

Fuzzy Eoq Model for Deteriorating Items with Exponential Membership Function

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Abstract In this Paper a multi item EOQ model with stock dependent demand for deteriorating items is considered in fuzzy environment. Inventory costs such as holding cost and setup cost have been represented by exponential membership function and profit, deteriorating rate and total investment constraint are represented by linear membership functions. The model has been solved by fuzzy non-linear programming (FNLP) method. Results have been presented along with those of corresponding crisp model and a sensitivity analysis.

Keywords: *fuzzy inventory, crisp model, non-linear membership function*

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1. Introduction

Now a classical EOQ is an old-age problem several researchers worked on the EOQ model by changing the parameters and variables of the model. All these models are developed in crisp environment. This literature is available in various reference journals and survey papers

Generally Demand rate is considered to be constant, time dependent, ramp type and selling price dependent. However in present competitive market stock-dependent demand plays an important role in increase its demand.

Deterioration is one of the important factors in inventory system. Some items like food grains, vegetables, milk, eggs etc. deteriorate during their storage time and retailer suffers loss. In an inventory system available storage space, budget, number of orders etc. are always limited hence multi-item classical inventory models under these constraints have great importance.

In realistic situation all the parameters, variables and resources are always considered as constant. But their value, instead of being at a fixed level varies within a range.

Due to this vagueness and uncertainty of inventory parameters, the fuzzy set theory is more appropriate to formulate inventory model. In fuzzy decision-making process first Bellman and Zadeh [1] introduced fuzzy set theory. Tanaka et. al. [6] used the concept of fuzzy sets to decision problems and Zimmerman [7] showed the classical algorithms could be used to solve fuzzy linear programming problems.

Normally both linear and non-linear shapes for the membership functions of the fuzzy objective and constraints are proposed. In this area Mandal and Maiti [5] and Mandal, Roy and Maiti [4] solved the classical EOQ model in a fuzzy environment with fuzzy objective, fuzzy

inventory costs with constraints like storage area, total budget etc., by non-linear programming method using linear as well as several types of non-linear membership functions for inventory parameters.

In this paper a multi item EOQ model with stock dependent demand for deteriorating items is considered in fuzzy environment. Here inventory costs such as holding cost and setup cost have been represented by exponential membership functions and profit, deteriorating rate and total investment constraint are represented by linear membership functions. The model has been solved by fuzzy non-linear programming (FNLP) method. Results have been presented along with those of corresponding crisp model and a sensitivity analysis.

2. Assumptions

1. The scheduling period is constant and no lead-time.
2. Replenishment rate is infinite.
3. Selling price is Known and constant.
4. Demand rate is stock dependent.
5. Shortages are not allowed.
6. Deteriorating rate is age specific failure rate.

3. Notations

T_i : Time period for each cycle for the i^{th} item.

R_i : Demand rate per unit time of i^{th} item; $[R_i = a_i + b_i q_i]$.

θ_i : Deterioration rate of i^{th} item.

$Q_i(t)$: Inventory level at time t of i^{th} item.

C_H : Total Holding cost.

C_{H_i} : Holding cost per unit of i^{th} item.

C_{S_i} : Set up cost for i^{th} item.

S_{d_i} : Total deteriorating units of i^{th} item.

P_i : Selling price per unit of i^{th} item.

Q_i : Initial stock level of i^{th} item.

PF(Q_i): Total profit of i^{th} item.

n : Number of items.

(wavy bar (\sim) represents the fuzzification of the parameters)

4. Mathematical Formulation

4.1. Crisp Model

As $Q_i(t)$ is the inventory level at time t of the i^{th} item, then the differential equation describing the state of inventory is given by

$$\frac{d}{dt} Q_i(t) + \theta_i Q_i(t) = -(a_i + b_i Q_i(t)) \quad 0 \leq t \leq T_i$$

solving the above differential equation using boundary condition $Q_i(t) = Q_i$ at $t=0$, we get

$$Q_i(t) = -\frac{a_i}{(\theta_i + b_i)} + \left[Q_i + \frac{a_i}{(\theta_i + b_i)} \right] e^{-(\theta_i + b_i)t} \quad (1.1)$$

and using boundary condition $Q_i(t) = 0$ at $t = T_i$

$$\therefore T_i = \frac{1}{(\theta_i + b_i)} \log \left\{ 1 + \frac{(\theta_i + b_i) Q_i}{a_i} \right\} \quad (1.2)$$

The holding cost of i^{th} item in each cycle is

$$C_H = C_{li} G_i(Q_i) \quad (1.3)$$

where,

$$G_i(Q_i) = \int_0^{Q_i} \frac{q_i dq_i}{a_i + (\theta_i + b_i) q_i} = \frac{Q_i}{(\theta_i + b_i)} + \frac{a_i}{(\theta_i + b_i)^2} \log \left\{ 1 + \frac{(\theta_i + b_i) Q_i}{a_i} \right\}$$

By neglecting the higher power terms, we get

$$G_i(Q_i) = \frac{Q_i^2}{2a_i} \left\{ 1 - 2 \frac{(\theta_i + b_i) Q_i}{3a_i} \right\}$$

The total number of deteriorating units of the i^{th} item is

$$S_{di}(Q_i) = \theta_i G_i(Q_i)$$

The net revenue for the i^{th} item is

$$\begin{aligned} N(Q_i) &= (P_i - C_i) Q_i - P_i S_{di}(Q_i) \\ N(Q_i) &= (P_i - C_i) Q_i - P_i \theta_i G_i(Q_i) \end{aligned} \quad (1.4)$$

The profit of i^{th} item is

$$\begin{aligned} PF(Q_i) &= N(Q_i) - C_{li} G_i(Q_i) - C_{3i}, i = 1, 2, \dots, n. \\ PF(Q_i) &= (P_i - C_i) Q_i - P_i \theta_i G_i(Q_i) - C_{li} G_i(Q_i) - C_{3i}, \quad i = 1, 2, \dots, n. \end{aligned} \quad (1.5)$$

Hence the problem is

$$\begin{aligned} \text{Max PF} &= \sum_{i=1}^n \left[(P_i - C_i) Q_i - (C_{li} + P_i \theta_i) G_i(Q_i) - C_{3i} \right] \\ \text{subject to} & \quad \sum_{i=1}^n C_i Q_i \leq B \\ Q_i &\geq 0, \quad i = 1, 2, 3, \dots, n \end{aligned} \quad (1.6)$$

4.2. Fuzzy Model

When above profit, costs, deteriorating rate and total budget become fuzzy, the said crisp model is transform to

$$\begin{aligned} \tilde{\text{Max PF}} &= \sum_{i=1}^n \left[(P_i - C_i) Q_i - (\tilde{C}_{li} + P_i \tilde{\theta}_i) G_i(Q_i) - \tilde{C}_{3i} \right] \\ \text{subject to} & \quad \sum_{i=1}^n C_i Q_i \leq \tilde{B} \\ Q_i &\geq 0, \quad i = 1, 2, 3, \dots, n \end{aligned} \quad (1.7)$$

5. Mathematical Analysis

A crisp non-linear programming problem may be defined as follows

$$\begin{aligned} \text{Max } g_0(x, c_0) \\ \text{subject to} \\ g_r(x, c_r) \leq b_r, \quad r = 1, 2, \dots, m. \\ x \geq 0 \end{aligned}$$

where $x = (x_1, x_2, \dots, x_n)^T$ is a variable vector g_0, g_r 's are algebraic expressions in x with coefficients $C_0 = (C_{01}, C_{02}, \dots, C_{0n})^T$ and $c_r = (c_{r1}, c_{r2}, \dots, c_{rn})^T$ respectively. Introducing fuzziness in the crisp parameters, the above problem in fuzzy environment becomes

$$\begin{aligned} \tilde{\text{Max}} g_0(x, \tilde{c}_0) \\ \text{subject to} \\ g_r(x, \tilde{c}_r) \leq \tilde{b}_r, \quad r = 1, 2, \dots, m. \\ x \geq 0 \end{aligned}$$

In fuzzy set theory the fuzzy objective goal, coefficients and resources are defined by their membership functions, which may be linear and / or non-linear. According to Bellman and Zadeh [1] and following Carlsson and Korhonen [2] above problem is transform to

$$\begin{aligned} \text{Max } \alpha \text{ subject to} \\ g_0(x, \mu C_0^{-1}(\alpha)) \leq \mu \alpha^{-1}(\alpha) \\ g_r(x, \mu C_r^{-1}(\alpha)) \geq \mu r^{-1}(\alpha), r = 1, 2, \dots, m \\ x \geq 0, \alpha \in [0, 1] \\ \text{where } \mu C_r^{-1} = \{ \mu C_1^{-1}, \mu C_2^{-1}, \dots, \mu C_k^{-1} \} \end{aligned}$$

Fuzzy goal, costs, deteriorating rate and total budget are representing by combinations of linear and non-linear membership functions:

Here Exponential membership function is considered to represent the fuzzy costs and fuzzy goal, deteriorating rate and total budget are represented by linear membership function.

Case I) Linear Membership Function

We assume continuous linear membership functions to be non-decreasing for inventory costs and non-increasing for fuzzy goal and constraints and these are

$$\mu_0(g_0(x)) = \begin{cases} 1 & ;g_0(x) < b_0 \\ 1 - \left(\frac{g_0(x) - b_0}{P_0} \right) & ;b_0 \leq g_0(x) \leq b_0 + P_0 \\ 0 & ;g_0(x) > b_0 + P_0 \end{cases}$$

and

$$\mu_{Coi} = \begin{cases} 1 & ;u > A_{oj} \\ 1 - \left(\frac{A_{oj} - u}{P_{oj}} \right) & ;A_{oj} - P_{oj} \leq u \leq A_{oj} \\ 0 & ;u \leq A_{oj} - P_{oj} \end{cases}$$

Here p_0 and P_{oj} 's are the maximally acceptable violation of the aspiration levels b_0 and A_{oj} 's.

Case II) Non - linear Membership Function

5.1. Exponential Membership Function

In this case

$$\mu_{Ci}(u) = \begin{cases} 1 & ;u > C_i \\ \left(\frac{t(C_i - u)/PC_i - q^t}{(1 - q^t)} \right) & ;C_i - PC_i \leq u \leq C_i \\ 0 & ;u < C_i - PC_i \end{cases}$$

where $0 < q < 1, t > 0, i = 1, 3$. It's pictorial representation is

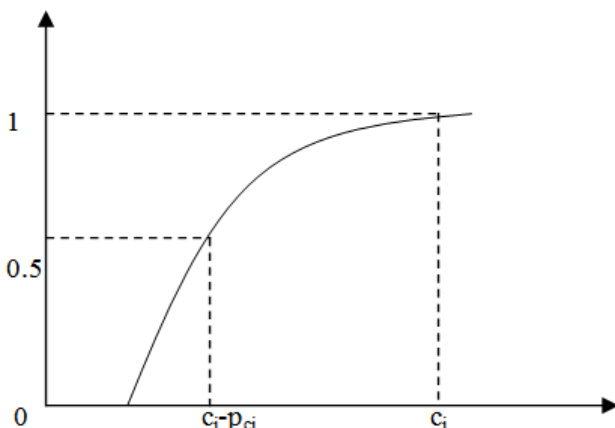


Figure. Exponential membership function of \tilde{C}_i

Let $q=0.5$ and $t=1$ then

$$\mu_{ci}^{-1}(\alpha) = c_i - P_{ci} \log_{0.5}(0.5 + 0.5\alpha)$$

Model: Fuzzy costs represented by exponential membership function and fuzzy goal, deteriorating rate and total budget are represented by linear membership function

Let

$$\mu_{cli}^{-1}(\alpha) = c_{li} - P_{cli} \log_{0.5}(0.5 + 0.5\alpha),$$

$$\mu_{c3i}^{-1}(\alpha) = c_{3i} - P_{c3i} \log_{0.5}(0.5 + 0.5\alpha),$$

$$\mu_{\theta i}^{-1}(\alpha) = \theta_i + P_{\theta i}(1 - \alpha),$$

$$\mu_{PF}^{-1}(\alpha) = PF_o - P_{PF}(1 - \alpha),$$

$$\mu_B^{-1}(\alpha) = B + P_B(1 - \alpha).$$

Here $P_{PF}, P_{\theta i}$ are the maximum acceptable violation of the aspiration levels PF_o and B .

Then the fuzzy model reduces to crisp model as:

$$\begin{aligned} & \text{Max } \alpha \\ & \text{subject to} \\ & PF(Q_i, \alpha) \geq PF_o - P_{PF}(1 - \alpha) \quad (1.8) \\ & \sum_{i=1}^n C_i Q_i \leq B + P_B(1 - \alpha) \\ & Q_i \geq 0, \alpha \in [0, 1] \end{aligned}$$

Where $Q_i > 0 (i=1, 2, \dots, n)$ are decision variables and

$$PF(Q_i, \alpha) = \sum_{i=1}^n [(P_i - C_i) Q_i - [(C_{1i} - P_{c1i} \log_{0.5}(0.5 + 0.5\alpha)) +$$

$$P_i(\theta_i + P_{\theta i}(1 - \alpha)) \left\{ \frac{Q_i^2}{2a_i} \left[1 - \frac{2[(\theta_i + P_{\theta i}(1 - \alpha)) + b_i]}{3a_i} \right] \right\} - (C_{3i} - P_{c3i} \log_{0.5}(0.5 + 0.5\alpha))]$$

6. Numerical Example

For $N=2$

$P_1=10, C_1=7, a_1=110, b_1=0.5, \theta_1=0.025, P_2=10, C_2=6.75, a_2=100, b_2=0.5, \theta_2=0.03,$

$PF_o=500, P_{PF}=50, B=1800, P_B=100, P_{\theta 1}=0.005, P_{\theta 2}=0.005, C_{11}=2, C_{31}=65, C_{12}=2.2, C_{32}=50, P_{C11}=0.05, P_{C31}=10, P_{C12}=0.05, P_{C32}=10$

The optimum results are:

$\alpha = 0.7256, Q_1=71.8234, Q_2=77.7292, PF=257.6075, B=1027.4379.$

From the above tables, it is observed that:

1. In Table 1.1, the optimum values are given for these fuzzy models with exponential membership function along with the crisp model. Here amount of profit varies from Rs. 193 to Rs. 299. As our permissible profit range is (210-260), only three crisp models profits fall within this range. Actually, if we make parametric studies on the crisp model with the different values of different parameters, results of some of those studies will coincide with the optimum values of fuzzy model. This laborious and time consuming parametric study can avoid by using fuzzy analysis. Depending on the experience or from past observed data, the exact form of membership functions for the variations of different inventory costs, objective and budget can be defined and then the optimum values are obtained for the appropriate fuzzy model.

2. In Table 1.2 it is seen that α increases but it never becomes one as it is expected. The values of decision variables (Q_i 's) become invariant but profit and budget decreases with the increase in tolerance of objective.

Table 1.1. Comparison Table

Model	C ₁₁	C ₁₂	C ₃₁	C ₃₂	θ_1	θ_2	B	Q ₁	Q ₂	PF	α
Crisp	2.2	2.4	80	70	0.03	0.035	1000	58.67146	87.30367	205.8378*	
	2.1	2.3	80	80	0.035	0.035	1000	56.40799	89.65098	199.1940*	
	2.2	2.4	65	60	0.02	0.03	1000	60.4744	85.8339	233.0279	
	2.4	2.2	65	50	0.02	0.03	1000	45.9012	100.547	236.7420	
	2	2.4	65	50	0.03	0.025	1100	65.6204	94.9122	275.0873*	
	2	2.2	65	60	0.03	0.025	1100	51.88589	109.1554	271.7629*	
	2	2.2	70	70	0.03	0.03	900	52.47932	78.91033	193.8238*	
	2	2.2	60	40	0.03	0.02	1100	50.5353	110.5561	298.0897*	
	2	2.2	80	70	0.03	0.03	1000	60.6636	85.2377	213.0223	
	Fuzzy	2	2.2	65	50	0.025	0.03	1000	71.823	77.7292	

* Not feasible with respect to the range considered for the objective goal (210 – 260)

Table 1.2. Effect of Variations in P_{PF}

P _{PF}	α	Q ₁	Q ₂	PF	B
0	0.41	74.18	79.89	259.01	1058.57
50	0.72	71.82	77.72	257.60	1027.43
100	0.82	71.10	77.03	254.69	1017.76
200	0.89	70.68	76.38	252.45	1010.39
500	0.95	69.75	76.49	250.78	1004.62
1000	0.97	69.92	75.98	250.05	1002.40

7. Concluding Remark

In this paper, a real life inventory problem in fuzzy environment has been proposed and presented its situations along with a sensitivity analysis approach. Here exponential membership function is considered to represent the nature of variations in inventory costs. Objective goal and budget are represented by linear membership functions. Depending on the earlier experience or past values of these parameters, the other types of membership functions such as piecewise linear, cubical parabolic, L-R fuzzy number, etc. can be considered to construct the membership functions for the above inventory parameters and then the model can be easily solved by the present method. The proposed methodology can be extended to other inventory problems including the ones with two storage, multi-objective, etc.

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