

A Study on New Sequence of Functions Involving $\bar{H}$ -Function

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Abstract A remarkably large number of operational techniques have drawn the attention of several researchers in the study of sequence of functions and polynomials. Very recently, Agarwal and Chand gave certain new sequence of functions involving the special functions in their series of papers. In this sequel, here, we aim to introduce a new sequence of functions involving the Generalized Mellin-Barnes Type of Contour Integrals by using operational techniques. Some generating relations and finite summation formulae of the sequence presented here are also considered. These generating relations and finite summation formulae are unified in nature and act as key formulae from which, we can obtain as their special cases.

Keywords: Special function, generating relations, $\bar{H}$ -function, sequence of functions

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1. Introduction

The idea of representing the processes of calculus, derivation, and integration, as operators is called operational technique; it is also known as operational Calculus. Many operational technique involving various special functions, have found significant importance and applications in various sub field of applicable mathematical analysis. Several applications of operational techniques can be found in the problems in analysis, in particular differential equations are transformed into algebraic problems, usually the problem of solving a polynomial equation. Since last four decades, a number of workers like Chak [6], Gould and Hopper [10], Chatterjea [9], Singh [23], Srivastava and Singh [24], Mittal [15,16,17], Chandal [7,8], Srivastava [21], Joshi and Parjapat [11], Patil and Thakare [18] and Srivastava and Singh [26] have studied in depth, the properties, applications and different extensions of the various operational techniques.

The key element of the operational technique is to consider differentiation as an operator acting on functions. Linear differential equations can then be recast in the form of an operator valued function $F(D)$ of the operator D acting on the unknown function equals the known function. Solutions are then obtained by making the inverse operator of F act on the known function.

Indeed, a remarkably large number of sequences of functions involving a variety of special functions have been developed by many authors (see, for example, [19]; for a very recent work, see also [2,3,4,22]). Here, we aim

at presenting a new sequence of functions involving the $\bar{H}$ -function, by using operational techniques. Some generating relations and finite summation formula are also obtained.

For our purpose, we begin by recalling some known functions and earlier works.

In 1971, the Rodrigues formula for generalized Lagurre polynomials is given by Mittal [15] as follows:

$$T_{kn}^{(\alpha)}(x) = \frac{1}{n!} x^{-\alpha} \exp(p_k(x)) D^n \left[x^{\alpha+n} \exp(-p_k(x)) \right], \quad (1)$$

where $p_k(x)$ is a polynomial in x of degree k .

Mittal [16] also proved following relation for (1) as follows:

$$\begin{aligned} T_{kn}^{(\alpha+s-1)}(x) \\ = \frac{1}{n!} x^{-\alpha-n} \exp(p_k(x)) T_s^n \left[x^\alpha \exp(-p_k(x)) \right], \end{aligned} \quad (2)$$

where s is constant and $T_s \equiv x(s + xD)$.

In 1979, Srivastava and Singh [24] studied a sequence of functions $V_n^{(\alpha)}(x; a, k, s)$ defined by:

$$\begin{aligned} V_n^{(\alpha)}(x; a, k, s) \\ = \frac{x^{-\alpha}}{n!} \exp\{p_k(x)\} \theta^n \left[x^\alpha \exp\{-p_k(x)\} \right]. \end{aligned} \quad (3)$$

By employing the operator $\theta \equiv x^a(s + xD)$, where s is constant and $p_k(x)$ is a polynomial in x of degree k .

A new sequence of function $\left\{V_n^{(M,N;P,Q;\alpha)}(x;a,k,s)\right\}_{n=0}^{\infty}$ involving the well-known $\bar{H}$ -function, introduced in this paper is defined as follows:

$$V_n^{(M,N;P,Q;\alpha)}(x;a,k,s) = \frac{1}{n!} x^{-\alpha} \bar{H}_{P,Q}^{M,N} [p_k(x)] \left(T_x^{a,s}\right)^n \left[x^{\alpha} \bar{H}_{P,Q}^{M,N} [-p_k(x)]\right], \quad (4)$$

where $T_x^{a,s} \equiv x^a (s + xD)$, $D \equiv \frac{d}{dx}$, a and s are constants, $\beta \geq 0$, k is finite and non-negative integer, $p_k(x)$ is a polynomial in x of degree k and $\bar{H}_{P,Q}^{M,N}[x]$ is a well known $\bar{H}$ -function is defined and represented in the following manner (see, [12] and see also, [1,5]):

$$\bar{H}_{P,Q}^{M,N}[x] = \bar{H}_{P,Q}^{M,N} \left[z \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j; B_j)_{1,M}, (b_j, \beta_j)_{M+1,Q} \end{matrix} \right. \right] \quad (5)$$

$$= \frac{1}{2\pi i} \int_L x^{\xi} \bar{\phi}(\xi) d\xi \quad (z \neq 0),$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^M \Gamma(b_j - \beta_j \xi) \prod_{j=1}^N \{\Gamma(1 - a_j + \alpha_j \xi)\}^{A_j}}{\prod_{j=1}^Q \{\Gamma(1 - b_j + \beta_j \xi)\}^{B_j} \prod_{j=1}^P \Gamma(a_j - \alpha_j \xi)}. \quad (6)$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and M, N, P, Q are integers such that $1 \leq M \leq Q$, $1 \leq N \leq P$ $(\alpha_j)_{1,P}, (\beta_j)_{1,Q}$ are positive real numbers and $(A_j)_{1,N}, (B_j)_{M+1,Q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,P}$ and $(\beta_j)_{1,Q}$ are complex numbers.

The nature of contour L , sufficient conditions of convergence of defining integral (5) and other details about the $\bar{H}$ -function can be seen in the papers [1,5,12,13].

The behavior of the $\bar{H}$ -function for small values of $|z|$ follows easily from a result given by Rathie [19]:

$$\bar{H}_{P,Q}^{M,N}[z] = o(|z|^{\alpha}); \quad (7)$$

where

$$\alpha = \min_{1 \leq j \leq m} \operatorname{Re} \left(\frac{b_j}{\alpha_j} \right), |z| \rightarrow 0 \quad (8)$$

$$\Omega = \left(\begin{matrix} \sum_{j=1}^M |B_j| + \sum_{j=M+1}^Q |b_j B_j| \\ - \sum_{j=1}^N |a_j A_j| - \sum_{j=N+1}^P |A_j| \end{matrix} \right) > 0, 0 < |z| < \infty. \quad (9)$$

The following function which follows as special cases of the $\bar{H}$ -function will be required in the sequel [12]

$$\begin{aligned} & {}^P\psi_Q \left[\begin{matrix} (a_j, \alpha_j; A_j)_{1,P} \\ (b_j, \beta_j; B_j)_{1,Q} \end{matrix} ; z \right] \\ &= \bar{H}_{P,Q+1}^{1,P} \left[-z \left| \begin{matrix} (1-a_j, \alpha_j; A_j)_{1,P} \\ (0,1), (1-b_j, \beta_j; B_j)_{1,Q} \end{matrix} \right. \right]. \end{aligned} \quad (10)$$

Some generating relations and finite summation formulae of class of polynomials or sequence of function have been obtained by using the properties of the differential operators. $T_x^{a,s} \equiv x^a (s + xD)$, $T_x^{a,1} \equiv x^a (1 + xD)$, where

$D \equiv \frac{d}{dx}$, is based on the work of Mittal [17], Patil and

Thakare [18], Srivastava and Singh [24].

Some useful Operational Techniques are given below:

$$\begin{aligned} & \exp(t T_x^{a,s}) (x^{\beta} f(x)) \\ &= x^{\beta} (1 - ax^a t)^{-\left(\frac{\beta+s}{a}\right)} f\left(x(1 - ax^a t)^{-1/a}\right). \end{aligned} \quad (11)$$

$$\begin{aligned} & \exp(t T_x^{a,s}) (x^{\alpha - an} f(x)) \\ &= x^{\alpha} (1 + at)^{-1 + \left(\frac{\alpha+s}{a}\right)} f\left(x(1 + at)^{1/a}\right). \end{aligned} \quad (12)$$

$$\left(T_x^{a,s}\right)^n (xuv) = x \sum_{m=0}^{\infty} \binom{n}{m} \left(T_x^{a,s}\right)^{n-m} (v) \left(T_x^{a,1}\right)^m (u). \quad (13)$$

$$\begin{aligned} & (1 + xD)(1 + a + xD)(1 + 2a + xD) \\ & \dots (1 + (m-1)a + xD) x^{\beta-1} = a^m \left(\frac{\beta}{a}\right)_m x^{\beta-1}. \end{aligned} \quad (14)$$

$$(1 - at)^{-\frac{\alpha}{a}} = (1 - at)^{-\frac{\beta}{a}} \sum_{m=0}^{\infty} \left(\frac{\alpha - \beta}{a}\right)_m \frac{(at)^m}{m!}. \quad (15)$$

2. Generating Relations

Here we start with presenting three generating relations involving new sequence of function (4), stated below.

First generating relation:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x;a,k,s) x^{-an} t^n \\ &= (1 - at)^{-\left(\frac{\alpha+s}{a}\right)} \bar{H}_{P,Q}^{M,N} (p_k(x)) \\ & \quad \bar{H}_{P,Q}^{M,N} \left(-p_k(x(1 - at)^{-1/a})\right). \end{aligned} \quad (16)$$

Second generating relation:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha-an)}(x;a,k,s) x^{-an} t^n \\ &= (1 + at)^{-1 + \left(\frac{\alpha+s}{a}\right)} \bar{H}_{P,Q}^{M,N} (p_k(x)) \\ & \quad \times \bar{H}_{P,Q}^{M,N} \left(-p_k(x(1 + at)^{1/a})\right). \end{aligned} \quad (17)$$

Third generating relation:

$$\sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n$$

$$= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{\overline{H}_{P,Q}^{M,N}(p_k(x))}{\overline{H}_{P,Q}^{M,N}\left(p_k\left(x(1-at)^{-1/a}\right)\right)} \quad (18)$$

$$V_n^{(M,N;P,Q;\alpha)}\left(x(1-at)^{-1/a}; a, k, s\right).$$

Proof of the first generating relation:

From (4), Let us consider

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \quad (19)$$

$$= x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) \exp(t T_x^{a,s}) \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right].$$

Using operational technique (11), above equation (19) reduces to

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n$$

$$= x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) x^{\alpha} (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \quad (20)$$

$$\overline{H}_{P,Q}^{M,N}\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right),$$

after replacing t by tx^{-a} , we get the desired result (16).

Proof of the Second generating relation:

Again from (4), we have

$$\sum_{n=0}^{\infty} x^{-an} V_n^{(M,N;P,Q;\alpha-an)}(x; a, k, s) t^n \quad (21)$$

$$= x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) \exp(t T_x^{a,s}) \left[x^{\alpha-an} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right].$$

Using the (12), we get

$$\sum_{n=0}^{\infty} x^{-an} V_n^{(M,N;P,Q;\alpha-an)}(x; a, k, s) t^n$$

$$= x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) x^{\alpha} (1+at)^{\frac{\alpha+s}{a}-1} \quad (22)$$

$$\times \overline{H}_{P,Q}^{M,N}\left[-p_k\left(x(1+at)^{1/a}\right)\right]$$

$$= (1+at)^{\frac{\alpha+s}{a}-1} \overline{H}_{P,Q}^{M,N}(p_k(x))$$

$$\times \overline{H}_{P,Q}^{M,N}\left[-p_k\left(x(1+at)^{1/a}\right)\right].$$

This complete the proof of second generating relation.

Proof of the third generating relation:

We can write (4) as follows

$$\left(T_x^{a,s}\right)^n \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]$$

$$= n! x^{\alpha} \frac{1}{\overline{H}_{P,Q}^{M,N}(p_k(x))} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s), \quad (23)$$

$$\exp\left(t\left(T_x^{a,s}\right)\right) \left\{ \left(T_x^{a,s}\right)^n \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] \right\}$$

$$= n! \exp\left(t T_x^{a,s}\right) \left[\frac{x^{\alpha} \frac{1}{\overline{H}_{P,Q}^{M,N}(p_k(x))}}{V_n^{(M,N;P,Q;\alpha)}(x; a, k, s)} \right], \quad (24)$$

$$\sum_{m=0}^{\infty} \frac{t^m}{m!} \left(T_x^{a,s}\right)^{m+n} \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]$$

$$= n! \exp\left(t T_x^{a,s}\right) \left[\frac{x^{\alpha} \frac{1}{\overline{H}_{P,Q}^{M,N}(p_k(x))}}{V_n^{(M,N;P,Q;\alpha)}(x; a, k, s)} \right].$$

On using the operational technique (11), (24) becomes

$$\sum_{m=0}^{\infty} \frac{t^m}{m!} \left(T_x^{a,s}\right)^{m+n} \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]$$

$$= n! x^{\alpha} (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \quad (25)$$

$$\times \frac{V_n^{(M,N;P,Q;\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right)}{\overline{H}_{P,Q}^{M,N}\left(p_k\left(x(1-ax^a t)^{-1/a}\right)\right)},$$

which upon using (23), gives

$$\sum_{m=0}^{\infty} \frac{t^m (m+n)!}{m! n!} x^{\alpha} \frac{V_{m+n}^{(M,N;P,Q;\alpha)}(x; a, k, s)}{\overline{H}_{P,Q}^{M,N}(p_k(x))}$$

$$= x^{\alpha} (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \quad (26)$$

$$\times \frac{V_n^{(M,N;P,Q;\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right)}{\overline{H}_{P,Q}^{M,N}\left[p_k\left(x(1-ax^a t)^{-1/a}\right)\right]}$$

Therefore, we get

$$\sum_{m=0}^{\infty} \binom{m+n}{n} V_{m+n}^{(M,N;P,Q;\alpha)}(x; a, k, s) t^m$$

$$= (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{\overline{H}_{P,Q}^{M,N}(p_k(x))}{\overline{H}_{P,Q}^{M,N}\left(p_k\left(x(1-ax^a t)^{-1/a}\right)\right)} \quad (27)$$

$$V_n^{(M,N;P,Q;\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right).$$

Finally by replacing t by tx^{-a} , we get the (18). This completes the proof of third generation relation.

Remark: If we set some suitable parametric replacement in (16), (17) and (18) respectively, then we can arrive at the known results (see, [2-11,15,16,17,18,21,22])

3. Finite Summation Formulas

or

In this section, we establish certain finite summation formulas, some of which are presumably (new) ones.

First finite summation formula:

$$\begin{aligned} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) \\ = \sum_{m=0}^{\infty} \frac{1}{m!} (ax^a)^m \left(\frac{\alpha}{a}\right)_m V_{n-m}^{(M,N;P,Q;0)}(x; a, k, s). \end{aligned} \quad (28)$$

Second finite summation formula:

$$\begin{aligned} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) \\ = \sum_{m=0}^{\infty} \frac{1}{m!} (ax^a)^m \left(\frac{\alpha-\beta}{a}\right)_m V_{n-m}^{(M,N;P,Q;\beta)}(x; a, k, s). \end{aligned} \quad (29)$$

Proof of First finite summation formula:

From equation (4), we have

$$\begin{aligned} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) \\ = \frac{1}{n!} x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) \left(T_x^{(a,s)}\right)^n \\ \times \left[x x^{\alpha-1} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]. \end{aligned} \quad (30)$$

Using the operational technique (13) on (30), we get

$$\begin{aligned} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) \\ = \frac{1}{n!} x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) x \sum_{m=0}^n \binom{n}{m} \left(T_x^{(a,s)}\right)^{n-m} \\ \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] \left(T_x^{a,1}\right)^m (x^{\alpha-1}), \\ = \frac{1}{n!} x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) x \sum_{m=0}^n \frac{n!}{m!(n-m)!} x^{a(n-m)} \times \\ \left[\frac{(s+xD)(s+a+xD)(s+2a+xD)}{\dots(s+(n-m-1)a+xD)} \right] \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] x^{am} \\ \times \left[\frac{(1+xD)(1+a+xD)(1+2a+xD)}{\dots(1+(m-1)a+xD)} \right] (x^{\alpha-1}). \end{aligned} \quad (31)$$

Using the result (14), The equation (31) becomes

$$\begin{aligned} = \overline{H}_{P,Q}^{M,N}(p_k(x)) \sum_{m=0}^n \frac{1}{m!(n-m)!} x^{an} \prod_{i=0}^{n-m-1} (s+ia+xD) \times \\ \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] a^m \left(\frac{\alpha}{a}\right)_m. \end{aligned} \quad (32)$$

Put $\alpha = 0$ and replacing n by $n-m$ in (30), we get

$$\begin{aligned} V_{n-m}^{(M,N;P,Q;0)}(x; a, k, s) \\ = \frac{1}{(n-m)!} \overline{H}_{P,Q}^{M,N}(p_k(x)) \left(T_x^{a,s}\right)^{n-m} \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right], \\ \frac{1}{(n-m)!} \left(T_x^{a,s}\right)^{n-m} \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] \\ = \frac{1}{\overline{H}_{P,Q}^{M,N}(p_k(x))} V_{n-m}^{(M,N;P,Q;0)}(x; a, k, s). \end{aligned} \quad (33)$$

After little simplification, we get

$$\begin{aligned} \frac{1}{(n-m)!} \prod_{i=0}^{n-m-1} (s+ia+xD) \left[\overline{H}_{P,Q}^{M,N}(-p_k(x)) \right] \\ = x^{a(n-m)} \frac{1}{\overline{H}_{P,Q}^{M,N}(p_k(x))} V_{n-m}^{(M,N;P,Q;0)}(x; a, k, s). \end{aligned} \quad (34)$$

Finally from (32) and (34), we get the desired result (28).

Proof of second finite summation formula:

Equation (4) can be written as

$$\begin{aligned} \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \\ = x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) \exp\left(t T_x^{(a,s)}\right) \left[x^{\alpha} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]. \end{aligned} \quad (35)$$

Applying the (11) on (35), we get

$$\begin{aligned} \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \\ = x^{-\alpha} \overline{H}_{P,Q}^{M,N}(p_k(x)) x^{\alpha} \left(1-ax^a t\right)^{-\left(\frac{\alpha+s}{a}\right)} \\ \times \overline{H}_{P,Q}^{M,N}\left(-p_k\left(x\left(1-ax^a t\right)^{-1/a}\right)\right), \\ = \left(1-ax^a t\right)^{-\left(\frac{\alpha+s}{a}\right)} \overline{H}_{P,Q}^{M,N}(p_k(x)) \\ \times \overline{H}_{P,Q}^{M,N}\left(-p_k\left(x\left(1-ax^a t\right)^{-1/a}\right)\right). \end{aligned} \quad (36)$$

Applying (15) on (36) then equation (36) reduces in to the form

$$\begin{aligned} \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \\ = \left(1-ax^a t\right)^{-\left(\frac{\beta+s}{a}\right)} \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} \\ \overline{H}_{P,Q}^{M,N}(p_k(x)) \overline{H}_{P,Q}^{M,N}\left(-p_k\left(x\left(1-ax^a t\right)^{-1/a}\right)\right), \\ = \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} x^{-\beta} \overline{H}_{P,Q}^{M,N}(p_k(x)) \\ \exp\left(T_x^{a,s}\right) \left[x^{\beta} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right], \\ = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m t^{n+m}}{m!n!} x^{-\beta} \\ \overline{H}_{P,Q}^{M,N}(p_k(x)) \left(T_x^{a,s}\right)^n \left[x^{\beta} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right], \\ = \sum_{n=0}^{\infty} \sum_{m=0}^n \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m t^n}{m!(n-m)!} x^{-\beta} \overline{H}_{P,Q}^{M,N}(p_k(x)) \\ \left(T_x^{a,s}\right)^{n-m} \left[x^{\beta} \overline{H}_{P,Q}^{M,N}(-p_k(x)) \right]. \end{aligned} \quad (37)$$

Now equating the coefficient of t^n both the sides, we get

$$\begin{aligned} & V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) \\ &= \sum_{m=0}^n \left(\frac{\alpha - \beta}{a} \right)_m \frac{(ax^a)^m}{m!(n-m)!} x^{-\beta} \bar{H}_{P,Q}^{M,N}(p_k(x)) \\ & \quad \left(T_x^{a,s} \right)^{n-m} \left[x^{\beta} \bar{H}_{P,Q}^{M,N}(-p_k(x)) \right]. \end{aligned} \quad (38)$$

Finally, by using the equation (4) on (37), we get the result (29).

4. Special Cases

a. If we put $A_j = B_j = 1$, -function reduces to Fox's H-function [25], p. 10, Eqn. (2.1.1)], then the equation (16), (17) and (18) takes the following form

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) x^{-an} t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} H_{P,Q}^{M,N}(p_k(x)) \\ & \quad \times H_{P,Q}^{M,N}\left(-p_k\left(x(1-at)^{-1/a}\right)\right) \end{aligned} \quad (39)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha-an)}(x; a, k, s) x^{-an} t^n \\ &= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} H_{P,Q}^{M,N}(p_k(x)) \\ & \quad \times H_{P,Q}^{M,N}\left(-p_k\left(x(1+at)^{1/a}\right)\right) \end{aligned} \quad (40)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{H_{P,Q}^{M,N}(p_k(x))}{H_{P,Q}^{M,N}\left(p_k\left(x(1-at)^{-1/a}\right)\right)} \\ & \quad V_n^{(M,N;P,Q;\alpha)}\left(x(1-at)^{-1/a}; a, k, s\right) \end{aligned} \quad (41)$$

b. If we put $A_j = B_j = \alpha_j = \beta_j = 1$, then the $\bar{H}$ -function reduces to general type of G-function [14] i.e.

$$\bar{H}_{P,Q}^{M,N} \left[z \left| \begin{matrix} (a_j, 1; 1)_{1,N} \\ (b_j, 1)_{1,M} \end{matrix} \right. \right] = G_{P,Q}^{M,N} \left[z \left| \begin{matrix} (a_j, 1)_{1,p} \\ (b_j, 1)_{1,q} \end{matrix} \right. \right]$$

the equation (16), (17) and (18) takes the following form:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) x^{-an} t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} G_{P,Q}^{M,N}(p_k(x)) \\ & \quad \times G_{P,Q}^{M,N}\left(-p_k\left(x(1-at)^{-1/a}\right)\right) \end{aligned} \quad (42)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q;\alpha-an)}(x; a, k, s) x^{-an} t^n \\ &= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} G_{P,Q}^{M,N}(p_k(x)) \\ & \quad \times G_{P,Q}^{M,N}\left(-p_k\left(x(1+at)^{1/a}\right)\right) \end{aligned} \quad (43)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q;\alpha)}(x; a, k, s) t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{G_{P,Q}^{M,N}(p_k(x))}{G_{P,Q}^{M,N}\left(p_k\left(x(1-at)^{-1/a}\right)\right)} \end{aligned} \quad (44)$$

$$V_n^{(M,N;P,Q;\alpha)}\left(x(1-at)^{-1/a}; a, k, s\right)$$

If we put $A_j = B_j = 1$, $N = P, M = 1$, $Q = Q + 1$, $b_1 = 0, \beta_1 = 1, a_j = 1 - a_j$, $b_j = 1 - b_j$, then the $\bar{H}$ -function reduces to generalized wright hypergeometric function [12] i.e.

$$\begin{aligned} & \bar{H}_{p,q+1}^{1,p} \left[z \left| \begin{matrix} (1-a_j, \alpha_j; A_j)_{1,P} \\ (0,1), (1-b_j, \beta_j; B_j)_{1,Q} \end{matrix} \right. \right] \\ &= {}_P\bar{\Psi}_Q \left[\begin{matrix} (a_j, \alpha_j; A_j)_{1,P} \\ (b_j, \beta_j; B_j)_{1,Q} \end{matrix} ; -z \right], \end{aligned}$$

the equation (16), (17) and (18) takes the following form:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(P,Q;\alpha)}(x; a, k, s) x^{-an} t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} {}_P\bar{\Psi}_Q(-p_k(x)) \\ & \quad \times {}_P\bar{\Psi}_Q\left(p_k\left(x(1-at)^{-1/a}\right)\right) \end{aligned} \quad (45)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(P,Q;\alpha-an)}(x; a, k, s) x^{-an} t^n \\ &= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} {}_P\bar{\Psi}_Q(-p_k(x)) \\ & \quad \times {}_P\bar{\Psi}_Q\left(p_k\left(x(1+at)^{1/a}\right)\right) \end{aligned} \quad (46)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(P,Q;\alpha)}(x; a, k, s) t^n \\ &= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{{}_P\bar{\Psi}_Q(-p_k(x))}{{}_P\bar{\Psi}_Q\left(-p_k\left(x(1-at)^{-1/a}\right)\right)} \end{aligned} \quad (47)$$

$$V_n^{(P,Q;\alpha)}\left(x(1-at)^{-1/a}; a, k, s\right)$$

5. Conclusion

In this paper, we have presented a new sequence of functions involving the $\bar{H}$ -function by using operational

techniques. With the help of our main sequence formula, some generating relations and finite summation formulae of the sequence are also presented here. Our sequence formula is important due to presence of $\bar{H}$ -function. On account of the most general nature of the $\bar{H}$ -function a large number of sequences and polynomials involving simpler functions can be easily obtained as their special cases but due to lack of space we cannot mention here.

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References

- [1] Agarwal, P., On multiple integral relations involving generalized Mellin-Barnes type of contour integral, *Tamsui Oxf. J. Math. Sci.* 2011, 27(4), 449-462.
- [2] Agarwal, P. and Chand, M., On new sequence of functions involving pFq , *South Asian J Math*, 2013, 3(3), 199-210.
- [3] Agarwal, P. and Chand, M., Graphical Interpretation of the New Sequence of Functions Involving Mittag-Leffler Function Using Matlab, *American Journal of Mathematics and Statistics* 2013, 3(2): 73-83.
- [4] Agarwal, P. and Chand, M., On new sequence of functions involving pFq , *South Asian Journal of Mathematics* 2013, Vol. 3 (3): 199-210.
- [5] Agarwal, P. and Jain, S., On unified finite integrals involving a multivariable polynomial and a generalized Mellin Barnes type of contour integral having general argument, *Nat. Acad. Sci. Lett.* 2009, 32(9-10), 281-286.
- [6] Chak, A. M., A class of polynomials and generalization of stirling numbers, *Duke J. Math.* 1956, 23, 45-55.
- [7] Chandel, R.C.S., A new class of polynomials, *Indian J. Math.* 1973, 15(1), 41-49.
- [8] Chandel, R.C.S., A further note on the class of polynomials $T_n^{ak}(x, r, p)$, *Indian J. Math.* 1974, 16(1), 39-48.
- [9] Chatterjea, S. K., On generalization of Laguerre polynomials, *Rend. Mat. Univ. Padova.* 1964, 34, 180-190.
- [10] Gould, H. W. and Hopper, A. T., Operational formulas connected with two generalizations of Hermite polynomials, *Duck Math. J.* 1962, 29, 51-63.
- [11] Joshi, C. M. and Prajapat, M. L., The operator $T_{a,k}$ and a generalization of certain classical polynomials, *Kyungpook Math. J.* 1975, 15, 191-199.
- [12] K.C. Gupta and R.C. Soni, On a basic integral formula involving the product of the H-function and Fox H-function, *J.Raj.Acad.Phy. Sci.* 2006, 4 (3), 157-164.
- [13] K.C. Gupta, R. Jain and R. Agarwal, On existence conditions for a generalized Mellin-Barnes type integral *Natl Acad Sci Lett.* 2007, 30(5-6), 169-172.
- [14] Meijer, C.S., On the G-function, *Proc. Nat. Acad. Wetensch* 1946, 49, p. 227.
- [15] Mittal, H. B., A generalization of Laguerre polynomial, *Publ. Math. Debrecen* 1971, 18, 53-58.
- [16] Mittal, H. B., Operational representations for the generalized Laguerre polynomial, *GlasnikMat.Ser III* 1971, 26(6), 45-53.
- [17] Mittal, H. B., Bilinear and Bilateral generating relations, *American J. Math.* 1977, 99, 23-45.
- [18] Patil, K. R. and Thakare, N. K., Operational formulas for a function defined by a generalized Rodrigues formula-II, *Sci. J. Shivaji Univ.* 1975, 15, 1-10.
- [19] Prajapati, J. C. and Ajudia, N. K., On New Sequence of Functions and Their MATLAB Computation *International Journal of Physical, Chemical & Mathematical Sciences*, 1(2), Accepted On: 27.08.2012.
- [20] Rathie, A.K., A new generalization of generalized hypergeometric functions, *Le Mathematic he Fasc. II* 1997, 52, 297-310.
- [21] Shrivastava, P. N., Some operational formulas and generalized generating function, *The Math. Education* 1974, 8, 19-22.
- [22] Shukla, A. K. and Prajapati J. C., On some properties of a class of Polynomials suggested by Mittal, *Proyecciones J. Math.* 2007, 26(2), 145-156.
- [23] Singh, R. P., On generalized Truesdell polynomials, *Rivista de Mathematica* 1968, 8, 345-353.
- [24] Srivastava, A. N. and Singh, S. N., Some generating relations connected with a function defined by a Generalized Rodrigues formula, *Indian J. Pure Appl. Math.* 1979, 10(10), 1312-1317.
- [25] Srivastava, H.M., Gupta, K.C. and Goyal, S.P., The H-function of one and two variables with applications, *South Asian Publishers, New Dehli, Madras*, 1982.
- [26] Srivastava, H. M. and Singh, J. P., A class of polynomials defined by generalized, Rodrigues formula, *Ann. Mat. Pura Appl.* 1971, 90(4), 75-85.