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A Generalized Multiplier Transform on a P-valent Integral Operator with Application

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Abstract The aim of this paper is to obtain coefficient estimates of the integral operator of the form: $F_{p,\mu} = \int_0^z \prod_{i=1}^k \left(\frac{I_{\alpha,\beta,\gamma} f_{i,p}(t)}{t} \right)^{\frac{i}{\mu}} dt, \mu \in C, p \geq 1$ and $|\mu| \leq 1$ using the relationship between starlike and convex functions and give its implication to disease control. Also, we obtain the growth and distortion theorems for the operator.

Keywords: *p-valent; differential operator, integral operator, growth and distortion, disease control*

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1. Introduction and Preliminaries

Let A denote the class of normalized univalent functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1)$$

which are analytic in the unit disc $U = \{z \in C: |z|\}$ with $f(0) = f(0)' - 1 = 0$.

In the class of analytic functions in (1) above, the coefficients a_n is the quotient of the nth derivative of the function been expressed in Taylor series expansion and n!. Derivatives, whether partial or total, describe the slope of a function which is the steepness of a line. It is a known fact that the larger the slope, the steeper the line and vice versa. The coefficients a_n play prominent role in analytic functions. Thus, in applying analytic functions to any area of life, one needs to consider it's coefficient bounds. The author in [6] stated that starlike and convex functions, which are aspect of analytic functions have its applications in human physiology, physical and natural phenomenon. Analytic functions can also be applied to curtailing the spread of any disease, or its occurrence. Here, we think of applying analytic functions to prevention of population of carrier of disease organism from being transformed to infectious population and thereby preventing them from being transformed to disease population. In [10], major categories of infectious agents and common vectors and vehicles of disease were given while Harvard health publishing in [9], gave some ways of preventing infections, World health Organization in [11] stated that there is an urgent need to re-establish basic infection control measures and in [12] gave measures of controlling the spread of infectious diseases, which is still very relevant today. Different authors have derived several differential operators. Author in [4], derived a new multiplier differential operator, in an attempt to get a class of analytic functions with finer coefficients bounds which give a better result in an application mentioned above.

For the function f of the form (1) in A, the following results are well known: f is said to be starlike respectively convex if and only if

$$\mathrm{Re}\frac{zf(z)'}{f(z)} > 0, |z| < 1.$$

And

$$\mathrm{Re}\,1 + \frac{zf''(z)}{f'(z)} > 0, |z| < 1.$$

Swamy [7], introduced a multiplier differential operator of the form $I_{\alpha,\beta}^s f$ defined by:

$$I_{\alpha,\beta}^s f(z) = f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + k\beta}{\alpha + \beta} \right)^n a_n z^n$$

which is analytic and univalent in the unit disk. For details see [7].

Author in [5] defined a linear operator $I_{\alpha,\beta,\gamma}^n f$ by:

$$I_{\alpha,\beta,\gamma}^s f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i z^n, \qquad (2)$$

$$\alpha \geq 1, \beta, \gamma \geq 0$$

and the class $\Gamma_\alpha(\zeta_1,\zeta_{2;}\gamma)$ by:

$$\Gamma_\alpha(\zeta_1,\zeta_{2;}\gamma)=\left\{f\in A:\left|\frac{G(z)+\frac{1}{\alpha}-1}{\zeta_1\left(G(z)+\frac{1}{\alpha}+\zeta_2\right)}\right|<\lambda\right\}$$

Where

$$G(z)=\sum_{i=1}^{k}\frac{1}{\alpha}\left(\frac{zf_i'(z)}{f_i(z)}-1\right).$$

And

$$f_i(z)=z+\sum_{n=p+1}^{\infty}a_n^i z^n. \qquad (3)$$

Now, let

$$I_{\alpha,\beta,\gamma}^s f_{i,p}(z)=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s a_n^i z^n, \qquad (4)$$

$\alpha,p\ge 1,\beta,\gamma\ge 0.$

We define $F_{p,\mu}$ by:

$$F_{p,\mu}=\int_0^z\prod_{i=1}^{k}\left(\frac{I_{\alpha,\beta,\gamma}^s f_{i,p}(t)}{t}\right)^{\frac{1}{\mu}}dt, \qquad (5)$$

$\mu\in C$ and $|\mu|\le 1;\ s,\beta,\gamma\ge 0$

and the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$ by:

$$\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)=\left\{I_{\alpha,\beta,\gamma}^s f_{i,p}\in A:\left|\frac{H(z)+\frac{1}{\mu}-p}{\zeta_1\left(H(z)+\frac{1}{\mu}+\zeta_2\right)}\right|<\delta\right\} \qquad (6)$$

Where $H(z)=\dfrac{zF_\mu''(z)}{F_\mu'(z)}.$

Furthermore, let

$$I_{\alpha,\beta,\gamma}^s f_{i,p}(z)=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s a_n^i z^n,$$

and

$$I_{\alpha,\beta,\gamma}^s g_{i,p}(z)=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s b_n^i z^n,$$

we define the convolution of $I_{\alpha,\beta,\gamma}^s f_{i,p}$ and $I_{\alpha,\beta,\gamma}^s g_{i,p}$ by:

$$\left(I_{\alpha,\beta,\gamma}^s f_{i,p}*I_{\alpha,\beta,\gamma}^s g_{i,p}\right)(z)=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s a_n^i b_n^i z^n. \qquad (7)$$

In this paper, we obtain the coefficients bound for the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$ and apply it to disease control measure.

Lemma [3]: Let f_i be as in (3). Then f_i is in the class $\Gamma_\alpha(\zeta_1,\zeta_{2;}\gamma)$ if and only if

$$\sum_{i=1}^{k}\sum_{n=2}^{\infty}n\left[(1+\gamma\zeta_1)+\alpha(\gamma\zeta_2-1)\right]\left|a_n^i\right| \le\lambda(\zeta_1+\alpha\zeta_2)+|\alpha-1|,$$

$0\le\lambda<1;0\le\zeta_1;\zeta_2\le 1;0<\alpha\le 1.$

Now we state and prove the main results of this paper.

2. Main Results

Theorem 1: Let $F_{p,\mu}$ be as in (5). Then $I_{\alpha,\beta,\gamma}^s f_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$ if

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s\left[n(1+\delta\zeta_1)+\mu(\delta\zeta_2-1)\right]\left|a_n^i\right| \le\delta(p\zeta_1+\mu\zeta_2)+|\mu-p|,$$

$0\le\zeta_1;\zeta_2\le 1;0<\delta<1;\beta,\gamma\ge 0;\mu\in C$ and $|\mu|\le 1.$

Proof: Given that

$$F_{p,\mu}=\int_0^z\prod_{i=1}^{k}\left(\frac{I_{\alpha,\beta,\gamma}^s f_{i,p}(t)}{t}\right)^{\frac{1}{\mu}}dt,$$

$\mu\in C$ and $|\mu|\le 1;s,\beta,\gamma\ge 0.$

And

$$I_{\alpha,\beta,\gamma}^s f_{i,p}(z)=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s a_n^i z^n,$$

$\alpha,p\ge 1;s,\beta,\gamma\ge 0,$

with simple calculation, we have

$$\left|\frac{H(z)+\frac{1}{\mu}-p}{\zeta_1\left(H(z)+\frac{1}{\mu}+\zeta_2\right)}\right|$$

$$=\left|\frac{\dfrac{\sum_{i=1}^{k}\left(pz^p+\sum_{n=p+1}^{\infty}nx^s a_n^i z^n\right)-\mu\left(z^p+\sum_{n=p+1}^{\infty}x^s a_n^i z^n\right)}{\sum_{i=1}^{k}\mu\left(z^p+\sum_{n=p+1}^{\infty}x^s a_n^i z^n\right)}}{\dfrac{\left[\sum_{i=1}^{k}\zeta_1\left(pz^p+\sum_{n=p+1}^{\infty}nx^s a_n^i z^n\right)+\zeta_2\sum_{i=1}^{k}\mu\left(z^p+\sum_{n=p+1}^{\infty}x^s a_n^i z^n\right)\right]}{\sum_{i=1}^{k}\mu\left(z^p+\sum_{n=p+1}^{\infty}x^s a_n^i z^n\right)}}\right|$$

Where $x = \frac{\alpha + k\beta + n^2\gamma}{\alpha+\beta+\gamma}$,

$$H(z) = \frac{zF''_{p,\mu}(z)}{F'_{p,\mu}(z)} = \sum_{i=1}^{k} \frac{1}{\mu}\left(\frac{zI^s_{\alpha,\beta,\gamma} f_{i,p}{}'(z)}{I^s_{\alpha,\beta,\gamma} f_{i,p}(z)} - 1 \right).$$

For $z \to 1^-$, we have

$$\left| \frac{H(z) + \frac{1}{\mu} - p}{\zeta_1\left(H(z) + \frac{1}{\mu} + \zeta_2 \right)} \right|$$

$$\le \frac{p - \mu + \sum_{i=1}^{k}\sum_{n=p+1}^{\infty} x^s (n-\mu) | a_n^i |}{p\zeta_1 + \mu\zeta_2 - \sum_{i=1}^{k}\sum_{n=p+1}^{\infty} x^s (n\zeta_1 + \mu\zeta_2) | a_n^i |}.$$

This is bounded by δ if

$$p - \mu + \sum_{i=1}^{k}\sum_{n=p+1}^{\infty} x^s (n-\mu) | a_n^i |$$

$$\le p\delta\zeta_1 + \mu\delta\zeta_2 - \sum_{i=1}^{k}\sum_{n=p+1}^{\infty} \delta x^s (n\zeta_1 + \mu\zeta_2) | a_n^i |.$$

Fixing the value of x and restructuring, we have the result.

Remark: The above theorem shows the relationship between the convexity of the integral operator in (5) and the starlikeness of the differential operator in (4).

Corollary 1: Let $F_{p,\mu}$ be as defined in (5) and $I^s_{\alpha,\beta,\gamma} f_{i,p}$ belongs to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. Also, let f_i belongs to the class $\Gamma_\alpha(\zeta_1,\zeta_{2;}\gamma)$ as given in lemma 1. Then $f_i \subset I^s_{\alpha,\beta,\gamma} f_{i,p}$, $0 < \delta < 1$; $\beta,\gamma \ge 0$; $\mu \in C$ and $|\mu| \le 1$.

Proof: Let $F_{p,\mu}$ be as defined in (5) and $I^s_{\alpha,\beta,\gamma} f_{i,p}$ belongs to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. From theorem 1, we have for $\alpha, p \ge 1; s,\beta,\gamma \ge 0; 0 \le \zeta_1;\ \zeta_2 \le 1;\ \mu \in C$ and $|\mu| \le 1$.

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty} n\left[(1+\delta\zeta_1) + \mu(\delta\zeta_2 - 1) \right] \left| a_n^i \right|$$

$$\le \frac{(\alpha+\beta+\gamma)^s \left(\delta(p\zeta_1 + \mu\zeta_2) + |\mu - p| \right)}{\left(\alpha + n\beta + n^2\gamma\right)^s}$$

$$\le \lambda(\zeta_1 + \alpha\zeta_2) + |\alpha - 1|.$$

For $0 \le \lambda < 1;\ 0 \le \zeta_1;\ \zeta_2 \le 1; 0 < \alpha \le 1;\ n \ge 2$.

This proves the result.

Corollary 2: Let $I^s_{\alpha,\beta,\gamma} f_{i,p}$ belongs to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. Then

$$\left| a_n^i \right| \le \frac{(\alpha+\beta+\gamma)^s \left(\delta(p\zeta_1 + \mu\zeta_2) + |\mu - p| \right)}{\left(n\left[(1+\delta\zeta_1) + \mu(\delta\zeta_2 - 1) \right] \right)\left(\alpha + n\beta + n^2\gamma\right)^s},$$

$\alpha, p \ge 1; s,\beta,\gamma \ge 0; 0 \le \zeta_1;\ \zeta_2 \le 1;\ \mu \in C$ and $|\mu| \le 1$ $n \ge 2$.

Corollary 3: Let $I^s_{\alpha,\beta,\gamma} f_{i,1}$ belongs to the class $\Gamma_{1,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_2;\delta)$. Then

$$\left| a_n^i \right| \le \frac{(\alpha+\beta+\gamma)^s \left(\delta(\zeta_1 + \mu\zeta_2) + |\mu - 1| \right)}{\left(n\left[(1+\delta\zeta_1) + \mu(\delta\zeta_2 - 1) \right] \right)\left(\alpha + n\beta + n^2\gamma\right)^s},$$

$\alpha \ge 1;\ s,\beta,\gamma \ge 0; 0 \le \zeta_1;\ \zeta_2 \le 1; n \ge 2$.

Corollary 4: Let $I^s_{\alpha,\beta,\gamma} f_{i,1}$ belongs to the class $\Gamma_{1,1;\alpha,\beta,\gamma}(\zeta_1,\zeta_2;\delta)$. Then

$$\left| a_n^i \right| \le \frac{(\alpha+\beta+\gamma)^s \delta(\zeta_1 + \zeta_2)}{\left(n\left[(1+\delta\zeta_1) + (\delta\zeta_2 - 1) \right] \right)\left(\alpha + n\beta + n^2\gamma\right)^s},$$

$\alpha \ge 1;\ s,\beta,\gamma \ge 0; 0 \le \zeta_1;\ \zeta_2 \le 1; n \ge 2$.

Corollary 5: Let $I^s_{1,1,1} f_{i,1}$ belongs to the class $\Gamma_{1,1;\alpha,\beta,\gamma}(\zeta_1,\zeta_2;\delta)$. Then

$$\left| a_n^i \right| \le \frac{(3)^s \delta(\zeta_1 + \zeta_2)}{\left(n\left[(1+\delta\zeta_1) + (\delta\zeta_2 - 1) \right] \right)\left(\alpha + n + n^2\right)^s},$$

$\alpha \ge 1;\ 0 \le \zeta_1;\ \zeta_2 \le 1; n \ge 2$.

Corollary 6: Let $I^0_{\alpha,\beta,\gamma} f_{i,1}$ belongs to the class $\Gamma_{1,1;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. Then $\left| a_n^i \right| \le \frac{\delta(\zeta_1 + \zeta_2)}{\left(n\left[(1+\delta\zeta_1) + (\delta\zeta_2 - 1) \right] \right)}$,

$\alpha \ge 1; 0 \le \zeta_1;\ \zeta_2 \le 1;\ n \ge 2$.

Corollary 7: Let $I^0_{\alpha,\beta,\gamma} f_{i,1}$ belongs to the class $\Gamma_{1,1;\alpha,\beta,\gamma}(0,1;\delta)$. Then $\left| a_n^i \right| \le \frac{\delta\mu}{\left(n\left[1 + (\delta - 1) \right] \right)}$, $\alpha \ge 1$; $\beta,\gamma \ge 0; n \ge 2$.

Theorem 2: Let $F_{p,\mu}$ be as in (5) and $I^s_{\alpha,\beta,\gamma} f_{i,p}$, $I^s_{\alpha,\beta,\gamma} g_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. Then $\left(I^s_{\alpha,\beta,\gamma} f_{i,p} * I^s_{\alpha,\beta,\gamma} g_{i,p}\right)$(z) belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$ if

$$\lambda \le \frac{\left\{ \begin{array}{l} (n-\mu)|\mu - p|\left[2(v-u) + n - \mu - |\mu - p| \right] \\ +\delta^2 (v^2 |\mu - p| - u^2 (n-\mu) \end{array} \right\}}{\left\{ \begin{array}{l} \delta uv(\delta(u-v) + 2\left[|\mu - p| - (n-\mu) \right] \\ +v|\mu - p|^2 - \mu(n-\mu)^2 \end{array} \right\}}$$

Where $u = p\zeta_1 + \mu\zeta_2$ and $v = n\zeta_1 + \mu\zeta_2$; $\alpha, p \ge 1;\ s,\beta,\gamma \ge 0; 0 \le \zeta_1;\ \zeta_2 \le 1;\ \mu \in C$ and $|\mu| \le 1; n \ge 2$.

Proof: Let $I^s_{\alpha,\beta,\gamma} f_{i,p}$, $I^s_{\alpha,\beta,\gamma} g_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_{2;}\delta)$. Then from Theorem 1, and for real value of z we have:

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty} \frac{x^s\left[\delta(n\zeta_1 + \mu\zeta_2) + |n - \mu| \right] a_n^i}{\delta(p\zeta_1 + \mu\zeta_2) + |\mu - p|} \le 1.$$

And

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty} \frac{x^s\left[n\delta(\zeta_1 + \mu\zeta_2) + |n - \mu| \right] b_n^i}{\delta(p\zeta_1 + \mu\zeta_2) + |\mu - p|} \le 1.$$

We need to find the smallest λ such that

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty} \frac{x^s\left[\lambda(n\zeta_1 + \zeta_2) + |n - \mu| \right] a_n^i b_n^i}{\lambda(p\zeta_1 + \mu\zeta_2) + |\mu - p|} \le 1. \quad (8)$$

By Cauchy Schwartz inequality, we have:

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\frac{x^s\left[\delta\left(n\zeta_1+\mu\zeta_2\right)+|n-\mu|\right]\sqrt{a_n^ib_n^i}}{\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|}\le 1. \quad (9)$$

Thus, it suffices to show that:

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\frac{x^s\left[\lambda\left(n\zeta_1+\zeta_2\right)+|n-\mu|\right]a_n^ib_n^i}{\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|}$$
$$\le\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\frac{x^s\left[\delta\left(n\zeta_1+\zeta_2\right)+|n-\mu|\right]\sqrt{a_n^ib_n^i}}{\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|}.$$

From where we have:

$$\sqrt{a_n^ib_n^i}\le\frac{\left\{x^s\begin{bmatrix}\delta\left(n\zeta_1+\mu\zeta_2\right)\\+|n-\mu|\end{bmatrix}\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right\}}{\left\{\delta\left(\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)x^s\begin{bmatrix}\lambda\left(n\zeta_1+\mu\zeta_2\right)\\+|n-\mu|\end{bmatrix}\right\}}.$$

And by (10), we have:

$$\frac{\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|}{\delta\left(n\zeta_1+\mu\zeta_2\right)+|n-\mu|}$$
$$\le\frac{\left[\delta\left(n\zeta_1+\mu\zeta_2\right)+|n-\mu|\right]\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|}{\delta\left(\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)\left[\lambda\left(n\zeta_1+\mu\zeta_2\right)+|n-\mu|\right]}.$$

And by simple simplification, with $u=p\zeta_1+\mu\zeta_2$ and $v=n\zeta_1+\mu\zeta_2$, we have:

$$\lambda\le\frac{\left\{\begin{matrix}(n-\mu)|\mu-p|\left[2(v-u)+n-\mu-|\mu-p|\right]\\+\delta^2(v^2|\mu-p|-u^2(n-\mu)\end{matrix}\right\}}{\left\{\begin{matrix}\delta uv(\delta(u-v)+2\left[|\mu-p|-(n-\mu)\right]\\+v|\mu-p|^2-\mu(n-\mu)^2\end{matrix}\right\}}. \quad (10)$$

This proves the result.

Corollary 8: Let $F_{p,\mu}$ be as in (5) and $I_{\alpha,\beta,\gamma}^sf_{i,p}*I_{\alpha,\beta,\gamma}^sg_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$. Then

$$|a_n^ib_n^i|\le\frac{q^s\left(\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{p^s\left(\lambda\left(n\zeta_1+\zeta_2\right)+|n-\mu|\right)}$$

$\alpha,p\ge1;s,\beta,\gamma\ge0;0\le\zeta_1;\ \zeta_2\le1;\ \mu\in C\ and\ |\mu|\le1;\ n\ge2$

Where λ is as defined in (10) and $p;q=\alpha+n\beta+n^2\gamma$; $p\ \alpha+\beta+\gamma$ respectively.

Corollary 9: Let $F_{p,\mu}$ be as in (5) and $I_{\alpha,\beta,\gamma}^sf_{i,p}*I_{\alpha,\beta,\gamma}^sg_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$. Then

$$|a_n^i|\le\frac{q^s\left(\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{p^s\left(\lambda\left(n\zeta_1+\zeta_2\right)+|n-\mu|\right)|b_n^i|}$$

$\alpha,p\ge1;\ s,\beta,\gamma\ge0;0\le\zeta_1;\ \zeta_2\le1;\ \mu\in\ C\ and|\mu|\le1;\ n\ge2$

Where λ is as defined in (10) and $p;q=\alpha+n\beta+n^2\gamma$; $p\ \alpha+\beta+\gamma$ respectively.

Corollary 10: Let $F_{p,\mu}$ be as in (5) and $I_{\alpha,\beta,\gamma}^sf_{i,p}*I_{\alpha,\beta,\gamma}^sg_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$. Then

$$|b_n^i|\le\frac{q^s\left(\lambda\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{p^s\left(\lambda\left(n\zeta_1+\zeta_2\right)+|n-\mu|\right)|a_n^i|}$$

$\alpha,p\ge1;s,\beta,\gamma\ge0;0\le\zeta_1;\ \zeta_2\le1;\ \mu\in\ C\ and\ |\mu|\le1;\ n\ge2$

Where λ is as defined in (10) and $p;q=\alpha+n\beta+n^2\gamma$; $p\ \alpha+\beta+\gamma$ respectively.

In what follows, we show that the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_2,\delta\right)$ is close under convex combination.

Theorem 3: Let $F_{p,\mu}$ be as in (5) and $I_{\alpha,\beta,\gamma}^sf_{i,p}$, $I_{\alpha,\beta,\gamma}^sg_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$.

Then $G_i(z)$ given by

$$G_i(z)=(1-\lambda)I_{\alpha,\beta,\gamma}^sf_{i,p}(z)+\lambda I_{\alpha,\beta,\gamma}^sg_{i,p}(z)$$
$$=z^p+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^sC_n^iz^n,$$
$$\alpha,p\ge1;s,\beta,\gamma\ge0$$

belongs to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$. Where $C_n^i=(1-\lambda)|a_n^i|+\lambda|b_n^i|, 0\le\lambda\le1$.

Proof: Let $I_{\alpha,\beta,\gamma}^sf_{i,p}$ and $I_{\alpha,\beta,\gamma}^sg_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_{2;}\delta\right)$. Then we have

$$(1-\lambda)\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s\begin{bmatrix}n\left(1+\delta\zeta_1\right)\\+\mu\left(\delta\zeta_2-1\right)\end{bmatrix}\left|a_n^i\right|)$$
$$\le(1-\lambda)(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p| \quad (11)$$
$$0\le\zeta_1;\zeta_2\le1;0<\delta<1;\beta,\gamma\ge0;\mu\in C\ and\ |\mu|\le1.$$

And respectively

$$(1-\lambda)\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s\begin{bmatrix}n\left(1+\delta\zeta_1\right)\\+\mu\left(\delta\zeta_2-1\right)\end{bmatrix}\left|b_n^i\right|)$$
$$\le(1-\lambda)(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|, \quad (12)$$
$$0\le\zeta_1;\zeta_2\le1;0<\delta<1;\beta,\gamma\ge0;\mu\in C\ and\ |\mu|\le1$$

adding (11) and (12) gives

$$\sum_{i=1}^{k}\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma}\right)^s\begin{bmatrix}n\left(1+\delta\zeta_1\right)\\+\mu\left(\delta\zeta_2-1\right)\end{bmatrix}(1-\lambda)\begin{pmatrix}\left|a_n^i\right|\\+\left|b_n^i\right|\end{pmatrix}$$
$$\le(1-\lambda)(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|$$
$$+\lambda\left(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)$$
$$=\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|.$$

This proves the result.

Now, we establish two of the fundamental theorems about univalent functions in relation to function in the subclass $\Gamma_{p,\mu;\alpha,\beta,\gamma}\left(\zeta_1,\zeta_2;\delta\right)$; the growth and distortion theorems, which provide bounds on $|I_{\alpha,\beta,\gamma}^sf_{i,p}(z)|$ and $|I_{\alpha,\beta,\gamma}^sf_{i,p}'(z)|$ respectively. Theorems (4) and (5) below are the growth and distortion theorems respectively.

Theorem 4: Let $I^{s}_{\alpha,\beta,\gamma}f_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_2;\delta)$. Then

$$r-\frac{(\alpha+\beta+\gamma)^{s}\left(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{\left(n\left[(1+\delta\zeta_1)+\mu\left(\delta\zeta_2-1\right)\right]\right)\left(\alpha+n\beta+n^{2}\gamma\right)^{s}}r^{2}$$

$$\leq\left|I^{s}_{\alpha,\beta,\gamma}f_{i,p}(z)\right|$$

$$\leq r+\frac{(\alpha+\beta+\gamma)^{s}\left(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{\left(n\left[(1+\delta\zeta_1)+\mu\left(\delta\zeta_2-1\right)\right]\right)\left(\alpha+n\beta+n^{2}\gamma\right)^{s}}r^{2}.$$

Proof: For the function $f\in A$,

$$|f(z)|\leq r+\sum_{n=2}^{\infty}|a_n|r^{n}\leq r+r^{2}\sum_{n=2}^{\infty}|a_n|.$$

Similarly,

$$|f(z)|\leq r-r^{2}\sum_{n=2}^{\infty}|a_n|.$$

Fixing the value of a_n for the function in $I^{s}_{\alpha,\beta,\gamma}f_{i,p}$ and rearranging gives the result.

Theorem 5: Let $I^{s}_{\alpha,\beta,\gamma}f_{i,p}$ belong to the class $\Gamma_{p,\mu;\alpha,\beta,\gamma}(\zeta_1,\zeta_2;\delta)$. Then

$$1-\frac{(\alpha+\beta+\gamma)^{s}\left(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{\left(n\left[(1+\delta\zeta_1)+\mu\left(\delta\zeta_2-1\right)\right]\right)\left(\alpha+n\beta+n^{2}\gamma\right)^{s}}r$$

$$\leq\left|I^{s}_{\alpha,\beta,\gamma}f'_{i,p}(z)\right|$$

$$\leq 1+\frac{(\alpha+\beta+\gamma)^{s}\left(\delta\left(p\zeta_1+\mu\zeta_2\right)+|\mu-p|\right)}{\left(n\left[(1+\delta\zeta_1)+\mu\left(\delta\zeta_2-1\right)\right]\right)\left(\alpha+n\beta+n^{2}\gamma\right)^{s}}r.$$

Proof: The proof follows from theorem 4.

3. Conclusion

In this article, we take the coefficients of the analytic functions in the above theorems as the size of some dynamic parameters, which could be infectious agents, such as viruses, bacteria, fungi, protozoa, and helminthes, that need to be curtailed to achieve the above aim and the α,β and $\gamma$are infection control measures. Thus, to achieve reduction in the infectious agents, we need to reduce the slope of the class of functions

$$I^{s}_{\alpha,\beta,\gamma}f_{i,p}(z)=z^{p}+\sum_{n=p+1}^{\infty}\left(\frac{\alpha+n\beta+n^{2}\gamma}{\alpha+\beta+\gamma}\right)^{s}a^{i}_{n}z^{n},$$

$$\alpha,p\geq 1,\beta,\gamma\geq 0$$

which is the coefficients a^{i}_{n}. It is noted from the above that the higher the control measures, the smaller the coefficient bounds. And that the coefficient bounds in the theorems above is of less magnitude than those obtained in the previous literature, thus it will yield a better result in the prevention we aimed at.

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