_i \quad (22)$$

This leads us to write the following equation:

$$Q_{n,u} = -\frac{1}{h}F_{n,t} \quad (23)$$

With $F_{n,t} = \sum_{\Omega}\left(I_{p_t}\right)_i$ and $Q_{n,u} = \sum_{\Omega} u_i$

By using the equation (15), we have:

$$I_{N,t} = I_{0,t} - F_{n,t} \quad (24)$$

So, we deduct $F_{n,t}$ as the following:

$$F_{n,t} = I_{0,t} - I_{N,t} = t(x_0) - t(x_N) \quad (25)$$

So, to find $Q_{n,u}$ we form the following system (26) composed of the equations (23) and (25):

$$Q_{n,u} = -\frac{1}{h}F_{n,t} \quad (26)$$

$$F_{n,t} = I_{0,t} - I_{N,t} = t(x_0) - t(x_N)$$

$I_{N,t}$ is outside of the current mesh or ensemble Ω. The mesh is composed of elements from x_0 till x_{N-1}. $I(N,t)$ represents a boundary condition of $t(x)$.

As we have $Q_{n,u}$, we can find the semi-analytic function of u. The method remains the same like we did for the primitive. The first step is to represent $u(x)$ as a polynomial (27):

$$u(x) = ax + b \quad (27)$$

Now, we must find the constants a and b by applying:

- the compulsory condition:
  $Q_{n,u} = a(x_0 + x_1 + ... + X_{N-1}) + b*N$

And

- the initial condition or boundary condition:
  $u(x_0) = ax_0 + b$ / $u(x_N) = ax_N + b$

2.6. Multivariable Functions

In this section, we consider a function with two (02) variables: i and j. We note I its main current and I_p its residual current. $I_{p_i}^{j}$ is the residual current in the direction i at the node (i, j). $I_{p_j}^{i}$ is the residual current in the direction j at the node (i, j). That means $I_{p_{i-1}}^{j}$ is the residual current in the direction i at the node $(i-1, j)$. This is the rule of presenting the expressions in this article. The derivatives of I are given by the following equations:

$$\left(\frac{\partial I}{\partial i}\right)_{(i,j)} = -\frac{I_{p_i}^{j}}{h_i} \quad (28)$$

$$\left(\frac{\partial I}{\partial j}\right)_{(i,j)} = -\frac{I_{p_j}^{i}}{h_j} \quad (29)$$

$$\left(\frac{\partial^2 I}{\partial i \partial j}\right)_{(i,j)} = \frac{\partial}{\partial i}\left(\frac{\partial I}{\partial j}\right) = \frac{\partial}{\partial i}\left(-\frac{I_{p_j}^{i}}{h_j}\right) = \frac{I_{p_i}^{j} - I_{p_{i+1}}^{j}}{h_i h_j} \quad (30)$$

If $h_i = h_j = h$ (30) becomes (31):

$$\left(\frac{\partial^2 I}{\partial i \partial j}\right)_{(i,j)} = \frac{I_{p_i}^{j} - I_{p_{i+1}}^{j}}{h^2} \quad (31)$$

$$dI(i,j) = \frac{\partial I}{\partial i} + \frac{\partial I}{\partial j} = -\frac{I_{p_i}^{j}}{h_i} - \frac{I_{p_j}^{i}}{h_j} \quad (32)$$

If $h_i = h_j = h$ (32) becomes (33):

$$dI(i,j) = -\frac{I_{p_i}^{j} + I_{p_j}^{i}}{h} = -\frac{I_p(i,j)}{h} \quad (33)$$

$$\left(\frac{\partial^2 I}{\partial i^2}\right)_{(i,j)} = \frac{I_{p_{i-1}}^{j} - I_{p_i}^{j}}{h^2} \quad (34)$$

$$\left(\frac{\partial^2 I}{\partial j^2}\right)_{(i,j)} = \frac{I_{p_{j-1}}^{i} - I_{p_j}^{i}}{h^2} \quad (35)$$

$$d^2 I(i,j) = \left(\frac{\partial^2 I}{\partial i^2}\right)_{(i,j)} + \left(\frac{\partial^2 I}{\partial j^2}\right)_{(i,j)} = \frac{I_{p_{i-1}}^{j} - I_{p_i}^{j}}{h^2} + \frac{I_{p_{j-1}}^{i} - I_{p_j}^{i}}{h^2} = \frac{I_{p_{i-1}}^{j} + I_{p_{j-1}}^{i} - I_p(i,j)}{h^2} \quad (36)$$

With:

$$I_p(i,j) = I_{p_j}^{i} + I_{p_i}^{j}$$

2.7. Method for Solving Differential Equations

The resolution of a differential equation is very simple. The method consists to apply the derivatives equations in the differential equation like we propose in the following example (37):

$$\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} = f(x,y) \quad (37)$$

To solve the differential equation, we must begin to apply (28) and (29). So, we can write the following equation:

$$-\left(\frac{I^{j}_{p_i,g}}{h}+\frac{I^{i}_{p_j,g}}{h}\right)=f(i,j)$$

The variable i is associated to x, and j is associated to y.

$$I^{(i,j)}_{p,g}=-hf(i,j)$$

By summing this precedent equation, we find directly the solution $F_{n,g}$ like seen by (38):

$$F_{n,g}=-hQ_{n,f} \tag{38}$$

The reader is invited to transform $F_{n,g}$ to an analytic function. The method remains the same than what we explain in the subsection 2.4.

2.8. Mathematical Properties

2.8.1. Basic Properties

In this subsection, we establish the mathematical properties of the Functions Numbers. There are the followings:

If we consider two functions f and g defined in Ω and their respective Main Function Number $Q_{n,f}$ and $Q_{n,g}$, we can write:

$$\sum_{\Omega}(f_i+g_i)=Q_{n,f}+Q_{n,g} \tag{39}$$

$$\sum_{\Omega}(f_i-g_i)=Q_{n,f}-Q_{n,g} \tag{40}$$

$$\sum_{\Omega}(f_i*g_i)=Q_{n,f*g} \tag{41}$$

$$\sum_{\Omega}(\frac{f_i}{g_i})=Q_{n,\frac{f}{g}} \tag{42}$$

$$\sum_{\Omega}f^{\alpha}=Q_{n,f^{\alpha}} \tag{43}$$

To find the properties of $F_{n,f}$, $F_{n,g}$ and the corresponding functions f and g, we must replace $Q_{n,f}$ and $Q_{n,g}$ like shown by the equation (17) in (39), (40), (41), (42) and (43).

2.8.2. The Law of Nodes

The Figure 2 represents the mesh in which there are some nodes: the node A which is the intersection of two (02) elements, the node B which is the intersection of three (03) elements and the node C which is the intersection of four (04) elements. The element is a segment between two nodes.

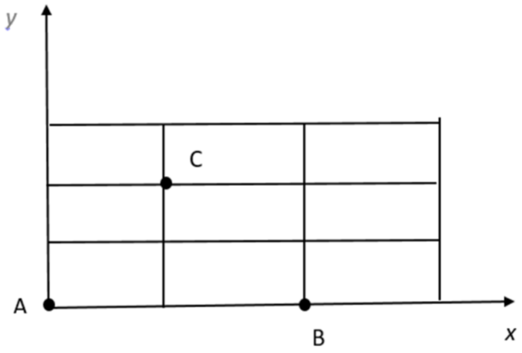

Figure 2. The mesh

We consider the residual current of these nodes I_p. We call $I^{j}_{p_i}$ the residual current of each node in the x axis or direction. And $I^{i}_{p_j}$, the residual current of the nodes in the y direction. So, we define I_p, $I^{j}_{p_i}$ and $I^{i}_{p_j}$ as the following:

$$I_p(i,j)=I^{j}_{p_i}+I^{i}_{p_j} \tag{44}$$

For the node A, we have:

$$I^{j}_{p_i}=I^{i}_{p_j}=\frac{I_p(i,j)}{2} \tag{45}$$

For the node B, we have:

$$I^{j}_{p_i}=\frac{2}{3}I_p(i,j) \tag{46}$$

$$I^{i}_{p_j}=\frac{1}{3}I_p(i,j) \tag{47}$$

For the node C, we find:

$$I^{j}_{p_i}=\frac{2}{4}I_p(i,j)=\frac{1}{2}I_p(i,j) \tag{48}$$

$$I^{i}_{p_j}=\frac{2}{4}I_p(i,j)=\frac{1}{2}I_p(i,j) \tag{49}$$

3. Applications

3.1. Function 1: Integral

Calculate:

$g(x)=\int \exp(x^2)dx$ on $\Omega=[0,1[$

$h=0,1$

$g(0)=1$

We call $f(x)=exp(x^2)$

- Variation Study of f:

f' is the derivative of f

$f'(x)=2x\exp(x^2)$

In the interval $\Omega = [0,1[$, $f' > 0 \Rightarrow f$ is monotonous in Ω. So, we do not have a piecewise primitive. We can apply the Function Number Method (FNM).

Remark: If the function f was not monotonous in Ω, we would consider g as a piecewise function defined in the semi-intervals of Ω.

- Integral calculus:

As g is the primitive of f, we can write:

$$\frac{dg}{dx} = f(x) \Rightarrow -\frac{I_{p_i,g}}{h} = f_i \Rightarrow I_{p_i,g} = -hf_i$$

By applying the sum, we get the following expressions:

$$\Sigma_\Omega I_{p_i,g} = -h\Sigma_\Omega f_i \Rightarrow F_{n,g} = -hQ_{n,f}$$

As the function f is known, we can calculate $Q_{n,f} = \sum_\Omega f_i$.

We find $Q_{n,f} = 13,812606$

So, we deduct $F_{n,g} = -1,3812606$

- Calculation of the homogeneous residual current $I_{ph,g}$:

$$I_{ph,g} = \frac{F_{n,g}}{N} = \frac{-1,3812606}{10} = -0,13812606$$

- Calculus of $Q_{n,g}$:

$$Q_{n,g} = \frac{2I_{0,g} + I_{ph,g} - F_{n,g}}{2} N$$

$N = 10$, $I_{0,g} = g(0) = 1$

We find $Q_{n,g} = 16,2156727$

- Initial Condition:

We propose g as a polynomial $g(x) = ax + b$. So, we must find a and b:

$g(0) = b = 1$

- Compulsory condition:

The compulsory condition is the following:

$$Q_{n,g} = a\sum_{i=0}^{9} x_i + b * N$$

We find $a = 1,3812606$

- Semi-analytic Solution:

The solution of our integral calculus is: $g(x) = 1,3812606x + 1$

The Figure 3 shows the numerical application for the calculus of the primitive g.

The Figure 4 represents the curves of our semi-analytic solution (blue) and the solution obtained by applying the Finite Difference Method (FDM) (orange).

Interval (x)	f(x)	Qn,f	Fn,g	Iph,g	Qn,g	a	b	our solution	FDM	Gap
0	1	13,812606	-1,3812606	-0,13812606	16,2156727	1,3812606	1	1	1	0
0,1	1,01005017							1,13812606	1,10100502	0,03712104
0,2	1,04081077							1,27625212	1,20508609	0,07116603
0,3	1,09417428							1,41437818	1,31450352	0,09987466
0,4	1,17351087							1,55250424	1,43185461	0,12064963
0,5	1,28402542							1,6906303	1,56025715	0,13037315
0,6	1,43332941							1,82875636	1,70359009	0,12516627
0,7	1,63231622							1,96688242	1,86682171	0,10006071
0,8	1,89648088							2,10500848	2,0564698	0,04853868
0,9	2,24790799							2,24313454	2,2812606	-0,03812606

Figure 3. Numerical application of the primitive $g(x)$

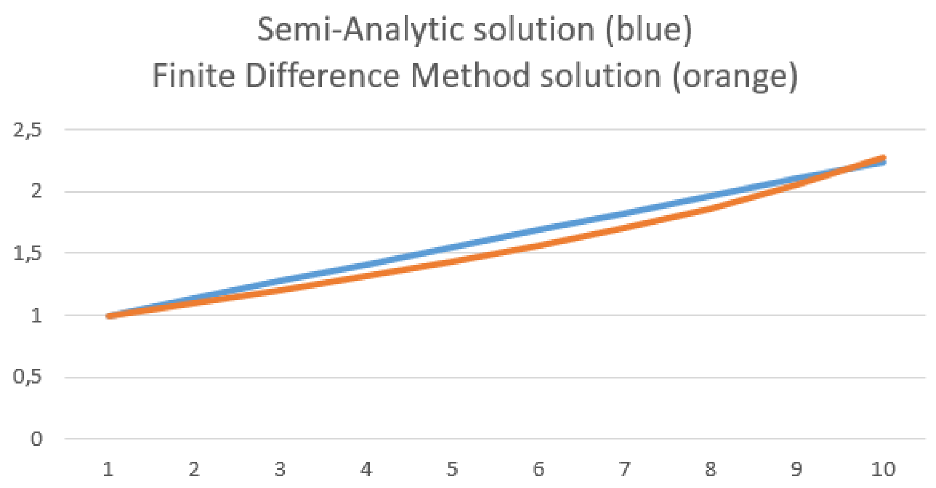

Figure 4. The curves of results found by FDM and FNM

3.2. Function 2: Derivative

Find u the derivative of t such as:

$$t(x) = \sqrt{x}$$

$u = \frac{dt}{dx}$ on $\Omega = [0,1[$

$u(1) = \frac{1}{2}$ (boundary condition[3]).

$h = 0,1$

- Variation Study of $t(x)$:

$t(x) = \sqrt{x}$, so $t(x) > 0$ in Ω.

The function $t(x)$ is monotonous in Ω. So, $u(x)$ is not a piecewise derivative function.

- Derivative calculus:

$$u_i = \left(\frac{dt}{dx}\right)_i = -\frac{I_{p_i,t}}{h} \Rightarrow$$

By applying the sum, we get:

$$Q_{n,u} = -\frac{1}{h} F_{n,t}$$

By (24) we know that:

$$F_{n,t} = I_{0,t} - I_{N,t} = t(0) - t(1) = 0 - 1 = -1$$

$$F_{n,t} = -1$$

So, $Q_{n,u} = -\frac{1}{0,1}(-1) = 10 \Rightarrow Q_{n,u} = 10$

- Conditions:

We consider u as a polynomial function: $u(x) = ax + b$

Compulsory condition:

$$Q_{n,u} = a\sum_{i=0}^{9} x_i + Nb \Rightarrow 10 = 4,5a + 10b$$

Boundary condition: $I_{N,u} = u(1) = \frac{1}{2}$

We find the following values:

$a = -0,9090909$ and $b = 1,40909090$

- The semi-analytic solution:

We find the semi-analytic solution:

$u(x) = -0,9090909x + 1,40909090$

The Figure 5 represents the numerical application details about the calculus of the semi-analytic solution $u(x)$. The gap represents the difference between the analytic solution $u(x) = \frac{1}{2\sqrt{x}}$ and our semi-analytic solution $u(x) = -0,9090909x + 1,40909090$.

The Figure 6 represents the curves of the semi-analytic derivative $u(x)$ (orange) and the exact derivative (blue).

3.3. Differential Equation 1

Find y such as:

$y'' + y' = 2x$ on $\Omega = [0,1[$

$y(0) = 0$

$h = 0,1$

- Variation Study:

The differential equation can be written as the following:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = f(x)$$

with $f(x) = 2x$

$f'(x) > 0$ in Ω

So, f is monotonous, y is not a piecewise function in Ω.

- General calculus:

To solve this problem, we apply the equations (7) and (6) in our differential equation. So, we get the following expression:

$$\frac{I_{p_{i-1},y} - I_{p_i,y}}{h^2} - \frac{I_{p_i,y}}{h} = f_i$$

By calculating and reducing to the same denominator, we get:

$$I_{p_{i-1},y} - (1+h)I_{p_i,y} = h^2 f_i$$

By applying the sum to the precedent expression, we get:

$$\sum_{i=1}^{N-1} I_{p_{i-1},y} - (1+h)\sum_{i=1}^{N-1} I_{p_i,y} = h^2 \sum_{i=1}^{N-1} f_i$$

We know that:

$$\sum_{i=1}^{N-1} I_{p_i,y} = I_{p_1,y} + I_{p_2,y} + \dots + I_{p_{N-2},y} + I_{p_{N-1},y}$$

$$= F_{n,y} - I_{p_0,y}$$

$$\sum_{i=1}^{N-1} I_{p_{i-1},y} = I_{p_0,y} + I_{p_1,y} + \dots + I_{p_{N-2},y}$$

$$= F_{n,y} - I_{p_{N-1},y}$$

$$\sum_{i=1}^{N-1} f_i = f_1 + f_2 + \dots + f_{N-1} = Q_{n,f} - f_0 = Q_{n,f} - I_{0,f}$$

By replacing these different relations in the main equation, we get:

$$F_{n,y} - I_{p_{N-1},y} - (1+h)(F_{n,y} - I_{p_0,y}) = h^2(Q_{n,f} - I_{0,f})$$

By calculating, we reach this following equation:

$$-hF_{n,y} + (1+h)I_{p_0,y} - I_{p_{N-1},y} = h^2(Q_{n,f} - I_{0,f})$$

- Approximation:

To solve the problem, we make the following approximation:

$$I_{p_0,y} = I_{p_{N-1},y} = I_{ph,y} = \frac{F_{n,y}}{N}$$

By replacing this approximation in the precedent equation, we find:

$$-hF_{n,y} + (1+h)\frac{F_{n,y}}{N} - \frac{F_{n,y}}{N} = h^2(Q_{n,f} - I_{0,f})$$

This leads to:

$$(\frac{1}{N} - 1)F_{n,y} = h(Q_{n,f} - I_{0,f})$$

[3] If the boundary or the initial condition is not given, consider $u(0) = t(0)$ or $u(1) = t(1)$

And we find the Residual Function Number $F_{n,y}$:

$$F_{n,y} = \frac{hN(Q_{n,f} - I_{0,f})}{1-N}$$

$$Q_{n,f} = \sum_{i=0}^{9} f(x_i) = 9$$

By applying the numerical application, we get:

$$F_{n,y} = -1$$

- Homogeneous residual current:

$$I_{ph,y} = \frac{F_{n,y}}{N} = -0{,}1$$

- Main Function Number $Q_{n,y}$:

$$Q_{n,y} = \frac{2I_{0,y} + I_{ph,y} - F_{n,y}}{2} N = 4{,}5$$

- Semi-analytic solution:

We consider the solution y as polynomial function:

$$y = ax + b$$

To find the constants a and b, we apply the initial condition and the compulsory condition:

$y(0) = b = 0$ (initial condition)

$Q_{n,y} = a\sum_{i=0}^{9} x_i$ (compulsory condition)

$a = 1$

Finally, the solution is $y(x) = x$

By the first view, it seems like our semi-analytic solution is not correct. But, it is right. The numerical application testifies its correctness. We precise that our solution is not an exact solution but a strong approximate solution. The Figure 7 shows the numerical application details concerning our semi-analytic solution $y(x) = x$. By the analytic method, we found the exact solution of the differential equation. The gap represents the difference between our solution and the analytic solution $y = \frac{1}{1-\exp(-1)} + \frac{1}{\exp(-1)-1}\exp(-x)$. The exact solution has been found by adding a new condition $y(1) = 1$. This result shows that the Function Number Method permits to find the solution of any mathematics problem.

Interval (x)	Io,t	In,t	Fn,t	Qn,u	sum of x	u= ax+b	Real Derivativ	Gap	a	b
0	0	1	-1	10	4,5	0			-0,9090909	1,4090909
0,1						1,31818181	1,58113883	0,26295702		
0,2						1,22727272	1,11803399	-0,10923873		
0,3						1,13636363	0,91287093	-0,2234927		
0,4						1,04545454	0,79056942	-0,25488512		
0,5						0,95454545	0,70710678	-0,24743867		
0,6						0,86363636	0,64549722	-0,21813914		
0,7						0,77272727	0,5976143	-0,17511297		
0,8						0,68181818	0,55901699	-0,12280119		
0,9						0,59090909	0,52704628	-0,06386281		

Figure 5. Numerical application of the derivative $u(x)$

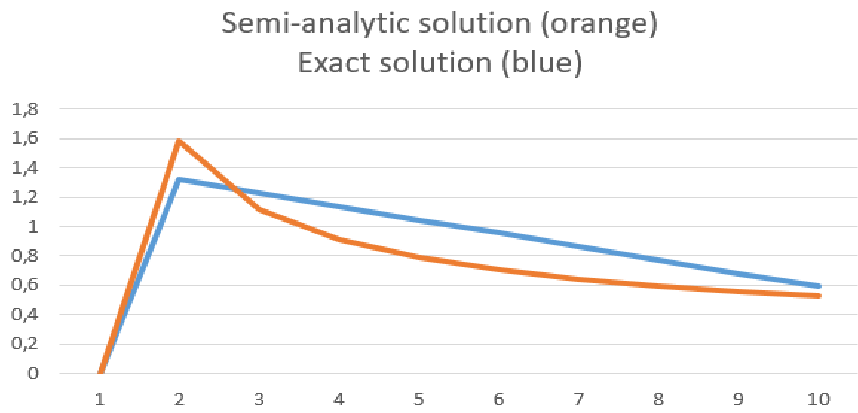

Figure 6. The curve of the semi-analytic and the exact solution

Interval x	f(x)	Qn,f	Fn,y	Iph,y	Qn,y	Sum of x	a	-1,58197671	y real	y solution	gap
0	0	9	-1	-0,1	4,5	4,5	1		0	0	0
0,1	0,2								0,15054499	0,1	0,05054499
0,2	0,4								0,28676373	0,2	0,08676373
0,3	0,6								0,41001954	0,3	0,11001954
0,4	0,8								0,52154601	0,4	0,12154601
0,5	1								0,62245933	0,5	0,12245933
0,6	1,2								0,71376948	0,6	0,11376948
0,7	1,4								0,79639032	0,7	0,09639032
0,8	1,6								0,87114875	0,8	0,07114875
0,9	1,8								0,93879298	0,9	0,03879298

Figure 7. Numerical application details of $y(x)$

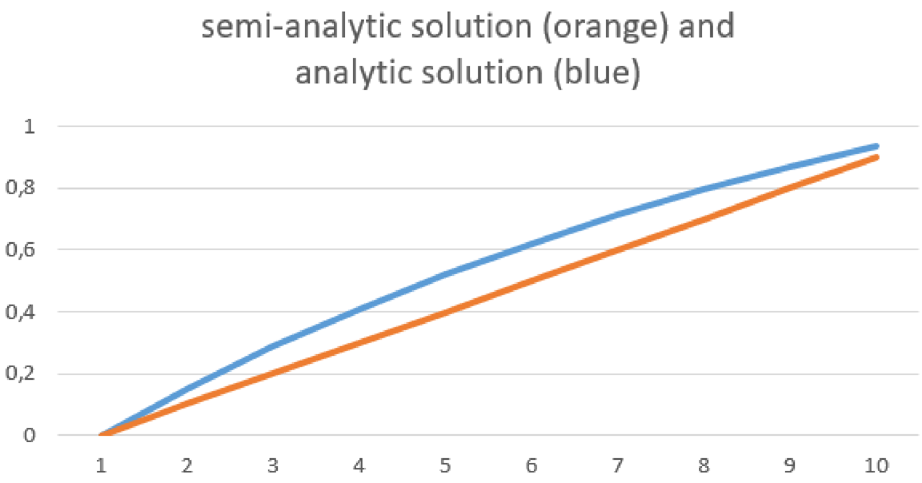

Figure 8. The curve of semi-analytic and analytic solution

The Figure 8 represents the curves of our semi-analytic solution (orange) and the analytic solution (blue).

3.4. Differential Equation 2

We consider the following differential equation:

$$y' + y = x^2 + 1 on \Omega = [0,1[$$

$y(0) = 1$

$h = 0,1$

- General calculus:

To solve this problem, the differential equation can be directly written as:

$$\frac{dy}{dx} + y = f(x)$$

with $f(x) = x^2 + 1$

$f'(x) > 0$ in $\Omega \Rightarrow$ $f(x)$ is monotonous in Ω. So, y is not a piecewise function.

So, we have:

$$-\frac{I_{p_i,y}}{h} + I_{i,y} = f_i$$

By applying the sum, we get:

$$-\frac{1}{h}\sum_{i=0}^{N-1} I_{p_i,y} + \sum_{i=0}^{N-1} I_{i,y} = \sum_{i=0}^{N-1} f_i \Rightarrow$$

$$-\frac{1}{h}F_{n,y} + Q_{n,y} = Q_{n,f}$$

By applying the relation (17) as the following, we can deduct $F_{n,y}$ (50):

$$Q_{n,y} = \frac{2I_{0,y} + I_{ph,y} - F_{n,y}}{2} N \Rightarrow$$

We also replace $I_{ph,y} = \frac{F_{n,y}}{N}$ in the expression of $Q_{n,y}$, we get $F_{n,y}$:

$$F_{n,y} = \frac{2(Q_{n,y} - I_{0,y}N)}{1-N}$$

By replacing $F_{n,y}$ in the last form of our differential equation, we get:

$$Q_{n,y} = \frac{h(1-N)Q_{n,f} - 2I_{0,y}N}{h(1-N) - 2}$$

After finding $Q_{n,y}$, it comes easy to transform it to a semi-analytic solution.

- Semi-analytic solution:

We calculate $Q_{n,f} = \sum_{i=0}^{9} f(x_i) = 12,85$

And $Q_{n,y} = 10,8844828$

We choose the solution y as the following polynomial:

$y(x) = ax + b$

By applying the initial condition $y(0) = 1$ and the compulsory condition $Q_{n,y} = \sum_{i=0}^{9} x_i + Nb$, we find the constants a and b:

$a = 0,19655172$ and $b = 1$

Finally the solution is: $y(x) = 0,19655172x + 1$.

x	f(x)	Qn,f	Qn,y	sum of x	a	b	Our solution	Real solution	Gap
0	1	12,85	10,8844828	4,5	0,19655172	1	1	1	0
0,1	1,01						1,01965517	1,00032516	0,01933001
0,2	1,04						1,03931034	1,00253849	0,03677185
0,3	1,09						1,05896552	1,00836356	0,05060196
0,4	1,16						1,07862069	1,01935991	0,05926078
0,5	1,25						1,09827586	1,03693868	0,06133718
0,6	1,36						1,11793103	1,06237673	0,05555431
0,7	1,49						1,13758621	1,09682939	0,04075681
0,8	1,64						1,15724138	1,14134207	0,01589931
0,9	1,81						1,17689655	1,19686068	-0,01996413

Figure 9. Numerical Application of $y(x)$

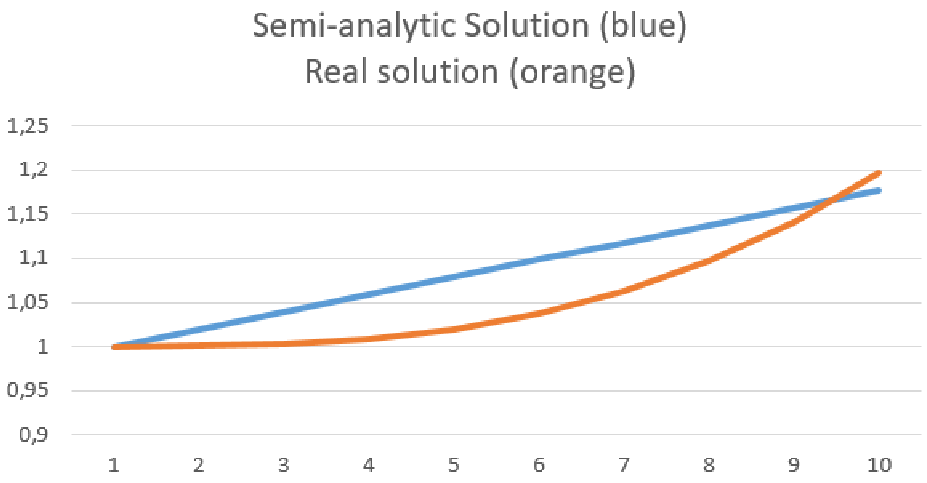

Figure 10. The Curve of the semi-analytic and analytic solution

The Figure 9 presents the numerical application details of the calculus of our semi-analytic solution y . The analytic solution of this differential equation is $y=-2\exp(-x)+x^2-2x+3$.

The Figure 10 represents the curves of the two functions: semi-analytic (blue) and analytic (orange) solution. We can see how the Function Number Method offers a precise approximation.

3.5. Differential Equation 3

We consider the following types of differential equations:

$$h(x)y''+t(x)y'+f(x)y=g(x)$$

or

$$t(x)y'+f(x)y=g(x)$$

For example, we have:

$$y'+(x-1)y=x^2 \text{ on } \Omega=[0,1[$$

$$h=0{,}1 \text{ and } y(0)=0$$

So,

$$f(x)=x-1,$$

$$g(x)=x^2$$

- Approximation:

To solve this kind of differential equation, we apply the same rule. So, we have:

$$-\frac{I_{p_i,y}}{h}+I_{i,f}I_{i,y}=I_{i,g}=g(x_i)$$

$I_{i,f} = f(x_i)$ in Ω . The value of $I_{i,f}$ is variable in Ω . So, it prevents us to continue to solve our differential equation. But, we can solve the problem by establishing an approximation of $I_{i,f}$ as $I_{h,f}$. $I_{h,f}$ represents the *homogeneous main current*. It is an average of $I_{i,f}$ in the ensemble Ω .

$$I_{h,f}=\frac{\sum_{\Omega} f_i}{N}=\frac{\sum_{\Omega} I_{i,f}}{N}=\frac{Q_{n,f}}{N}$$

By applying the sum to our precedent differential equation form, we get:

$$-\frac{1}{h}F_{n,y}+I_{h,f}Q_{n,y}=Q_{n,g}$$

By replacing $F_{n,y}$ by its expression of (50), we get $Q_{n,y}$:

$$Q_{n,y}=\frac{h(1-N)Q_{n,g}-2I_{0,y}N}{h(1-N)I_{h,f}-2}$$

- Semi-analytic solution:

$$Q_{n,g}=2{,}85$$

$$I_{h,f}=-0{,}55$$

$$Q_{n,y}=1{,}70431894$$

We consider our solution as a polynomial $y=ax+b$. By applying the compulsory condition and the initial condition, we get the values of a and b :

$$a=0{,}37873754$$

$$b=0$$

The solution of our differential equation is $y=0{,}37873754x$.

The Figure 11 presents the numerical application.

x	g(x)	Qn,g	f(x)	Ih,f	Qny	sum of x	a	b	our solution	real solution	gap
0	0	2,85	-1	-0,55	1,70431894	4,5	0,37873754	0	0	0	0
0,1	0,01		-0,9						0,03787375	0,00034114	0,03753261
0,2	0,04		-0,8						0,07574751	0,00278264	0,07296487
0,3	0,09		-0,7						0,11362126	0,00953838	0,10408288
0,4	0,16		-0,6						0,15149502	0,02287224	0,12862278
0,5	0,25		-0,5						0,18936877	0,04500859	0,14436019
0,6	0,36		-0,4						0,22724252	0,07803844	0,14920408
0,7	0,49		-0,3						0,26511628	0,12382662	0,14128966
0,8	0,64		-0,2						0,30299003	0,1839256	0,11906444
0,9	0,81		-0,1						0,34086379	0,25950176	0,08136203

Figure 11. Numerical application

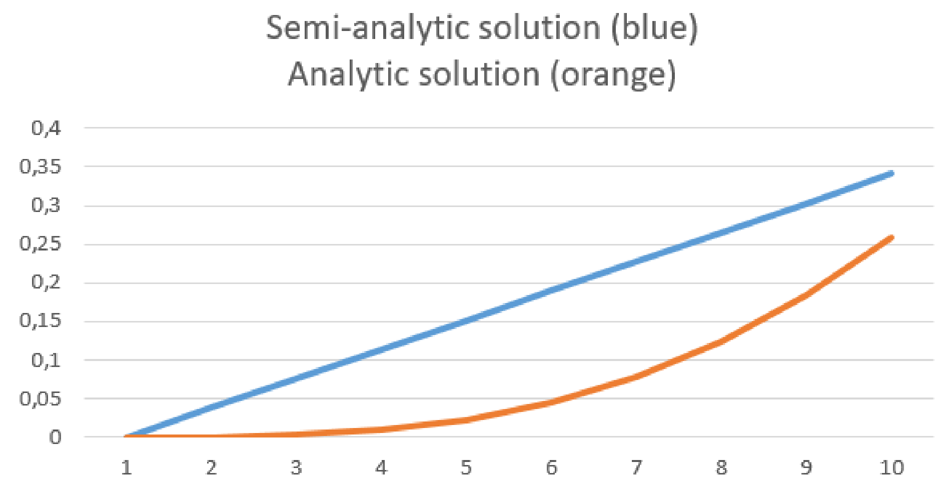

Figure 12. The curves of solutions

The Figure 12 represents the curves of our semi-analytic solution (blue) and the exact solution (orange) $y(x)=x+1-\exp(x-\frac{1}{2}x^2)$.

4. Conclusion

In this article, we have presented a new type of function that we called: the Function Number. The Function Number is simply a real number which represents a variable function. To transform any variable function to a Function Number, there is a simple methodology that we have presented in the section 2. The use of the Function Number is so easy that it permits to solve complex mathematical problems. The Function Number can be used for integral calculus, for deriving variable functions and also for solving differential equations. In the last section, we have proposed some applications to verify the precision of the results given by the Function Number Method. This new methodology permits to solve some unsolved mathematics problems; like integral calculus and differential equations in Physics and Engineering.

Data Availability Statement

No underlying data was collected or produced in this study.

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