American Journal of Applied Mathematics and Statistics, 2024, Vol. 12, No. 1, 10-14
Available online at http://pubs.sciepub.com/ajams/12/1/2
Published by Science and Education Publishing
DOI:10.12691/ajams-12-1-2

Assessing the Effectiveness of the APOS/ACE Instructional Treatment with the Help of Neutrosophic Triplets

Michael Gr. Voskoglou*

Department of Mathematical Sciences, Graduate T. E. I. of Western Greece, Patras, Greece
*Corresponding author: mvosk@hol.gr

Received December 20, 2023; Revised January 24, 2024; Accepted February 01, 2024

Abstract The APOS/ACE instructional treatment for teaching mathematics was introduced in the USA by Prof. Ed Dubinsky and his research team during the 1990's The central idea of the APOS/ACE treatment is that one can always find a suitable computer task for helping students to build the mental constructions that lead to the learning of the corresponding mathematical topic. In this work a method is presented for assessing the overall performance of a student group when the instructor is not sure about the accuracy of the individual grades assigned to the students. This method is developed using neutrosophic sets as tools and writing their elements in the form of neutrosophic triplets and it is used here for evaluating the effectiveness of the APOS/ACE instructional treatment for teaching mathematics. The outcomes of the classroom application performed for this purpose provide a strong indication that the APOS/ACE approach benefits the mediocre and the weak in mathematics students more than the good students, but this requires further experimental research.

Keywords: *APOS/ACE, Neutrosophic Set (NS), Neutrosophic Assessment, Neutrosophic Triplet (NT), GPA Index*

Cite This Article: Michael Gr. Voskoglou, "Assessing the Effectiveness of the APOS/ACE Instructional Treatment with the Help of Neutrosophic Triplets." American Journal of Applied Mathematics and Statistics, vol. 12, no. 1 (2024): 10-14. doi: 10.12691/ajams-12-1-2.

1. Introduction

In earlier works the author of the present article article has developed several methods for assessing the overall performance of a group of objects concerning a certain activity under fuzzy conditions. These methods, the most important of which are reviewed in [1], include the measurement of the corresponding system's uncertainty, the Rectangular Fuzzy Assessment Model (RFAM), which is based on the Center of Gravity (COG) defuzzification technique [2], the use of Fuzzy Numbers and of Grey Numbers as assessment tools, etc. Recently, in order to tackle assessment cases in which the evaluator is not sure about the accuracy of the grades assigned to the members of a group under assessment for evaluating their individual performance, the author introduced *neutrosophic sets (NSs)* as assessment tools [3]. The present work, uses the last method for evaluating the effectiveness of the APOS/ACE instructional treatment for teaching mathematics [4]

The rest of the paper is formulated as follows: Section 2 presents the basic ideas of the APOS/ACE theory. Section 3 contains the mathematical background about NSs, which is necessary for the understanding of the paper. In Section 4 a classroom application is developed evaluating the effectiveness of the APOS/ACE approach for teaching mathematics and the paper closes with the general conclusions and some hints for further research, which are included in the last Section 5.

2. Basic Ideas of the APOS/ACE Instructional Treatment for Teaching Mathematics

Computers have become nowadays a valuable tool for Education The animation of figures and representations, achieved by using the proper software, develop the students' imagination and enhance their problem solving skills. Several didactic methods have been developed in which computers play a dominant role. One of them is the *APOS/ACE* instructional treatment for teaching Mathematics, developed by Prof. Ed Dubinsky and his collaborators in the USA during the 1990's [4,5,6]. In earlier works we have applied this approach for teaching the rational numbers [7], the polar coordinates [8,9] and the derivatives [10,11] at university level.

APOS is a theory based on Piaget's principle that an individual learns by applying certain mental mechanisms to build specific cognitive structures and uses these structures to deal with problems connected to the corresponding situations [12]. According to the APOS theory, these mechanisms involve *interiorization* and *encapsulation*, while the cognitive structures involve

***A**ctions*, ***P**rocesses*, ***O**bjects* and ***S**chemas.* The first letters of the last four words form the acronym APOS.

A mathematical concept begins to be formed as one applies transformations on certain entities to obtain other entities. A transformation is first conceived as an action. For example, if an individual can think of a function only through an explicit expression and can do little more than substitute for the variable in the expression and manipulate it, he/she is considered to have an action understanding of functions.

As an individual repeats and reflects an action it may be interiorized to a mental process. A process performs the same operation as the action, but wholly in the mind of the individual, enabling him/her to imagine performing the transformation without having to execute each step explicitly. For example, an individual with a process understanding of a function thinks about it in terms of inputs, possibly unspecified, and transformations of those inputs to produce outputs.

If one becomes aware of a mental process as a totality and can construct transformations acting on this totality, then he/she has encapsulated the process into a cognitive object. In the case of functions, for example, encapsulation allows one to form sets of functions, to define operations on such sets, to equip them with a topology, etc.

A mathematical topic often involves many actions, processes and objects that need to be organized into a coherent framework that enables the individual to decide which mental processes to use in dealing with a mathematical situation. Such a framework is called a schema. In the case of functions, for example, the schema structure is used to recognize the need of using a specific function in a given mathematical or real-world situation

Dubinsky and his collaborators realized that for each mental construction that comes out from an APOS analysis, one can find a computer task such that, if a student engages in that task, he/she is fairly likely to build the mental construction that leads to the learning of the corresponding mathematical topic. As a consequence, a pedagogical approach was developed based on the APOS analysis, which is known as the *ACE teaching cycle*. ACE is a repeated cycle of three components: *Activities on the computer (**A**)*, *Classroom discussion **(C)*** and *exercises **(E)*** done outside the class.

In applying the ACE cycle the mathematical topic under consideration is divided into smaller subtopics and each iteration of the cycle corresponds to one of these subtopics. The implementation of the ACE cycle and its effectiveness in helping students make mental constructions and learn mathematics has been reported in several research studies of Dubinsky's team, e.g. [4,6,13,14], etc.

3. Neutrosophic Sets

Zadeh, on the purpose of tackling mathematically the existing in everyday life definitions with no clear boundaries (e.g. clever people, high mountains, etc.), generalized in 1965 the concept of crisp set to that of *fuzzy set (FS)*, in which every element of the universal set of the discourse has a membership degree in the interval [1,15]. Atanassov in 1986 added to Zadeh's membership the non-membership function and extended FS to the concept of *intuitionistic FS (IFS)* [16]. Smarandache, inspired by the various neutralities appearing in the real world - like <friend, neutral, enemy>, <win, draw, defeat>, <short, mediocre, high>, etc. - introduced in 1995 the degree of *indeterminacy* or *neutrality*, and extended further IFS to the concept of *neutrosophic set (NS) [17]*. The simplest form of a NS is the *single valued NS (SVNS)*, which is defined as follows [18]:

Definition 1: A SVNS A in the universe U is of the form

$A = \{(x,T(x),I(x),F(x)): x \in U,\ T(x),I(x),F(x) \in [0,1],\ 0 \leq T(x)+I(x)+F(x) \leq 3\}$ (1)

In (1) T(x), I(x), F(x) are the degrees of *truth* (or membership), *indeterminacy* (or neutrality) *and falsity* (or non-membership) respectively with respect to A, for all x in U, referred to as the *neutrosophic components* of x. For simplicity, we write A = <T, I, F>.

There is not any general rule for defining the membership, non-membership and indeterminacy functions of a NS, their definitions depending on the subjective criteria of each observer and, therefore, not being unique. The only restriction is to be compatible to common sense, otherwise the NS defined by them does not give a reliable description of the corresponding real situation.

The etymology of the term "neutrosophy" comes from the adjective "neutral´ and the Greek word "sophia" (wisdom) and, according to Smarandanche, who introduced it, means "knowledge of the neutral thought".

Example 1: Let U be the set of the players of a football team and let A be the SVNS of the good players of U. Then each player x of U is characterized by a *neutrosophic triplet (NT)* (t, i, f) with respect to A, with t, i, f in [1]. For instance, x(0.6, 0.2, 0.4) ∈ A, means that there is a 60% belief that x is a good player, but simultaneously a 20% doubt about and a 40% belief that x may not be a good player. In particular, x (0,1,0) ∈ A means that we do not know absolutely nothing about x's affiliation with A.

NSs are suitable for tackling the uncertainty due to ambiguity and inconsistency. Ambiguity is created when the existing information leads to several interpretations by different observers. For example, the statement "Boy no girl" written as "Boy, no girl" means boy, but written as "Boy no, girl" means girl. Inconsistency appears when two or more pieces of information cannot be true at the same time. As a result the obtainable in this case information is conflicted or undetermined. For example, "The probability of raining tomorrow is 80% but this does not mean that the probability of not raining is 20%, because they might be hidden weather factors". The concepts and operations defined on FSs can be extended in a natural way to NSs [15].

Remark 1: (i) Indeterminacy is understood to be everything which is between the opposites of truth and falsity [19]. In an IFS the indeterminacy is equal by default with the hesitancy, i.e. we have I(x) = 1- T(x) – F(x). Also, in a FS is I(x) = 0 and F(x) = 1 – T(x), whereas in a crisp set it is T(x)=1 (or 0) and F(x)= 0 (or 1). In other words, crisp sets, FSs and IFSs are special cases of SVNSs.

(ii) If the sum T(x) + I(x) + F(x) of the neutrosophic components of x ∈ U in a NS in U is < 1, then it leaves room for *incomplete* information about x, when it is equal to 1 for *complete* information and when is greater than 1 for *paraconsistent* (i.e. contradiction tolerant) information about x. A SVNS may contain simultaneously elements leaving room for all the previous types of information.

(iii) When $T(x) + I(x) + F(x) < 1$, $\forall$ x $\in$ U, then the corresponding SVNS is usually referred to as *picture FS (PiFS)* [20]. In this case 1- T(x)-I(x)-F(x) is called the degree of *refusal membership* of x in A. The PiFSs based models are suitable for describing situations where we face human opinions involving answers of types yes, abstain, no and refusal to express an opinion. Voting is a representative example of such a situation.

(iv) The difference between the *general definition of a* NS and of the previously given definition of a SVNS is that in the general definition T(x), I(x) and F(x) may take values in the *non-standard unit interval*]−0, 1+[, including values < 0 or > 1 [17]. This is something that can happen in everyday life situations. For example, assume that in a company the full-time working program for its employees is 40 hours per week. Then, an employee may cover by $\frac{40}{40} = 1$ the company's working program (full-time job), or by $\frac{30}{40} < 1$ (part-time job) or by $\frac{45}{40} > 1$ (over-time job). Assume further that a full-time employee caused damage to the company's equipment, the cost of which was decided to be subtracted from his salary. Then, if the cost of the damage is equal to $\frac{50}{40}$ of his weekly salary, the employee will be entitled to the $-\frac{10}{40}$ of his salary for this week. In this work we are going to use SVNs only. For general facts on SVNSs we refer to [16]

(v) Prof. Smarandache and his collaborators have applied the neutrosophic theory, apart from mathematics (e.g. see [3, 21-24]), to various other topics of human activity, like Philosophy, Physics, Biology, Economics, Linguistics, Psychology, Sociology, Literature, etc. [25].

(vi) A relatively recent extension of the concept of a NS is the *plithogenic set*, which is a set whose elements are characterized by one or more attributes and there may be several values for each attribute [26,27]. Plithogenic sets have already found some important practical applications [28,29].

Summation of NTs is equivalent to the union of NSs. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in several ways equivalently to the known in the literature neutrosophic union operators [30]. For the needs of the present work, writing the elements of a SVNS A in the form of NTs (t, i, f), with t, i, f in [1] and $0 \leq t + i + f \leq 3$ and considering them simply as ordered triplets we define addition and scalar multiplication as follows:

Definition 2: Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in the NS A and let r be a positive number. Then:

- The *sum* $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1 + t_2\ , i_1 + i_2, f_1 + f_2)$ (2)
- The *scalar product* $r(t_1, i_1, f_1) = (rt_1, ri_1, rf_1)$ (3)

The sum and the scalar product of two elements of a NS A with respect to Definition 2 need not be in A, because we may have that $(t_1 + t_2) + (i_1 + i_2) + (f_1 + f_2) > 3$ or $rt_1 + ri_1 + rf_1 > 3$. The *mean value* of a finite number of elements of A, however, which is defined below, is always an element of A.

Definition 3: Let A be A NS and let (t_1, i_1, f_1), (t_2, i_2, f_2), ….,, (t_k, i_k, f_k) be a finite number of elements of A. Then the *mean value* of these elements is the element of A

$$(t, i, f) = \frac{1}{k}[(t_1, i_1, f_1) + (t_2, i_2, f_2) + \ldots + (t_k, i_k, f_k)] \quad (4)$$

4. The Classroom Application

The purpose of the following classroom application was to evaluate the effectiveness of the APOS/ACE instructional treatment for teaching mathematics. The subjects were the first term students of two departments of the School of Engineering of my university during the course "Higher Mathematics I", which includes Complex Numbers, Differential and Integral Calculus in one variable and elements from Linear Algebra. According to the grades obtained in the PanHellenic examination for entrance in Higher Education, the potential of the two departments in mathematics was nearly the same. The course's instructor was also the same person, but the teaching methods followed were different. Namely, the APOS/ACE approach was applied for teaching the course to the 60 students of the first department (experimental group), whereas the classical method with lectures on the board was applied for the 60 students of the second department (control group). The grades used for evaluating the individual performance of each student were the commonly used qualitative grades A = Excellent, B = Very Good, C = Good, D = Fair (Pass) and F = Unsatisfactory (Failed).

The results of the final examination, after the end of the course, were the following:

- *Department I:* A: 9 students, B: 15, C: 18, D: 12, F: 6
- *Department II:* A: 12, B: 15, C: 9, D: 12, F: 12

Some of the students' answers in the examination, however, were not clearly presented or well justified. As a result, the instructor was not quite sure for the accuracy of the grades assigned to them. For this reason, he decided to use NTs for the assessment of the two departments' overall performance. For this, starting from the students with the highest grades, he denoted by Si, i = 1, 2,…., 60, the students of each department. Considering the NS of the good students, he assigned (according to his personal criteria) NTs to all students of the two departments as follows:

- *Department I:* $S_1 - S_{32}$: (1, 0, 0), $S_{33} - S_{38}$: (0.8, 0.1, 0.1), $S_{39} - S_{42}$: (0.7, 0.2, 0.1), $S_{43} - S_{46}$: (0.4, 0.2, 0.4), $S_{47} - S_{50}$: (0.3, 0.2, 0.5), $S_{51} - S_{53}$: (0.2, 0.2, 0.6), S_{54}- S_{55}: (0.1, 0.2, 0.7), $S_{56} - S_{57}$: (0, 0.2, 0.8), , $S_{57} - S_{60}$: (0, 0, 1).
- *Department II*: $S_1 - S_{31}$: (1, 0, 0), $S_{32} - S_{35}$: (0.8, 0.1, 0.1), S_{36}: (0.7, 0.1, 0.2), $S_{35} - S_{43}$: (0.4, 0.1, 0.5), S_{44} - S_{46}: (0.3, 0.2, 0.5), $S_{47} - S_{50}$: (0.2, 0.2, 0.6), S_{51} - S_{52}: (0.1, 0.2, 0.7), $S_{53} - S_{58}$: (0, 0.3, 0.7), , $S_{59} - S_{60}$: (0,0,1).

Then, according to equation (4), the mean value of the NTs of Department I is equal to $\frac{1}{60}$ [32(1,0,0)+6 (0.8,0.1,0.1)+ 4(0.7,0.2,0.1)+4 (0.4, 0.2,0.4)+4(0.3,0.2,0.5) +3(0.2,0.2,0.6)+ 2(0.1,0.2, 0.7)+2(0,0.2,0.0.8)+3 (0,0,1) ≈ (0.72, 0.07, 0.21). In the same way one finds that the mean value of the NTs of Department II is equal to (0.65, 0.08, 0.27).

Thus, a random student of Department I has a 72% chance to be a good student, but at the same time there exists a 7% doubt about it and a 21% chance to be not a good student. Also, the chance of a random student of Department II to be a good student is 65%, with a 8% doubt about it and a 27% chance to be not a good student. Consequently, the experimental group, despites the doubts of the instructor for the individual grades assigned to the students of the two departments, demonstrated a better overall performance than the control group.

Remark 2: (i) Denote by n_X the number of students of each department who got the grade X=A, B, C, D, F in the final examination of the module "Higher Mathematics I" Then one finds that the *Grade Point Average (GPA)* index [31] (Chapter 6, p. 125) for Department I is equal to $\frac{n_D+2n_C+3n_B+4n_A}{60}=\frac{12+2x18+3x15+4x9}{60}=2.12$ and similarly the GPA index for Department II is equal to 2.05. But, since the GPA index is a weighted average in which greater coefficients are assigned to the higher grades, this shows that the experimental group demonstrated a slightly better quality performance than the control group. In fact, since $0 \leq \text{GPA} \leq 4$, it is straightforward to check that the superiority of the experimental group in this case is only $0.07 \text{ x } \frac{100}{4} = 1.75\%$. In conclusion, the classroom application presented here demonstrates a superiority in general of the experimental (APOS/ACE) group with respect to the control group. This superiority, however, is more significant concerning the two groups' overall performance, but rather negligible concerning their quality performance. This provides a strong indication that the application of the APOS/ACE method benefits more the mediocre and the weak students in mathematics, and less the good students with higher grades. Further experimental research is needed, however, for obtaining a safer conclusion about this.

(ii) As we have already seen, the neutrosophic assessment method is suitable for evaluating the overall performance of a group, when the evaluator (e.g. the teacher) is not sure for the credibility of the individual grades assigned to its members (e.g. students). This frequently happens in practice; e.g. when a teacher is new in a class and has not formed yet a clear idea of each student's individual skills, when the students' answers in a written test are not clear or well justified, etc. Although the calculation of the mean value of the NTs used in such cases gives in general a fairly good idea about the group's overall performance, it does not provide an exact numerical value characterizing the group's mean performance, as the use of GNs (closed real intervals) or equivalently the use of TFNs does [1] (Sections 5.2 and 6.2). This means that in cases where the evaluator is quite sure about the accuracy of the individual grades assigned to the group's members, the use of GNs or TFNs as assessment tools is more suitable, than the use of the NTs.

5. Final Conclusions

A classroom application was presented in this work concerning the assessment of the effectiveness of the APOS/ACE instructional treatment for teaching mathematics using NTs as tools. It was emphasized that the use of NTs, although it does not provide an exact numerical value characterizing a group's mean performance concerning a certain activity, is more suitable in general than the use of GNs or TFNs as tools for the assessment of the overall performance of the group in cases where the evaluator is not sure for the credibility of the grades assigned to each of the group's members for evaluating its individual performance. On the other hand, a limitation of this approach and of all the methods in general using FSs or their generalizations involving membership functions as tools, is that the assessment of the individual performance of the members of the corresponding group is based on the personal criteria of the evaluator.

A strong indication was also obtained through the classroom application presented in this work that the application of the APOS/ACE instructional treatment for teaching mathematics benefits the mediocre and weak students more than the good students in mathematics, but more experimental research is needed for obtaining safer conclusions about it.

References

[1] Voskoglou, M.Gr., Methods for Assessing Human-Machine Performance under Fuzzy Conditions, *Mathematics*, *7*, 230, 2019.

[2] Van Broekhoven, E., De Baets, B., Fast and accurate centre of gravity defuzzification of fuzzy systems outputs defined on trapezoidal fuzzy partitions, *Fuzzy Sets Syst*., 6, 157, 904–918, 2006.

[3] Voskoglou, M.Gr., Broumi, S., Smarandache, F., A Combined Use of Soft and Neutrosophic Sets for Student Assessment with Qualitative Grades, *Journal of Neutrosophic and Fuzzy Systems*, 4(1), 15-20, 2022

[4] Asiala, M., Brown, A., DeVries, D.J., Dubinsky, E., Mathews, D. & Thomas, K., A framework for research and curriculum development in undergraduate mathematics education, *Research in Collegiate Mathematics Education II*, CBMS Issues in Mathematics Education, 6, 1-32, 1996.

[5] Dubinsky, E., McDonald, M.A., APOS: A constructivist theory of learning in undergraduate mathematics education research, in D. Holton et al. (Eds.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, Kluwer Academic Publishers, Dordrecht, Netherlands, pp. 273-280, 2001.

[6] Arnon, I., Cottrill, J., Dubinsky, E., Oktac¸ A., Roa, S., Trigueros, M. & Weller, K., *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*, Springer, N. Y., Heidelberg, Dordrecht, London, 2014.

[7] Voskoglou, M. Gr., An application of the APOS/ACE approach teaching the irrational numbers, *Journal of Mathematical Sciences and Mathematics Education*, 8(1), 30-47, 2013

[8] Voskoglou, M.Gr., Fuzzy Logic in the APOS/ACE Instructional Treatment of Mathematics, *American Journal of Educational Research*, 3(3), 330-339, 2015.

[9] Borji, V., Voskoglou, M.Gr., Applying the APOS Theory to Study the Student Understanding of the Polar Coordinates, *American Journal of Educational Research*, 4(16), 1149-1156, 2016.

[10] Borji, V., Voskoglou, M.Gr., Designing an ACE Approach for Teaching the Polar Coordinates, *American Journal of Educational Research*, 5(3), 303-309, 2017.

[11] Voskoglou, M. Gr., A Markov Chain Model for the APOS/ACE Instructional Treatment of Mathematics, *International Journal of Education and Learning Systems*, 4, 5(3), 1-6, 2019.

[12] Piaget, J., *Genetic Epistemology*, Columbia University Press, New York and London, 1970.

[13] Weller, K., Clark, J.M., Dubinsky, E., Loch, S., McDonald, M., Students performance and attitudes in courses based on APOS

theory and the ACE teaching cycle, in A. Selden et al. (Eds.), *Research in Collegiate Mathematics Education V*, Providence, RI, American Mathematical Society, pp. 97-181, 2003.

[14] Weller, K., Arnon, I.., Dubinsky, E., Pre - service Teachers' Understanding of the Relation between a Fraction or Integer and Its Decimal Expansion: Strength and Stability of Belief, *Canadian Journal of Science, Mathematics and Technology Education*, 11(2), 129-159, 2011.

[15] Zadeh, L.A., Fuzzy Sets. *Inf. Control*, 8, 338–353, 1965.

[16] Atanassov, K.T., Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, 20(1), 87-96, 1986.

[17] Smarandache, F., *Neutrosophy / neutrosophic probability, set, and logic*, Proquest, Michigan, USA, 1998.

[18] Wang, H., Smarandanche, F., Zhang, Y. & Sunderraman, R., Single valued neutrosophic sets, *Review of the Air Force Academy (Brasov)*, 1(16), 10-14, 2010.

[19] Smarandache, F., Indeterminacy in neutrosophic theories and their applications, *International Journal of Neutrosophic Science*, 15(2), 89-97, 2021.

[20] Cuong, B.C., Picture fuzzy sets, *Journal of Computer Science and Cybernetics*, 30(4), 409-420, 2014.

[21] Smarandache, F., *Algebra*, retrieved from https:// fs.unm.edu/Algebra.htm, 2023.

[22] Smarandache, F., *Geometries*, retrieved from https:// fs.unm.edu/Geometries.htm, 2023.

[23] Salama, A.A. & Alblowi, S.A., Neutrosophic sets and neutrosophic topological spaces, *IOSR Journal of Mathematics*, 3(4), 31-35, 2013.

[24] Voskoglou, M. Gr., An Application of Neutrosophic Sets to Decision Making, *Neutrosophic Sets and Systems*, 53, 1-9, 2023.

[25] Smarandache, F., Smarandache Notions, retrieved from https://fs.unm.edu, 2023.

[26] Smarandache, F., Plithogeny, *Plithogenic Set, Logic, Probability and Statistics*, Pons, Brussels, Belgium, 2017.

[27] Smarandache, F., Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic Set – Revisited, *Neutrosophic Sets and Systems*, 21, 153-166, 2018.

[28] Ganathy, S., Nagarajan, D., Broumi, S., & Lathamaheswari, M., Plithogenic sets and their applications in decision making, *Neutrosophic Sets and Systems*, 38, 453-459, 2020.

[29] Quek, S.G., Salvachandram, G., Smarandache, F., Vimala, J., Bui, Q.-T. & Gerogiannis, V., Entropy Measures for Plithogenic Sets and Applications in Multi – Attribute Decision Making, *Mathematics*, 8, 965, 2020.

[30] Smarandache, F., Subtraction and Division of Neutrosophic Numbers, *Critical Review*, XIII, 103-110, 2016.

[31] Voskoglou, M.Gr., *Finite Markov Chain and Fuzzy Logic Assessment Models*, Create Space Independent Platform (Amazon), Columbia, SC, 2017.