American Journal of Applied Mathematics and Statistics, 2018, Vol. 6, No. 5, 181-185
Available online at http://pubs.sciepub.com/ajams/6/5/2

DOI:10.12691/ajams-6-5-2

Limited Failure Censored Life Test Sampling Plan in Dagum Distribution

B. Srinivasa Rao[1], P. Sricharani[2,*], M.S. Ravikumar[3]

[1]Statistics, R.V.R & J.C College of Engineering, Guntur, A.P, India
[2]Department of Basic Sciences, Shri Vishnu Engineering College for Women, Bhimavaram, A.P., India
[3]Bio-Statistics, Konaseema Institute of Medical Sciences & Research Foundation, Amalapuram, A.P, India
*Corresponding author: charani.ravi657@gmail.com

Received August 05, 2018; Revised September 11, 2018; Accepted September 19, 2018

Abstract The Dagum distribution is considered as a life time random variable of a product whose lots are to be decided for acceptance or otherwise on the basis of sample lifetimes drawn from the lot. The sample is divided into various groups in order to develop a group sampling plan in such a way that the life testing experiment is terminated as soon as the first failure in each group is observed. The acceptance criterion based on the theory of order statistics is proposed.

Keywords: *single sampling, lot acceptance, group sampling plan, truncated life tests, reliability test plans, order statistics*

Cite This Article: B. Srinivasa Rao, P. Sricharani, and M.S. Ravikumar, "Limited Failure Censored Life Test Sampling Plan in Dagum Distribution." *American Journal of Applied Mathematics and Statistics*, vol. 6, no. 5 (2018): 181-185. doi: 10.12691/ajams-6-5-2.

1. Introduction

Acceptance sampling is concerned with inspection and decision making regarding products. Life tests are experiments carried out on sample products in order to assess the life time of an item (time to its failure or the time it stops working satisfactorily). A common practice in life test is to terminate the test at a prefixed time and record the number of failures that occurred during that time period or when a prefixed number of failures is realised. The former termination is generally called truncated life tests/time censored life test and the latter is called a failure censored life test. If the quality of a product is measured through the life time, sampling plans to determine acceptability of a product with respect to life time are called Reliability Sampling Plans.

In life test sampling plans a common constraint is the duration of total time spent on testing. Sampling plans based on time truncated life tests would address this constraint to some extent. When the life time random variable is assumed to follow a specific continuous probability distribution, sampling plans are developed by various researchers covering a wide spectrum of probability models.

Epstein (1954) [1] was one of the foremost works about acceptance sampling plans based on truncated life tests with the exponential distribution as the probability model. Other researchers in this direction are as follows: Goode and Kao (1961) [2] worked with the Weibull model which includes the exponential distribution as a particular case. Gupta and Groll (1961) and Gupta (1962) [3,4] considered the gamma and log-normal distributions, respectively. More recently, the studies of Kantam *et al.* (2001), Baklizi (2003) Baklizi and El-Masri (2004), Rosaiah and Kantam (2005), Balakrishanan *et al.*(2007), Aslam and Kantam (2008), Srinivasa Rao *et al.* (2009), Rosaiah *et al.* (2009), Srinivasa Rao and Kantam (2010), Lio *et al.*(2010a), Lio *et al.*(2010b), Wanbo Lu (2011), Kantam *et al.*(2012), Srinivasa Rao *et al.* (2012), Srinivasa Rao and Kantam (2013), Kantam and Sriram (2013), Subba Rao *et al.*(2013), Kantam *et al.*(2013), Rosaiah *et al.*(2014), Subba Rao *et al.* (2014) *[5-24]* and the references therein, are related to construction of acceptance sampling plans based on truncated life tests with different probability models. In all these works, given the termination time of a life test, the construction of the sampling plan consists of determining the minimum number of sample items that are to be life-tested and the acceptance number beyond which the observed failures out of the life-tested items of the sample lead to rejection of the submitted lot, conditioned on pre specified producer's and consumer's risks.

On the other hand, if a failure censored life test is under consideration, one has to wait till a pre specified number of failures out of the sample items that are being tested is realised. Sometimes the unknown life of product might be quite long possibly resulting in even a failure censored life-testing plan to be long time consuming. J*ohnson* (1964) [25] proposed a sampling plan in which the experimenter can decide to group the test units into several groups and then conduct the life-tests on all the groups simultaneously until the first failure in each group is realised. Based on the recorded first failure time in each group if a decision process about the acceptance or rejection of submitted lot is developed the procedure may be named as Limited Failure Censored Life Test Sampling Plan (LFCLTSP). Balasooriya (1995) [26] developed such a sampling plan for the two parameter exponential distribution though the specific name is not given as LFCLTSP. Wu and Tsai (2000) [27], Wu *et al.* (2001) [28], Jun *et al.* (2006) [29] have proposed LFCLTSP when the underlying lifetime random variable follows Weibull distribution, with respective distinct approaches in working out the parameters of the sampling plan. The scheme of life testing and termination process of LFCLTSP is named by some researchers as *Sudden Death Testing* (for example Pascual and Meeker – 1998 [30]; Jun *et al.* (2006) [29]). 'Limited failure censored life tests' is the name proposed by Wu *et al.* (2001) [28]. Kantam and Ravikumar (2016) [31] named it as LFCLTSP.

In this paper we attempt to develop LFCLTSP for Dagum distribution on lines of Kantam and Ravikumar (2016) [31]. Construction of LFCLTSP for Dagum distribution with various parameter combinations is presented in Section – 2. The results are illustrated in Section – 3.

2. Construction of LFCLTSP for Dagum Distribution:

Let the limited failure censored samples - $Y_1, Y_2, ..., Y_m$ which are *m* first order statistics in *m* independent random samples of size *n* each. If *Z* denotes the maximum of $Y_1, Y_2, ..., Y_m$ it may also be viewed as the total test time/experimental time as opined by Kantam and Srinivasa Rao (2004) [32]. Hence, larger realised value of *Z* can be considered as an indication that the products in the submitted lot have longer life prompting one to consider the lot as a good lot for acceptability. In other words "$Z > cL$" can be taken as a criterion of acceptance of the lot. Thus Kantam and Ravikumar (2016) [31] proposed following decision rule.

(i) Draw a random sample of size $N = m \times n$ and allocate *n* items to each of the *m* groups.

(ii) Observe Y_i the time to the first failure in the i^{th} group (i=1,2,….,m).

(iii) Identify the quantity Z = Max($Y_1, Y_2, ..., Y_m$)

(iv) Accept the lot if $Z \geq cL$ and reject the lot otherwise (*c* may be called acceptability constant – a concept similar to the acceptance number in time truncated reliability test plans).

Using the theory of order statistics we can get the cumulative distribution function (cdf) of *Z* in a closed form as long as the cumulative distribution function (cdf) of the base line distribution is in a closed form. Hence the percentiles of *Z* can be used to get the design parameters m & c analytically.

For our focal distribution namely Dagum distribution with shape parameters *a*, *p* and scale parameter b the following is the analytical procedure of calculating design parameters of LFCLTSP.

The Probability density function (pdf) of Dagum distribution is given by

$$f(x) = \frac{ap}{x}\left(\frac{\left(\frac{x}{b}\right)^{ap}}{\left(\left(\frac{x}{b}\right)^{a}+1\right)^{p+1}}\right), x>0, a>0, b>0, p>0. \quad (2.1)$$

Cumulative distribution function (cdf) of Dagum distribution is

$$F(x) = \left(1+\left(\frac{x}{b}\right)^{-a}\right)^{-p}, x>0, a>0, b>0, p>0. \quad (2.2)$$

The fraction non-conforming or unreliability is expressed by

$$k = Pr\{X < L\} = F(L). \quad (2.3)$$

If *k* is given, the corresponding L is obtained from

$$w = L = b\left[\frac{1}{\left(\frac{1}{k}\right)^{\frac{1}{p}}-1}\right]^{\frac{1}{a}}. \quad (2.4)$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size *n* from Equation (2.2)

The cdf of least of $X_1, X_2, \ldots, X_n$ is given by

$$F_{(1)}(x) = 1-[1-F(x)]^n. \quad (2.5)$$

That is

$$F_{(1)}(x) = 1-\left[1-\left(1+\left(\frac{x}{b}\right)^{-a}\right)^{-p}\right]^n \quad (2.6)$$

$Y_1, Y_2, \ldots, Y_m$ of the limited failure censored test are now a random sample of size *m* from $F_{(1)}(x)$. Hence, the cdf of *Z* – the largest of $Y_1, Y_2, \ldots, Y_m$ is given by

$$G_{(m)}(z) = [F_1(z)]^m \quad (2.7)$$

$$i.e., G_{(m)}(z) = \left[1-\left[1-\left(1+\left(\frac{x}{b}\right)^{-a}\right)^{-p}\right]^n\right]^m. \quad (2.8)$$

The design parameters *m* and *c* of LFCLTSP are obtained with the help of percentiles of $G_{(m)}(z)$ given in Equation (2.8). If α and β are respectively the producer's and consumer's risks for desirable /acceptable lot quality level k_0, undesirable/lot tolerance quality level k_1 then *m* and *c* are the solutions of the following two inequalities.

$$G_m(cw_0) \leq \alpha \quad (2.9)$$

$$G_m(cw_1) \geq 1-\beta \quad (2.10)$$

where w_0 and w_1 are the solution of Equation (2.4).

The inequalities (2.7), (2.8) respectively imply

$$cw_0 \leq G_m^{-1}(1-\alpha) \tag{2.11}$$

$$cw_1 \geq G_m^{-1}(\beta) \tag{2.12}$$

which jointly lead to

$$\frac{w_0}{w_1} \leq \frac{G_m^{-1}(1-\alpha)}{G_m^{-1}(\beta)}. \tag{2.13}$$

Therefore, m can be obtained by the smallest integer satisfying In Equation (2.13). The acceptability constant c can be obtained from the equality case in either of the expressions (Inequations (2.11), (2.12)). We have tabulated the values of m and c analytically determined for the selected combinations of k_0, k_1 and are presented in Table 1 through Table 3 for p=0.25, a=2, b=3; p=0.50, a=2, b=3; p=1, a=2, b=3. The values of m obtained by LFCLTSP can be seen to be consistently smaller, So the sampling plan indicating less number of items to be put to life test.

Table 1. Design Parameters of LFCLTSP at p=0.25,a=2, b=3, α=0.05 and β=0.1

(Min-Max) Approach for Dagum Distribution at p=0.25, a=2, b=3 and α=0.05 and β=0.1					
k_0	k_1	m		c	
		n=5	n=10	n=5	n=10
0.005	0.02	5	6	868.4421	317.9703
	0.04	2	3	97.45442	80.6788
	0.06	2	2	97.45442	24.9840
	0.08	2	2	97.45442	24.9840
	0.10	2	2	97.45442	24.9840
	0.14	2	2	97.45442	24.9840
	0.20	2	2	97.45442	24.9840
0.01	0.04	5	6	217.1105	79.4925
	0.06	3	4	77.09835	38.4701
	0.08	3	3	77.09835	20.1697
	0.10	2	2	24.3636	6.2460
	0.14	2	2	24.3636	6.2460
	0.20	2	2	24.3636	6.2460
0.02	0.06	8	10	107.8785	39.9541
	0.08	5	6	54.2776	19.8731
	0.10	4	4	36.1245	9.6175
	0.14	3	3	19.2745	5.0424
	0.20	2	2	6.0909	1.5615
0.03	0.08	10	13	62.4247	23.7599
	0.10	7	8	40.2385	13.4087
	0.14	4	5	16.0553	6.5193
	0.20	3	3	8.5664	2.2410
0.04	0.10	12	16	42.5610	16.4112
	0.14	6	7	18.1478	6.2669
	0.20	4	4	9.0311	2.4043
0.05	0.14	9	11	19.9239	7.1379
	0.20	5	6	8.6843	3.1796
0.07	0.20	9	11	10.1651	3.6417

Table 2. Design Parameters of LFCLTSP at p=0.50, a=2, b=3, α=0.05 and β=0.1

(Min-Max) Approach for Dagum Distribution at p=0.50, a=2, b=3 and α=0.05 and β=0.1					
k_0	k_1	m		c	
		n=5	n=10	n=5	n=10
0.005	0.02	6	6	34.5781	17.9025
	0.04	3	3	17.6287	8.9910
	0.06	2	2	9.8838	4.9999
	0.08	2	2	9.8838	4.9999
	0.10	2	2	9.8838	4.9999
	0.14	2	2	9.8838	4.9999
	0.20	2	2	9.8838	4.9999
0.01	0.04	6	6	17.2883	8.9509
	0.06	4	4	12.1072	6.2140
	0.08	3	3	8.8140	4.4953
	0.10	2	2	4.9417	2.4998
	0.14	2	2	4.9417	2.4998
	0.20	2	2	4.9417	2.4998
0.02	0.06	11	11	12.8691	6.7377
	0.08	6	6	8.6429	4.4747
	0.10	4	5	6.0527	3.8404
	0.14	3	3	4.4063	2.2473
	0.20	2	2	2.4704	1.2497
0.03	0.08	16	15	10.3930	5.3000
	0.10	9	9	7.6234	3.9778
	0.14	5	5	4.9629	2.5596
	0.20	3	3	2.9368	1.4978
0.04	0.14	8	8	5.3022	2.7607
	0.20	4	5	3.0245	1.9190
0.05	0.14	13	13	5.6248	2.9523
	0.20	6	6	3.4535	1.7880
0.07	0.20	12	12	3.8474	2.0171

Table 3. Design Parameters of LFCLTSP at p=1, a=2, b=3, α=0.05 and β=0.1

(Min-Max) Approach for Dagum Distribution at p=1, a=2, b=3 and α=0.05 and β=0.1					
k_0	k_1	m		c	
		n=5	n=10	n=5	n=10
0.005	0.02	17	10	9.3562	5.3661
	0.04	4	4	5.2142	3.6275
	0.06	3	3	4.3766	3.0590
	0.08	2	2	3.2144	2.2585
	0.10	2	2	3.2144	2.2585
	0.14	2	2	3.2144	2.2585
	0.20	2	2	3.2144	2.2585
0.01	0.04	16	10	6.4793	3.7849
	0.06	6	5	4.5088	2.8657
	0.08	4	3	3.6777	2.1576
	0.10	3	3	3.0869	2.1576
	0.14	2	3	2.2672	2.1576
	0.20	2	2	2.2672	1.5929
0.02	0.08	14	9	4.3721	2.5671
	0.10	8	6	3.5832	2.1900
	0.14	4	4	2.5873	1.8000
	0.20	2	3	1.5950	1.5179
0.03	0.10	27	14	4.2909	2.4059
	0.14	9	6	3.0466	1.7790
	0.20	5	4	2.3636	1.4622
0.04	0.14	19	11	3.3572	1.9235
	0.20	7	5	2.3745	1.4109
0.05	0.20	11	7	2.5119	1.4540

3. Illustration

The quality assurance in a bearing manufacturing process states that k_0=0.02, k_1=0.14, α=0.05, β=0.1 the number of test positions (size of each group), *n*=10. For this information Table 1 of suggests *m*=3, *c*=5.04243. Accordingly a random sample of size *N*=50 items are put to test in five groups with 10 items in each group. The observed first failure times in the five groups are Y_1=120, Y_2=200, Y_3=185, Y_4=55, Y_5=265. Assuming that the life times follow Dagum distribution with shape parameter 0.25, 2 and a lower specification of L=100 they have at the above k_0, k_1, α, β, *n*=10, and acceptability constant *c*=5.04243then cL =504.243. Z = The maximum of {55,120,185} = 185. Since $Z < cL.\ i.e.$,185 < 504.243, the lot is to be rejected.

From this example, we see that our approach reached the decision of rejecting the lot by conducting limited failure censored life test for only three groups of 10 items each, resulting in low cost of experimentation and lower number of destructions.

More over it may be recalled that Z are defined as $Z = Max(Y_1, Y_2, \ldots, Y_m)$. If *c* is the acceptability constant and *L* is the lower specification, Z>cL. That is acceptance decision of LFCLTSP is considered and gives a stronger conclusion with this illustration.

4. Conclusion

This paper provides the number of groups into which a sample given size is to be divided in order to arrive at a conclusion of accepting or rejecting a submitted lot with a given risk. The tables of this paper provide the actual number of products whose failure is to be tolerated in a life testing experiment. For instance, the first row of the Table 3 indicates that a sample of 85 products is to be divided into 17 groups of size 5 each. A sample of 100 groups of size 10 each. The methodology of this plan indicates that in the first case the testing is to be stopped as soon as the first failure occurs in each of the 17 groups in succession. i.e., the experimenter has to bear the loss lives of 17 products in a sample of Size 85. In the second case, the experimenter has to bear the loss of lives of 10 products in a sample of size 100. Evidently, the second situation is to be preferred when the failed product number is less. With this reliability, the sampling plans given in the table are named as Limited Failure Censored Life Test Sampling plan.

References

[1] Epestein, B. *Truncated Life Tests in the Exponential Case.* Annals of Mathematical Statistics, Vol. 25, 555-564, 1954.

[2] Goode, H.P. and Kao, J.H.K. *Sampling Plans Based on the Weibull Distribution.* Proceedings of Seventh National Symposium on Reliability and Quality Control, Philadelphia, 24-40, 1961.

[3] Gupta, S.S. and Groll, P.A. *Gamma Distribution in Acceptance Sampling Based on Life Tests.* Journal of the American Statistical Association, Vol. 56, 942-970, 1961.

[4] Gupta, S.S. *Life Test Sampling Plans for Normal and Lognormal Distribution.* Technometrics, Vol. 4, 151-175, 1962.

[5] Kantam, R.R.L., Rosaiah, K. and Srinivasa Rao, G. *Acceptance Sampling Based on Life Tests: Log-Logistic Model.* Journal of Applied Statistics, Vol. 28(1), 121-128, 2001.

[6] Baklizi, A.. Acceptance Sampling Based on Truncated Life Tests in the Pareto Distribution of the Second Kind. *Advances and Applications in Statistics*, Vol. 3(1), 33-48, 2003.

[7] Baklizi, A. and El Qader El Masri, A. Acceptance Sampling Based on Truncated Life Tests in the Birnbaum –Saunders Model. *Risk Analysis*, Vol. 24(6), 1453-1457, 2004.

[8] Rosaiah, K. and Kantam, R.R.L. *Acceptance Sampling Based on the Inverse Rayleigh Distribution*, Economic Quality Contro*l*, Vol. 20(2), 277-286, 2005.

[9] Balakrishnan, N., Leiva, V. and Lopez, J. *Acceptance Sampling Plans from Truncated Life Tests Based on the Generalised Birnbaum –Saunders Distribution*, Communications in statistics – Simulation and Computation, Vol. 36, 643-656, 2007.

[10] Aslam.,M. and Kantam, R.R.L. EconomicReliability Acceptance Sampling Based on Truncated Life Tests in the Birnbaum - Saunders Distribution *Pakistan Journal of Statistics*, Vol. 24(4), 269-276, 2008.

[11] Srinivasa Rao, G., Ghitany, M.E., and Kantam, R.R.L. *Marshall – Olkin Extended Lomax Distribution: An Economic Reliability Test Plan*, International Journal of Applied Mathematics, Vol. 22(1), 139-148, 2009.

[12] Rosaiah, K., Kantam, R.R.L. and Srinivasa Rao, B. Reliability Test Plan for Half Logistic Distribution,. *Calcutta Statistical Association Bulletin*, Vol.61, 241-244, 2009.

[13] Srinivasa Rao, G. and Kantam, R.R.L. *Acceptance Sampling Plans from Truncated Life Tests Based on the Log – Logistic Distributions for Percentiles*, Economic Quality Control, Vol. 25, 153-167, 2010.

[14] Lio, Y.L., Tzong- Ru, Tsai and Shuo-Jye Wu. *Acceptance Sampling Plans from Truncated Life Tests Based on the Birnbaum - Saunders Distribution for Percentiles,* Communications in Statistics – Simulation and Computation, Vol. 39, 119-136, 2010a.

[15] Lio, Y.L., Tzong- Ru, Tsai and Shuo-Jye Wu. Acceptance Sampling Plans from Truncated Life Tests Based on the Burr Type XII Percentiles, Journal of the Chinese Institute of Industrial Engineers, Vol. 27(4), 270-280, 2010b.

[16] Wanbo Lu., *Acceptance Sampling Plans based on Truncated Lifetests for Maxwell Distribution*, Pakistan Journal of Statistics, Vol.27(2), 159-170, 2011.

[17] Kantam, R.R.L., Sriram, B. and Suhasini, A. *Reliability Test Plans: Exponentiated Exponential Distribution*. Mathematical Sciences, International Research Journal, Vol.1(3), 1022-1032, 2012.

[18] Srinivasa Rao, G., Kantam, R.R.L., Rosaiah, K. and Pratapa Reddy, J. *Acceptance Sampling Plans for Percentiles Based on The Inverse Rayleigh Distribution,* Electronic Journal of Applied Statistical Analysis, Vol.5(2),164-177, 2012.

[19] Srinivasa Rao, B. and Kantam, R.R.L. *Acceptance Sampling Plans for Percentiles of Half Logistic Distribution*. International Journal of Reliability Quality and Safety Engineering,Vol. 20(5), 1-13, 2013.

[20] Kantam, R.R.L., Sriram, B. and Suhasini, A. Reliability Test Plans Based on Log Logistic Distribution, *International Journal of Research in Commerce and Management*, Vol.3(9), 49-57, 2013.

[21] Subba Rao,R., Prasad,G. and Kantam, R.R.L. *Acceptance Sampling on Life Tests: Exponentiated Pareto Model*,Engineering Sciences International Research Journal, Vol.1(1), 152-158, 2013.

[22] Kantam, R.R.L. and Sriram, B. Economic Reliability Test Plans Based on Rayleigh Distribution. *Journal of Statistics*, Vol.20, 88-101, 2013.

[23] Rosaiah, K., Kantam, R.R.L., Rama Krishna, V. and Siva Kumar, D.C.U. *An Economic eliability Test Plan for Type – I Generalized Half Logistic Distribution*, Journal of Chemical, Biological and Physical Sciences, Vol.4(2), 1486-1493, 2014.

[24] Subba Rao, R., Naga Durgamamba, A. and Kantam, R.R.L. Acceptance Sampling Plans: Size Biased Lomax Model, *Universal Journal of Applied Mathematics*, Vol.2(4), 176-183, 2014.

[25] Johnson, L, G. *Theory and Technique of Variation Research.* Amsterdam: Elsevier. 3.1964.

[26] Balasooriya, U. *Failure-Censored Reliability Sampling Plans for the Exponential Distribution*. Journal of Statistical Computation and Simulation, Vol.52, 337-349, 1995.

[27] Wu, J.-W. and Tsai, W.-L.*, Failure – Censored Sampling Plan for the Weibull Distribution*., Information and Management Sciences, Vol.11(2), 13-25, 2000.

[28] Wu, J.-W., Tsai, T.-R. and Ouyang, L.-Y.., *Limited Failure – Censored Life Test for the Weibull Distribution.,* IEEE Transactions on Reliability, Vol.50(1), 107-111, 2001.

[29] Jun, Ch.-H., Balamurali, S. and Lee, S.-H. *Variables Sampling Plans for Weibull Distributed Lifetimes Under Sudden Death, Testing.* IEEE Transactions on Reliability, Vol.55, No.1, 53-58, 2006.

[30] Pascual, F.G. and Meeker, W.Q. *The Modified Sudden Death Test: Planning life tests with a limited number of test positions,* Journal of Testing and Evaluation, Vol. 26(5), 434-443,1998.

[31] Kantam, R.R.L. and Ravikumar, M.S. *Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution.*,Journal of Modern Applied Statistical Methods, Vol. 15(2),428-454,2016.

[32] Kantam, R.R.L. and Srinivasa Rao, G. *A Note on Savings in Experimental Time Under Type II Censoring*, Economic Quality Control, Vol.19(1), 91-95, 2004.