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A Result on "Common Fixed Points" for Pair of OWC- Mappings in C-MS

K. Prudhvi*

Department of Mathematics, University College of Science, Saifabad, Osmania University, Hyderabad, Telangana State, India
*Corresponding author: prudhvikasani@rocketmail.com

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Abstract In this paper, we obtain a unique common fixed point theorem for pair of self-mappings of OWC(Occasionally Weakly Compatible) pair of mappings in C-MS(Cone -Metric Space) and also given the example for supporting this result. Our result is a generalization and improvement of the some results they are present in this references.

Keywords: *fixed point, OWC(Occasionally Weakly Compatible), C-MS(Cone -Metric Space)*

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1. Introduction

Before 1968 fixed point theorems are used the Banach contraction principle, but in 1968, Kannan [9] proved a fixed point theorem for self- mapping satisfying the contractive condition which did not required the continuity condition at each point. Later on many mathematicians studied in this fixed point theory and established fixed point theorems. In 1997 Huang and Zhang [7] proved fixed point results in cone metric space they replaced the real numbers by an ordered Banach space. Later on many authors (for e.g., see, 1-4, 6, 8, &10-13) were proved in many ways of fixed point results and they extended, generalized and improved the results in different ways. In the like manner, Bhatt and Chandra [5] proved some fixed point theorems in C-MS using the OWC. In this paper, we obtained a unique common fixed point theorem for pair of OWC-mappings in C-MS and also given one example for supporting of the results.

2. Preliminaries

We need some of useful Definitions and Lemma's for our main results, they are in [7].

Definition 2.1. Let a real Banach space S. And subset Q of S is called a cone iff

(d1) Q is non –empty and closed and $Q \neq \{0\}$;
(d2) $\alpha u + \beta v \in Q$, for, $\alpha, \beta \in \mathbb{R}$, $\alpha,\beta \geq 0$, $u, v \in Q$;
(d3) $Q \cap (-Q) = \{0\}$.

A cone $Q \subset S$, and define a partial ordering '$\leq$' w. r. to Q by $\alpha \leq \beta$ iff $\beta - \alpha \in Q$. A cone Q is said to be a normal if there exists a number $L > 0$ such that for all $\alpha, \beta \in S$,
" $0 \leq \alpha \leq \beta$ implies that $\|\alpha\| \leq L \|\beta\|$".

Then the least + ve number satisfying the above inequality is said to be a normal constant of Q, while $\alpha \ll \beta$ stands for $\beta - \alpha \in$ interior of Q.

Definition 2.2. Suppose that "X" be a nonempty set of "S". And let the map $\rho: X \times X \to S$ satisfying

(i). $0 \leq \rho(x, y)$ for all $x, y \in X$ and
$\rho(x, y) = 0$ iff $x = y$;
(ii). $\rho(x, y) = \rho(y, x)$ for all $x, y \in X$;
(iii). $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$, for all $x, y, z \in X$.
Then ρ said to be a cone metric on "X" and (X, ρ) is said to be a cone metric space.

Definition 2.3. Let (X, ρ) be a cone metric space.

(i) If $\{x_n\}$ is said to be a convergent sequence if for any $b \gg 0$, there exists a natural number N such that $\rho(x_n, x) \ll b$, for all $n > N$ and for some fixed x in X. We denote this $x_n \to x$, as $n \to \infty$.
(ii) If $\{x_n\}$ is said to be a Cauchy sequence if for every b in S with $b \gg 0$, there exists a natural number N such that $\rho(x_n, x_m) \ll b$, for all $n, m > N$.

Definition 2.4. A cone metric space (X, ρ) is said to be complete if every Cauchy sequence is convergent in it.

Definition 2.5 [8]. Let A and B be self-mappings of a set X. If $u = Ax = Bx$ for some x in X, then x is said to be a coincidence point of A and B, and u is called a point of coincidence of A and B.

Proposition 2.1. Let A and B be OWC-mappings of a set X iff there is a point x in X which is coincidence point of A and B at which A and B are commute.

Lemma 2.1. Let X be a set, A, B are OWC-mappings of X. If A and B have a unique point of coincidence $u = Ax = Bx$, then u is the unique common fixed point of A and B.

3. Main Results

Now we are proving our main result theorem.

Note: Suppose that $\Phi: \mathbb{R}^{+} \to \mathbb{R}^{+}$ is a function satisfying the condition $\Phi(t) < t$, for each $t > 0$.

Theorem3.1. Let (X, ρ) be a C-MS and "S" be a normal cone. And suppose p and q are two self- mappings of X and satisfying the following conditions"

(i) $\rho(px, py) \leq \Phi(\max\{\rho(qx, qy), \rho(qx, py) + \rho(qy, px)/2, \rho(qy, py)\})$,

for all x, y ϵ X.

(ii) p and q are OWC.

Then, p and q are having a unique common fixed point in X.

Proof. By the condition (ii) p and q are OWC, then there exists a point α ϵ X such that pα = qα, pqα = qpα.

Claim: "pα" is a unique common fixed point of p and q. Now we assert that "pα" is a fixed point of p. For if

ppα ≠ pα, then form (i) we get that

$$\begin{aligned}\rho(p\alpha, pp\alpha) &\leq \Phi(\max\{\rho(q\alpha, qp\alpha), \rho(q\alpha, pp\alpha) + \rho(qp\alpha, p\alpha)/2, \rho(qp\alpha, pp\alpha)\}) \\ &\leq \Phi(\max\{\rho(p\alpha, pq\alpha), \rho(p\alpha, pp\alpha) + \rho(pq\alpha, p\alpha)/2, \rho(pq\alpha, pp\alpha)\}) \\ &\leq \Phi(\max\{\rho(p\alpha, pp\alpha), \rho(p\alpha, pp\alpha) + \rho(pp\alpha, p\alpha)/2, \rho(pp\alpha, pp\alpha)\}) \\ &\leq \Phi(\max\{\rho(p\alpha, pp\alpha), 2\rho(p\alpha, pp\alpha)/2, 0\}) \\ &\leq \Phi(\max\{\rho(p\alpha, pp\alpha), \rho(p\alpha, pp\alpha)\}) \\ &< \rho(p\alpha, pp\alpha), \text{ which is a contradiction.}\end{aligned}$$

Hence ppα = pα and ppα = pqα = qpα = pα. Thus pα is a common fixed point of p and q.

Uniqueness: Suppose that α, β ϵ X such that pα = qα = α and pβ = qβ = β and α ≠ β, then by (i) we get that

$$\begin{aligned}\rho(\alpha, \beta) &= \rho(p\alpha, p\beta) \\ &\leq \Phi(\max\{\rho(q\alpha, q\beta), \rho(q\alpha, p\beta) + \rho(q\beta, p\alpha)/2, \rho(q\beta, p\beta)\}) \\ &\leq \Phi(\max\{\rho(\alpha, \beta), \rho(\alpha, \beta) + \rho(\beta, \alpha)/2, \rho(\beta, \beta)\}) \\ &\leq \Phi(\max\{\rho(\alpha, \beta), 2\rho(\alpha, \beta)/2, 0)\}) \\ &\leq \Phi(\max\{\rho(\alpha, \beta), \rho(\alpha, \beta)\}) < \rho(\alpha, \beta),\end{aligned}$$

which is a contradiction.

Therefore α = β. Therefore p and q having a unique common fixe point. Hence proved.

Example 3.2. Let $B = \mathbb{R}^2$, $S = \{(\alpha, \beta) \epsilon B / \alpha, \beta \geq 0 \square \mathbb{R}^2\}$ and define $\rho: \mathbb{R} \times \mathbb{R} \to B$ by

$\rho(\alpha, \beta) = (|\alpha - \beta|, \lambda|\alpha - \beta|)$, where $\lambda > 0$is a constant.

Define p, q: X → X by, $p(\alpha) = \frac{1+4\alpha}{5}$ and $q(\alpha) = \frac{1+6\alpha}{7}$, α ϵ X. And clearly (X, ρ) is a C- MS, p and q are OWC and p, q satisfy the condition (i) and also 1 is the unique common fixed point of p and q.

4. Conclusion

In this research article we obtained some results, these results are more general and improved than the results of [5].

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