

Assessing the Effectiveness of Flipped Learning for Teaching Mathematics to Management Students

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Received February 07, 2023; Revised March 14, 2023; Accepted March 27, 2023

Abstract The present work focuses on a classroom application for evaluating the effectiveness of the Flipped Learning methodology for teaching mathematics to management students. Using linguistic (qualitative) grades, the assessment of the mean student performance is realized with the help of grey numbers and the assessment of their quality performance by calculating the Grade Point Average (GPA) index. A neutrosophic assessment method is also applied for evaluating the overall student performance, because the instructor had doubts about the accuracy of the grades assigned to some students.

Keywords: *flipped learning, fuzzy assessment methods, grey numbers (GNs), GPA index, neutrosophic sets (NSs), neutrosophic triplets*

Cite This Article: Michael. Gr. Voskoglou, “Assessing the Effectiveness of Flipped Learning for Teaching Mathematics to Management Students.” *American Journal of Applied Mathematics and Statistics*, vol. 11, no. 1 (2023): 30-34. doi: 10.12691/ajams-11-1-5.

1. Introduction

Computers have become nowadays a valuable tool for Education. The animation of figures and representations, achieved by using the proper software, develop the students' imagination and enhance their problem-solving skills. In the forthcoming era of the Fourth Industrial Revolution computers will provide, through the advanced Internet of Things (IoT), a wealth of information for students and teachers.

Several didactic methods have been already developed in which computers play a dominant role. One of them is the *Flipped Learning (FL)* methodology, which has its roots to the work of Lage, Piatt and Tregla [1].

The present paper focuses on a classroom application for the assessment, using qualitative (linguistic) grades, of the effectiveness of the FL approach for teaching mathematics to management students.

The rest of the paper is formulated as follows: Section 2 is devoted to a brief presentation of the basic principles of the FL approach. The necessary information about fuzzy sets, neutrosophic sets and grey numbers, needed for the purposes of this work, is presented in Section 3. The fuzzy methods used for the student assessment are developed in Section 4 and the classroom application is presented in Section 5. The paper closes with the final conclusions and some hints for future research, which are included in the last Section 6.

2. The Flipped Learning Methodology

FL is a mixed process that involves both online and face-to-face teaching and requires turning around the daily didactic processes to which we are accustomed. In fact, the acquisition of the new knowledge from the students is done outside the classroom, through the use of digital platforms and technological tools that the teacher or other specialists have developed. Sams and Bergmann [2] developed an online audiovisual teaching material so that students could do it regardless of factors such as place and time. On the contrary, what is traditionally undertaken as homework is done in class with the supervision of the teacher in order to favor the adequacy of learning and autonomy of students and increase the time devoted to practice, problem solving and deepening of content [3].

FL is highly based on *social constructivism* [4,5], a learning theory which is combination of the principles of constructivism [6] with the Vygotsky's social development theory [7]. The constructivist view involves two principles: First, that knowledge is actively constructed by the learner, not passively received from the environment, and second the importance of the “*coming to know*”, which is a process of adaptation based on and constantly modified by the learner's experience of the world. Further, according to the socio-cultural ideas learning takes place within some socio-cultural setting. Shared meanings are formed through negotiation in the learning environment, leading to the development of common knowledge [8,9].

3. Mathematical Background

3.1. Fuzzy Sets and Logic

The development of human science and civilization owes a lot to Aristotle's (384-322 BC) *bivalent logic (BL)*, which was in the center of human reasoning for centuries. BL is based on the "*Principle of the Excluded Middle*", according to which each proposition is either true or false.

Opposite views, however, appeared also early in the human history supporting the existence of a third area between true and false, where these two notions can exist together; e.g. by Buddha Siddhartha Gautama (India, around 500 BC), by Plato (427-377 BC), more recently by the Marxist philosophers, etc. Integrated propositions of multi-valued logics reported, however, only during the early 1900's, mainly by Lukasiewicz and Tarski [[10], Section 2]. According to the Lukasiewicz's "*Principle of Valence*" propositions are not only either true or false, but they may have intermediate truth-values too.

Zadeh, replacing the characteristic function of a crisp subset of the universe U with the *membership function* $m: U \rightarrow [0, 1]$, introduced in 1965 the concept of *fuzzy set (FS)* [11], in which each element x of U has a membership degree $m(x)$ in the unit interval. The closer $m(x)$ to 1, the better x satisfies the characteristic property of the corresponding FS. For example, if A is the FS of the tall men of a country and $m(x) = 0.8$, then x is a rather tall man. On the contrary, if $m(x) = 0.4$, then x is a rather short man. Formally, a FS A in U can be written as a set of ordered pairs in the form

$$F = \{(x, m(x)) : x \in U\} \quad (1)$$

Zadeh also introduced, with the help of FS, the infinite-valued in the unit interval *fuzzy logic (FL)* [12], on the purpose of dealing with the existing in the everyday life partial truths. FL, in which truth values are modelled by numbers in the unit interval, embodies the Lukasiewicz's "*Principle of Valence*".

Uncertainty can be defined as the shortage of precise knowledge or complete information on the data that describe the state of a situation. It was only in a second moment that FS theory and FL were used to embrace uncertainty modelling. This happened when membership functions were reinterpreted as possibility distributions [13,14]. Zadeh [13] articulated the relationship between possibility and probability, noticing that what is probable must preliminarily be possible.

Probability theory used to be for many years the unique tool in hands of the specialists for dealing with problems connected to uncertainty. Probability, however, was proved to be suitable only for tackling the cases of uncertainty which are due to *randomness* [15]. Randomness characterizes events with known outcomes which, however, cannot be predicted in advance, e.g. the games of chance. FSs, apart from randomness, tackle also successfully the uncertainty due to *vagueness*, which is created when one is unable to distinguish between two properties, such as "a good player" and "a mediocre player". For general facts on FSs and the connected to them uncertainty we refer to the book [16].

3.2. Neutrosophic Sets

Several generalizations and extensions of the theory of FSs have been developed during the last years for the purpose of tackling more effectively all the forms of the existing in real world uncertainty. The most important among them are briefly reviewed in [17].

Atanassov in 1986, considered, in addition to Zadeh's membership degree, the degree of *non-membership* and extended FS to the notion of *intuitionistic FS (IFS)* [20]. Smarandache in 1995, inspired by the frequently appearing in real life neutralities - like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc. - generalized IFS to the concept of *neutrosophic set (NS)* by adding the degree of *indeterminacy* or *neutrality* [18]. The word "neutrosophy" is a synthesis of the word "neutral" and the Greek word "sophia" (wisdom) and means "the knowledge of the neutral thought". The simplest form of a NS is defined as follows:

Definition 1: A *single valued NS (SVNS)* A in the universe U is of the form

$$A = \left\{ \begin{array}{l} (x, T(x), I(x), F(x)) : \\ x \in U, T(x), I(x), F(x) \in [0, 1], \\ 0 \leq T(x) + I(x) + F(x) \leq 3 \end{array} \right\} \quad (2)$$

In equation (2) $T(x)$, $I(x)$, $F(x)$ are the degrees of *truth* (or membership), *indeterminacy* (or neutrality) and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x . For simplicity, we write $A \langle T, I, F \rangle$. Indeterminacy is defined to be in general everything that exists between the opposites of truth and falsity [20].

Example 1: Let U be the set of the players of a soccer club and let A be the SVNS of the good players of the club. Then each player x is characterized by a *neutrosophic triplet* (t, i, f) with respect to A , with t, i, f in $[0, 1]$. For example, $x(0.7, 0.1, 0.4) \in A$ means that there exists a 70% belief that x is a good player, but at the same time there exist a 10% doubt about it and a 40% belief that x is not a good player. In particular, $x(0, 1, 0) \in A$ means that we do not know absolutely nothing about the quality of player x (new player).

If the sum $T(x) + I(x) + F(x) < 1$, then it leaves room for incomplete information about x , if it is equal to 1 for complete information and if it is > 1 for *inconsistent* (i.e. contradiction tolerant) information about x . A SVNS may contain simultaneously elements leaving room to all the previous types of information. All notions and operations defined on FSs are naturally extended to SVNSs [21].

Summation of neutrosophic triplets is equivalent to the union of NSs. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in many ways, equivalently to the known in the literature neutrosophic union operators [22]. For the needs of the present work, writing the elements of a SVNS A in the form of neutrosophic triplets and considering them simply as ordered triplets we define addition and scalar product as follows:

Definition 2: Let $(t_1, i_1, f_1), (t_2, i_2, f_2)$ be in A and let k be a positive number. Then:

- The *sum*

$$(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1 + t_2, i_1 + i_2, f_1 + f_2) \quad (3)$$

- The *scalar product*

$$k(t_1, i_1, f_1) = (kt_1, ki_1, kf_1) \quad (4)$$

Remark 1: Summation and scalar product of the elements of a SVNS A with respect to Definition 2 need not be closed operations in A, since it may happen that $(t_1 + t_2) + (i_1 + i_2) + (f_1 + f_2) > 3$ or $kt_1 + ki_1 + kf_1 > 3$. With the help of Definition 2, however, one can define in A the *mean value* of a finite number of elements of A as follows:

Definition 3: Let A be a SVNS and let $(t_1, i_1, f_1), (t_2, i_2, f_2), \dots, (t_k, i_k, f_k)$ be a finite number of elements of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, $i = 1, 2, \dots, k$. Set $n = n_1 + n_2 + \dots + n_k$. Then the *mean value* of all these elements of A is defined to be the element (t_m, i_m, f_m) of A calculated by

$$\frac{1}{n} [n_1(t_1, i_1, f_1) + n_2(t_2, i_2, f_2) + \dots + n_k(t_k, i_k, f_k)] \quad (5)$$

3.3. Grey Numbers

The theory of *grey systems* [23] introduces an alternative way for managing the uncertainty in case of approximate data. A grey system is understood to be any system which lacks information, such as structure message, operation mechanism or/and behavior document.

Closed real intervals are used for performing the necessary calculations in grey systems. In fact, a closed real interval $[x, y]$ could be considered as representing a real number T, termed as a GN, whose exact value in $[x, y]$ is unknown. We write then $T \in [x, y]$. A GN T, however, is frequently accompanied by a *whitenization function* $f: [x, y] \rightarrow [0, 1]$, such that, if $f(a)$ approaches 1, then a in $[x, y]$ approaches the unknown value of T. If no whitenization function is defined, it is logical to consider as a representative crisp approximation of the GN T the real number

$$V(T) = \frac{x+y}{2} \quad (6)$$

The arithmetic operations on GNs are introduced with the help of the known arithmetic of the real intervals [24]. In this work we are going to make use only of the addition of GNs and of the scalar multiplication of a GN with a positive number, which are defined as follows:

Definition 4: Let $A \in [x_1, y_1], B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: $A+B$ is the GN

$$A + B \in [x_1 + y_1, x_2 + y_2] \quad (7)$$

- The scalar product kA is the GN

$$kA \in [kx_1, ky_1] \quad (8)$$

4. Fuzzy Assessment Methods with Qualitative Grades

In many cases it is a common practice to assess the student performance by using qualitative (linguistic)

instead of numerical grades. A widely accepted scale of such grades is the following: A=excellent, B=very good, C=good, D=mediocre and F=unsatisfactory. Here we present a series of fuzzy assessment methods that we are going to use in this work for assessing the overall performance of a student group (see also [10,25]).

4.1. Mean Performance

In the case of using qualitative grades the mean performance of a student group cannot be evaluated with the classical method of calculating the mean value of the student individual grades. To overcome this difficulty, we assign to each grade a GN, denoted for simplicity with the same letter, as follows: $A = [85, 100], B = [75, 84], C = [60, 74], D = [50, 59], F = [0, 49]$. The choice of the above GNs, although it corresponds to generally accepted standards, is not unique. For example, for a more strict assessment, one may choose $A = [90, 100], B = [80, 89], C = [70, 79], D = [60, 69], F = [0, 59]$, or otherwise. Such changes, however, does not affect the generality of our method.

Assume now that, from the n in total students of the group, n_X obtained the grade $X=A, B, C, D, F$. It is logical then to accept that the crisp approximation $V(M)$ of the GN

$$M = \frac{1}{n}(n_A A + n_B B + n_C C + n_D D + n_F F) \quad (9)$$

can be used for estimating the mean performance of the student group.

4.2. Quality Performance

A very popular in USA and other countries method for evaluating the quality performance of a group is the use of the *Grade Point Average (GPA)* index [26], p.125], which is calculated by the formula

$$GPA = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} \quad (10)$$

In other words, the GPA index is a weighted average in which greater coefficients (weights) are assigned to the higher grades. Note that, since in the worst case ($n=n_F$) is $GPA=0$ and in the ideal case ($n=n_A$) is $GPA=4$, we have in general that $0 \leq GPA \leq 4$ (7).

When two groups have the same GPA index, however, this method is not sufficient to show which of them performs better. In such cases the *Rectangular Fuzzy Assessment Model (RFAM)*, which is based on the *Center of Gravity (COG) defuzzification technique* can be used [25], pp. 126-130].

4.3. Neutrosophic Assessment

Frequently in practice the teacher has doubts about the grades assigned to some students, either because he/she had not the opportunity to evaluate their skills explicitly during a course, or because they didn't clarify their answers properly in a written test. In such cases, the most suitable method for assessing the overall performance of a student group is to use NSs as tools. In our case, considering the NS of the good students of the group, we

introduce neutrosophic triplets characterizing the individual performance of each student and we calculate the mean value of all these triplets with the help of equation (4) for obtaining the proper conclusions about the group's overall performance. In order to have complete information for each student's performance, the sum of the component of each triplet must be equal to 1.

5. The Classroom Application

The target of the following classroom application was to evaluate the effectiveness of the FL methodology for teaching mathematics to management students. The subjects were the first term students of two departments of the School of Management and Economics of the University of Patras during the teaching of the course "Higher Mathematics I", which includes Complex Numbers, Differential and Integral Calculus in one variable and elements from Linear Algebra. According to the grades obtained in the PanHellenic examination for entrance in Higher Education, the potential of the two departments in mathematics was about the same. The course's instructor was also the same person, but the teaching methods followed were different. Namely, the FL methodology was applied for teaching the course to the 60 students of the first department (experimental group), whereas the classical method with lectures on the board was applied for the 60 students of the second department (control group).

The results of the final examination, after the end of the course, were the following:

- Department I: A: 9 students, B: 15, C: 18, D: 12, F: 6
- Department II: A: 12, B: 15, C: 9, D: 12, F: 12

Therefore, applying the assessment methods of Section 4, we evaluated the performance of the two departments as follows:

Mean performance

By equation (9) one finds that

$$M_I = \frac{1}{60} \left(9[85,100] + 15[75,84] + 18[60,74] + 12[50,59] + 6[0,49] \right) = \frac{1}{60} [3570, 4994] \approx [59.5, 83.23].$$

Therefore, equation (6) gives that $V(M_I) \approx 71.36$, which shows that the experimental group demonstrated a good (C) mean performance.

In the same way one finds that $V(M_{II}) \approx 62.56$, which shows that the control group also demonstrated a good (C) mean performance, which, however, was 8.8% worse than that of the experimental group.

Quality performance

Equation (10) gives that

$$GPA_I = \frac{12+2*18+3*15+4*9}{60} = 2.12$$

and similarly $GPA_{II} = 2.05$, which shows that the experimental group demonstrated a slightly better quality performance. In fact, with the help of equation (7) it is easy to check that the superiority of the experimental group in this case is only $0.07*25 = 1.75\%$.

Note that, some of the student answers in the final examination were not clearly presented or well justified. As a result, the instructor was not quite sure for the accuracy of the grades assigned to them. For this reason, we decided to apply the neutrosophic method of section 4.3 too for the assessment of the two departments' overall performance. For this, starting from the students with the higher grades, let us denote by $S_i, i=1,2,\dots,60$, the students of each department. Considering the NS of the good students, we assigned neutrosophic triplets to all students of the two departments as follows:

- Department I: $S_1-S_{32}: (1,0,0), S_{33}-S_{38}: (0.8,0.1,0.1), S_{39}-S_{42}: (0.7,0.2,0.1), S_{43}-S_{46}: (0.4,0.2,0.4), S_{47}-S_{50}: (0.3,0.2,0.5), S_{51}-S_{53}: (0.2,0.2,0.6), S_{54}-S_{55}: (0.1,0.2,0.7), S_{56}-S_{57}: (0.0,2,0.0.8), S_{578}-S_{60}: (0,0,1).$
- Department II: $S_1-S_{31}: (1,0,0), S_{32}-S_{35}: (0.8,0.1,0.1), S_{36}: (0.7,0.1,0.2), S_{35}-S_{43}: (0.4,0.1,0.5), S_{44}-S_{46}: (0.3,0.2,0.5), S_{47}-S_{50}: (0.2,0.2,0.6), S_{51}-S_{52}: (0.1,0.2,0.7), S_{53}-S_{58}: (0,0.3,.0.7), S_{59}-S_{60}: (0,0,1).$

Then, by equation (5), the mean value of the neutrosophic triplets of Department I is equal to $\frac{1}{60} [32(1,0,0) + 6 (0.8,0.1,0.1)+ 4(0.7,0.2,0.1)+ 4(0.4,0.2,0.4) +4(0.3,0.2,0.5)+3(0.2,0.2,0.6)+2(0.1,0.2,0.7) + 2(0,0.2,0.0.8) +3 (0,0,1)] \approx (0.72, 0.07, 0.21)$. In the same way one finds that the mean value of the neutrosophic triplets of Department II is equal to $(0.65, 0.08, 0.27)$.

Thus, the probability for a random student of Department I to be a good student is 72%, but at the same time there exists a 7% doubt about it and a 21% probability to be not a good student. Also, the probability for a random student of Department II to be a good student is 65%, with a 8% doubt about it and a 27% probability to be not a good student. Consequently, the experimental group, despite the doubts of the instructor for the accuracy of the grades assigned to the students, demonstrated a better overall performance.

6. Conclusion

The classroom application presented in this work demonstrated a superiority of the experimental group with respect to the control group. This superiority was significant concerning the two groups' mean and overall (in terms of the neutrosophic method) performance, but rather negligible concerning their quality performance. This gives a strong indication that the application of the FL methodology benefits more the mediocre and the weak in mathematics students, but less the good students. Much more experimental research is needed, however, for obtaining safer conclusions.

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