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The Improved (G'/G)-Expansion Method to the (3+1)-Dimensional Kadomstev-Petviashvili Equation

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Abstract In this article, the improved (G'/G)-expansion method has been implemented to generate travelling wave solutions, where $G(\xi)$ satisfies the second order linear ordinary differential equation. To show the advantages of the method, the (3+1)-dimensional Kadomstev-Petviashvili (KP) equation has been investigated. Higher-dimensional nonlinear partial differential equations have many potential applications in mathematical physics and engineering sciences. Some of our solutions are in good agreement with already published results for a special case and others are new. The solutions in this work may express a variety of new features of waves. Furthermore, these solutions can be valuable in the theoretical and numerical studies of the considered equation. Also, in order to understand the behaviour of solutions, the graphical representations of some obtained solutions have been presented.

Keywords: *the improved (G'/G)-expansion method, the Kadomstev-Petviashvili equation, traveling wave solutions, nonlinear evolution equations*

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1. Introduction

Partial differential equations (PDEs) are widely used as models for describing important physical phenomena arising in scientific and engineering fields, such as, plasma physics, fluid mechanics, solid state physics, quantum mechanics, nonlinear optics, chemical physics and many others. It is more significant to construct analytical solutions of nonlinear evolution equations (NLEEs) to disclose more information for better understanding various nonlinear complex phenomena as well as further applications in real time scientific fields. During recent past, many important and powerful methods have been developed to obtain traveling wave solutions and to reveal their properties. For example, the inverse scattering method [1], the Hirota's bilinear method [2], the homogeneous balance method [3], the Backlund transformation method [4,5], the Jacobi elliptic function expansion method [6], the tanh-coth method [7,8], the F-expansion method [9], the Exp-function method [10,11,12,13,14], the first integral method [15,16] and others [17,18,19,20].

Another important method presented to construct exact solutions of nonlinear PDEs is the basic (G'/G)-expansion method. The concept of this method was first proposed by Wang *et al.* [21], consequently, many researchers applied the (G'/G)-expansion method to solve different kinds of NLEEs [22,23,24,25,26]. More recently, Zhang *et al.* [27] extended the basic (G'/G)-expansion method which is called the improved (G'/G)-expansion method to establish abundant traveling wave solutions of nonlinear PDEs. This method is one of the most powerful and effective method to handle different NLEEs. They employed $B(\xi)=\sum_{j=-w}^{w} c_j (G'/G)^j$, as traveling wave solutions, where either c_{-w} or c_w may be zero, but both c_{-w} and c_w cannot be zero at the same time. Subsequently, it has been successfully implemented to solve several classes of nonlinear problems [28-33].

Many researchers applied a variety of methods to obtain exact solutions of the (3+1)-dimensional KP equation. For instance, Peng and Krishnan [34] studied this equation by using extended mapping method to construct analytical solutions. In Ref. [35], Khalfallah implemented homogeneous balance method to establish traveling wave solutions of the same equation. Bekir and Uygun [36] investigated this equation for obtaining traveling wave solutions via the (G'/G)-expansion method. In this basic (G'/G)-expansion method, they utilized $u(\xi)=\sum_{i=0}^{m} a_i (G'/G)^i$, where $a_m \neq 0$, as traveling wave solutions, instead of $B(\xi)=\sum_{j=-w}^{w} c_j (G'/G)^j$, where either c_{-w} or c_w may be zero, but both c_{-w} and c_w cannot be zero at a time.

The significance of our present work is, in order to generate abundant traveling wave solutions, the (3+1)-dimensional KP equation has been considered by applying the improved (G'/G)-expansion method.

2. The Improved (G'/G) -Expansion Method

Suppose the general nonlinear partial differential equation:

$$Q\left(u,u_t,u_x,u_y,u_z,u_{tt},u_{xt},u_{xx},u_{xxx},...\right)=0, \tag{1}$$

where $u=u(x,y,z,t)$ is an unknown function, Q is a polynomial in $u(x,y,z,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. The main steps of the method are as follows:

Step 1. Consider the traveling wave variable:

$$u(x,y,z,t)=B(\xi), \qquad \xi=x+y+z-Ft, \tag{2}$$

where F is the wave speed. Now using Eq. (2), Eq. (1) is converted into an ordinary differential equation for $B(\xi)$:

$$H\left(B,B',B'',B''',...\right)=0, \tag{3}$$

where the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2. According to possibility, Eq. (3) can be integrated term by term one or more times, yields constant(s) of integration. The integral constant may be zero, for simplicity.

Step 3. Suppose that the traveling wave solution of Eq. (3) can be expressed in the following form:

$$B(\xi)=\sum_{j=-w}^{w} c_j\left(G'/G\right)^j \tag{4}$$

with $G=G(\xi)$ satisfies the second order linear ODE:

$$G''+\delta G'+\eta G=0, \tag{5}$$

where $c_j\,(j=0,\pm1,\pm2,...,\pm w)$, δ and η are constants.

Step 4. To determine the positive integer w, taking the homogeneous balance between the highest order nonlinear terms and the highest order derivatives appearing in Eq. (3).

Step 5. Substituting Eq. (4) and Eq. (5) into Eq. (3) with the value of w obtained in Step 4, yield polynomials in $(G'/G)^r$, $(r=0,\pm1,\pm2,...)$, then setting each coefficient of the resulted polynomials to zero, we obtain a set of algebraic equations for $c_j\,(j=0,\pm1,\pm2,...,\pm w)$, F,β and η.

Step 6. Solving the system of algebraic equations which are obtained in step 5 with the aid of algebraic software Maple and we obtain values for $c_j\,(j=0,\pm1,\pm2,...,\pm w)$ and F. Then, we substitute obtained values in Eq. (4) along with the general solution of Eq. (5) which is well known to us, we can obtain the traveling wave solutions of Eq. (1).

3. Application of the Method

In this section, the (3+1)-dimensional KP equation has been studied to obtain abundant traveling wave solutions including solitons, periodic and rational solutions by applying the improved (G'/G)-expansion method.

3.1. The (3+1)-Dimensional KP Equation

The KP equation is used to model shallow-water waves with weakly nonlinear restoring forces.

The (3+1)-dimensional KP equation is being considered which is followed by Bekir and Uygun [36]:

$$u_{xt}+6u_x^2+6uu_{xx}-u_{xxxx}-u_{yy}-u_{zz}=0. \tag{6}$$

Now, we use the wave transformation Eq. (2) into the Eq. (6), which yields:

$$-(F+2)B''+6(B')^2+6BB''-B^{(4)}=0. \tag{7}$$

Integrating twice and setting the constants of integration to zero, we obtain

$$-(F+2)B+3B^2-B''=0. \tag{8}$$

Taking the homogeneous balance between the nonlinear term B^2 and the highest order derivative B'' in Eq. (8), we obtain $w=2$.

Therefore, the solution of Eq. (8) is of the form:

$$\begin{aligned}B(\xi)=&\,c_{-2}\left(G'/G\right)^{-2}+c_{-1}\left(G'/G\right)^{-1}\\&+c_0+c_1\left(G'/G\right)+c_2\left(G'/G\right)^2,\end{aligned} \tag{9}$$

where c_{-2},c_{-1},c_0,c_1 and c_2 are constants to be determined.

Substituting Eq. (9) together with Eq. (5) into the Eq. (8), the left-hand side of Eq. (8) is converted into polynomials in $(G'/G)^r,(r=0,\pm1,\pm2,...)$. Then, equating each coefficient of the resulted polynomials to zero, yields a set of algebraic equations (for simplicity, which are not presented) for $c_{-2},c_{-1},c_0,c_1,c_2,F,\beta$ and η. Solving the system of obtained algebraic equations with the aid of algebraic software Maple, we obtain four different values.

Case 1:

$$\begin{aligned}&c_{-2}=0,c_{-1}=0,\ c_0=2\eta,\ c_1=2\beta,\\&c_2=2,F=-\left(\beta^2-4\eta+2\right),\end{aligned} \tag{10}$$

where β and η are free parameters.

Case 2:

$$\begin{aligned}&c_{-2}=0,c_{-1}=0,\ c_0=\frac{1}{3}\left(\beta^2+2\eta\right),\\&c_1=2\beta,\ c_2=2,F=\left(\beta^2-4\eta-2\right),\end{aligned} \tag{11}$$

where β and η are free parameters.

Case 3:

$$c_{-2} = 2\eta^2, c_{-1} = 2\beta\eta,\ c_0 = 2\eta,\quad c_1 = 0,\ c_2 = 0, F = -\left(\beta^2 - 4\eta + 2\right), \tag{12}$$

where β and η are free parameters.

Case 4:

$$c_{-2} = 2\eta^2, c_{-1} = 2\beta\eta,\ c_0 = \frac{1}{3}\left(\beta^2 + 2\eta\right),\quad c_1 = 0,\ c_2 = 0, F = \left(\beta^2 - 4\eta - 2\right), \tag{13}$$

where β and η are free parameters.

Substituting the general solution Eq. (5) into Eq. (9), we obtain three different families of traveling wave solutions of Eq. (8):

Family 1: Hyperbolic function solution: When $\beta^2 - 4\eta > 0$, yields

$$\begin{aligned} B(\xi) &= c_{-2}\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{\beta^2-4\eta}\,\frac{P_1\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}{P_1\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}\right)^{-2} \\ &+ c_{-1}\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{\beta^2-4\eta}\,\frac{P_1\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}{P_1\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}\right)^{-1} \\ &+ c_0 \\ &+ c_1\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{\beta^2-4\eta}\,\frac{P_1\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}{P_1\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}\right) \\ &+ c_2\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{\beta^2-4\eta}\,\frac{P_1\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}{P_1\cosh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi + P_2\sinh\frac{1}{2}\sqrt{\beta^2-4\eta}\,\xi}\right)^{2}. \end{aligned} \tag{14}$$

If P_1 and P_2 are taken particular values, various known solutions can be rediscovered.

Family 2: Trigonometric function solution: When $\beta^2 - 4\eta < 0$, we obtain

$$\begin{aligned} B(\xi) &= c_{-2}\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{4\eta-\beta^2}\,\frac{-P_1\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}{P_1\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}\right)^{-2} \\ &+ c_{-1}\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{4\eta-\beta^2}\,\frac{-P_1\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}{P_1\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}\right)^{-1} \\ &+ c_0 \\ &+ c_1\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{4\eta-\beta^2}\,\frac{-P_1\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}{P_1\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}\right) \\ &+ c_2\left(\frac{-\beta}{2} + \frac{1}{2}\sqrt{4\eta-\beta^2}\,\frac{-P_1\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}{P_1\cos\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi + P_2\sin\frac{1}{2}\sqrt{4\eta-\beta^2}\,\xi}\right)^{2}. \end{aligned} \tag{15}$$

If P_1 and P_2 are taken particular values, various known solutions can be rediscovered.

Family 3: Rational function solution: When $\beta^2 - 4\eta = 0$, we obtain

$$\begin{aligned} B(\xi) &= c_{-2}\left(\frac{-\beta}{2} + \frac{P_2}{P_1 + P_2\xi}\right)^{-2} + c_{-1}\left(\frac{-\beta}{2} + \frac{P_2}{P_1 + P_2\xi}\right)^{-1} \\ &+ c_0 + c_1\left(\frac{-\beta}{2} + \frac{P_2}{P_1 + P_2\xi}\right) + c_2\left(\frac{-\beta}{2} + \frac{P_2}{P_1 + P_2\xi}\right)^{2}. \end{aligned} \tag{16}$$

Substituting Eqs. (10), (11) (12) and (13) together with the general solution Eq. (5) into the Eq. (9), yields the hyperbolic function solution Eq. (14), we obtain following solutions respectively (if $P_1 = 0$ but $P_2 \neq 0$):

$$B_1(\xi) = \frac{\left(\beta^2 - 4\eta\right)}{2}\left(\coth\left(\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^2 - 1\right),$$

where $\xi = x + y + z + \left(\beta^2 - 4\eta + 2\right) t.$

$$B_2(\xi)=\frac{\left(\beta^2-4\eta\right)}{6}\left(3\coth\left(\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^2-1\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta-2\right)t.$

$$B_3(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\coth\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\coth\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-1}+1\end{array}\right)$$

where $\xi=x+y+z+\left(\beta^2-4\eta+2\right)t.$

$$B_4(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\coth\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\coth\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-1}\end{array}\right)+\frac{1}{3}\left(2\eta+\beta^2\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta-2\right)t.$

Again, substituting Eqs. (10), (11), (12) and (13) together with the general solution Eq. (5) into the Eq. (9), we obtain the hyperbolic function solution Eq. (14), our traveling wave solutions become respectively (if $P_2=0$ but $P_1\neq 0$):

$$B_5(\xi)=\frac{\left(\beta^2-4\eta\right)}{2}\left(\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^2-1\right).$$

$$B_6(\xi)=\frac{\left(\beta^2-4\eta\right)}{6}\left(3\tanh\left(\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^2-1\right).$$

$$B_7(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\tanh\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\tanh\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-1}+1\end{array}\right).$$

$$B_8(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\tanh\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{\beta^2-4\eta}}{2}\tanh\frac{1}{2}\sqrt{\beta^2-4\eta}\ \xi\right)^{-1}\end{array}\right)+\frac{1}{3}\left(2\eta+\beta^2\right).$$

Substituting Eqs. (10), (11), (12) and (13) together with the general solution Eq. (5) into the Eq. (9), yields the trigonometric function solution Eq. (15), our exact solutions become respectively (if $P_1=0$ but $P_2\neq 0$):

$$B_9(\xi)=\frac{\left(4\eta-\beta^2\right)}{2}\left(\cot\left(\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^2+1\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta+2\right)t.$

$$B_{10}(\xi)=\frac{\left(4\eta-\beta^2\right)}{6}\left(3\cot\left(\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^2+1\right),$$

where $\xi=x+y+z+\left(\beta^2-4\eta-2\right)t.$

$$B_{11}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{4\eta-\beta^2}}{2}\cot\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{4\eta-\beta^2}}{2}\cot\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-1}+1\end{array}\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta+2\right)t.$

$$B_{12}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}+\frac{\sqrt{4\eta-\beta^2}}{2}\cot\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}+\frac{\sqrt{4\eta-\beta^2}}{2}\cot\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-1}\end{array}\right)+\frac{1}{3}\left(2\eta+\beta^2\right),$$

where $\xi=x+y+z+\left(\beta^2-4\eta-2\right)t.$

Also, substituting Eqs. (10), (11), (12) and (13) together with the general solution Eq. (5) into the Eq. (9), yields the trigonometric function solution Eq. (15), we construct following solutions respectively (if $P_2=0$ but $P_1\neq 0$):

$$B_{13}(\xi)=\frac{\left(4\eta-\beta^2\right)}{2}\left(\tan\left(\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^2+1\right).$$

$$B_{14}(\xi)=\frac{\left(4\eta-\beta^2\right)}{6}\left(3\tan\left(\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^2+1\right).$$

$$B_{15}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}-\frac{\sqrt{4\eta-\beta^2}}{2}\tan\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}-\frac{\sqrt{4\eta-\beta^2}}{2}\tan\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-1}+1\end{array}\right).$$

$$B_{16}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\frac{-\beta}{2}-\frac{\sqrt{4\eta-\beta^2}}{2}\tan\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-2}\\+\beta\left(\frac{-\beta}{2}-\frac{\sqrt{4\eta-\beta^2}}{2}\tan\frac{1}{2}\sqrt{4\eta-\beta^2}\ \xi\right)^{-1}\end{array}\right)+\frac{1}{3}\left(2\eta+\beta^2\right).$$

Substituting Eqs. (10), (11), (12) and (13) together with the general solution Eq. (5) into the Eq. (9), we obtain the rational function solution Eq. (16), and we construct following traveling wave solutions respectively (if $\beta^2-4\eta=0$):

$$B_{17}(\xi)=\frac{1}{2}\left(\left(\frac{2P_2}{P_1+P_2\xi}\right)^2-\left(\beta^2-4\eta\right)\right),$$

where $\xi=x+y+z+\left(\beta^2-4\eta+2\right)t$.

$$B_{18}(\xi)=\frac{1}{6}\left(3\left(\frac{2P_2}{P_1+P_2\xi}\right)^2-\left(\beta^2-4\eta\right)\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta-2\right)t$.

$$B_{19}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\dfrac{-\beta}{2}+\dfrac{P_2}{P_1+P_2\xi}\right)^{-2}\\+\beta\left(\dfrac{-\beta}{2}+\dfrac{P_2}{P_1+P_2\xi}\right)^{-1}+1\end{array}\right),$$

where $\xi=x+y+z+\left(\beta^2-4\eta+2\right)t$.

$$B_{20}(\xi)=2\eta\left(\begin{array}{l}\eta\left(\dfrac{-\beta}{2}+\dfrac{P_2}{P_1+P_2\xi}\right)^{-2}\\+\beta\left(\dfrac{-\beta}{2}+\dfrac{P_2}{P_1+P_2\xi}\right)^{-1}\end{array}\right)+\frac{1}{3}\left(\beta^2+2\eta\right),$$

where $\xi=x+y+z-\left(\beta^2-4\eta-2\right)t$.

4. Results and Discussion

It is important to point out that some of constructed solutions are in good agreement with already published results which have been shown in the Table 1. Furthermore, some of obtained traveling wave solutions are expressed in Figure 1 to Figure 8.

Table 1. Comparison between Bekir and Uygun [36] solutions and Our obtained solutions

Bekir and Uygun [36] solutions	Our solutions
i. If $C_1=0, C_2\neq 0, \lambda=3$ and $\mu=2$ solution Eq. (5.14) (from section 5) becomes: $u_1(\xi)=\frac{1}{2}\left(\coth^2\frac{1}{2}\xi-1\right)$.	i. If $\beta=3,\eta=2$ and $B_1(\xi)=u_1(\xi)$, solution $B_1(\xi)$ becomes: $u_1(\xi)=\frac{1}{2}\left(\coth^2\frac{1}{2}\xi-1\right)$.
ii. If $C_1\neq 0, C_2=0, \lambda=4$ and $\mu=3$ solution Eq. (4.14) (from section 5) becomes: $u_1(\xi)=2\left(\tanh^2\xi-1\right)$.	ii. If $\beta=4,\eta=3$ and $B_5(\xi)=u_1(\xi)$, solution $B_5(\xi)$ becomes: $u_1(\xi)=2\left(\tanh^2\xi-1\right)$.
iii. If $C_1=0, C_2\neq 0, \lambda=4$ and $\mu=5$ solution Eq. (5.16) (from section 5) becomes: $u_3(\xi)=2\left(\cot^2\xi+1\right)$.	iii. If $\beta=4,\eta=5$ and $B_9(\xi)=u_3(\xi)$, solution $B_9(\xi)$ becomes: $u_3(\xi)=2\left(\cot^2\xi+1\right)$.
iv. If $C_1\neq 0, C_2=0, \lambda=1$ and $\mu=1$ solution Eq. (5.16) (from section 5) becomes: $u_3(\xi)=\frac{3}{2}\left(\tan^2\left(\sqrt{3}/2\right)\xi+1\right)$.	iv. If $\beta=1,\eta=1$ and $B_{13}(\xi)=u_3(\xi)$, solution $B_{13}(\xi)$ becomes: $u_3(\xi)=\frac{3}{2}\left(\tan^2\left(\sqrt{3}/2\right)\xi+1\right)$.

Beside above table, many new traveling wave solutions have been constructed, such as, B_2 to B_4, B_6 to B_8, B_{10} to B_{12} and B_{14} to B_{20} which are not being revealed in the previous literature.

4.1. Graphical Representations of the Solutions

The graphical depiction of some solutions has been described in the figures with the aid of commercial software Maple:

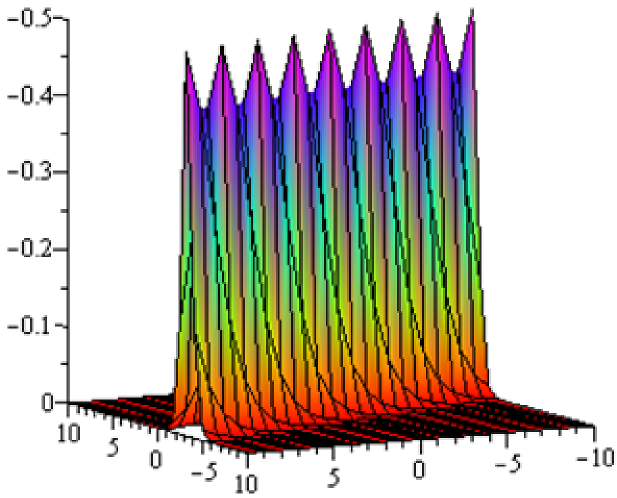

Figure 1. Solitons solution for $\beta=3,\eta=2$

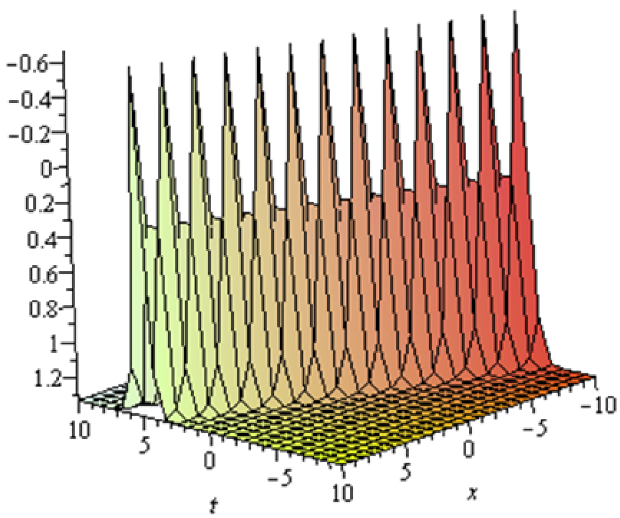

Figure 2. Solitons solution for $\beta=6,\eta=8$

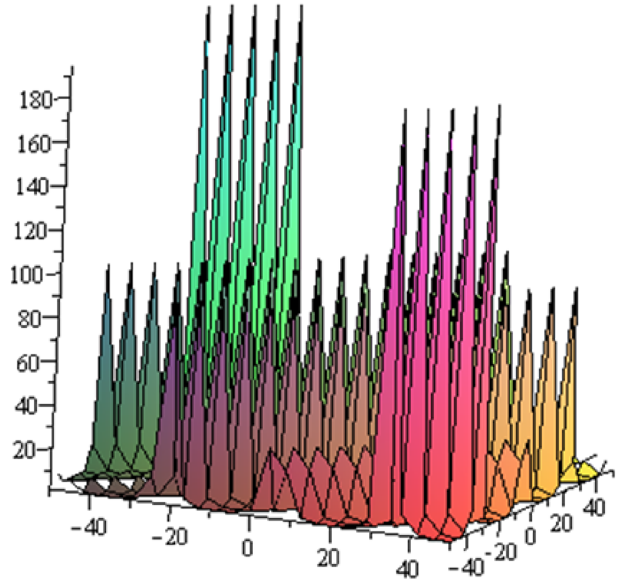

Figure 3. Solitons solution for $\beta = 3, \eta = 3$

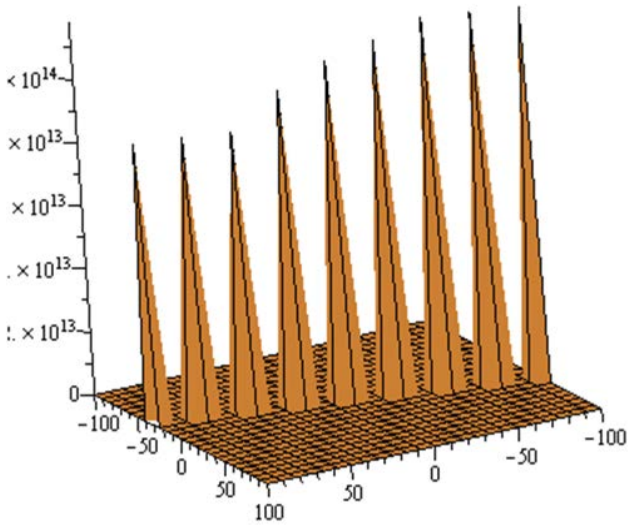

Figure 4. Solitons solution for $\beta = 5, \eta = 6$

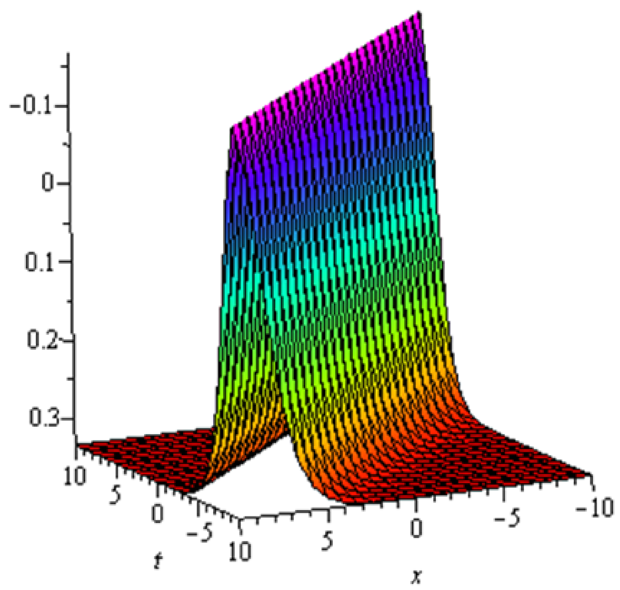

Figure 5. Periodic solution for $\beta = 7, \eta = 12$

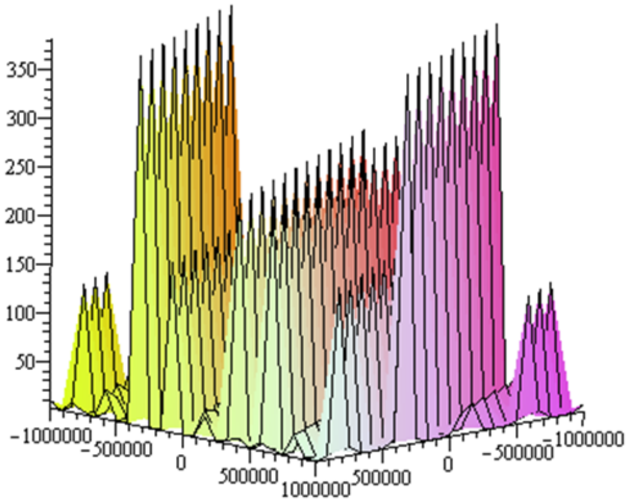

Figure 6. Solitons solution for $\beta = 4, \eta = 5$

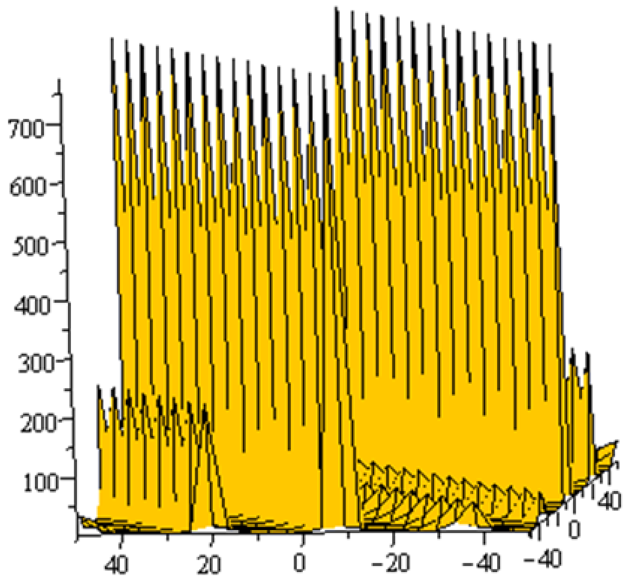

Figure 7. Solitons solution for $\beta = 5, \eta = 7$

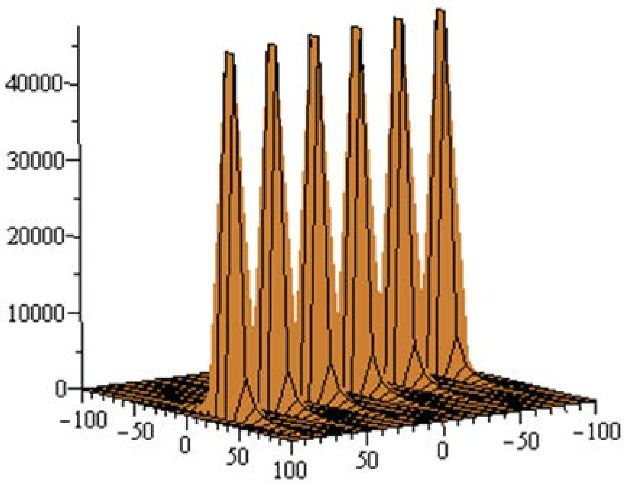

Figure 8. Solitons solution for $\beta = 8, \eta = 17.5$

5. Conclusions

In this article, we apply the improved (G'/G) -expansion method to generate a rich class of new traveling wave solutions of the highly nonlinear PDE, namely, the (3+1)-dimensional Kadomstev-Petviashvili equation. The presented solutions may express a variety of new features of waves. Moreover, the obtained exact solutions reveal that the improved $\left(G'/G\right)$ -expansion method is a promising mathematical tool, because, it can establish abundant new traveling wave solutions with different physical structures. Subsequently, the used method could lead to construct many new traveling wave solutions for various nonlinear PDEs which frequently arise in scientific real time application fields.

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