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A Unique Common Fixed Point Theorem for a Metric Space with the Property (E.A)

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Abstract In this paper, we prove a unique common fixed point theorem for metric space with (E.A) property. Our result is a generalization and improvement of some recent results existing in the literature [1].

Keywords: *fixed point, common fixed point, property (E. A), weakly compatible mappings*

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1. Introduction

Banach contraction [2] was the result of fixed point theorem for the contractive type mapping and which is the most celebrated fixed point theorem. Since then, a lot of fixed point theorems with applications have been studied under different types of contractive type conditions (see, e.g., [3-8]). Before 1968, all results in fixed points used the Banach contraction principle. In 1968 Kannan [9] has obtained a fixed point theorem for a mapping satisfying a contractive condition which did not require continuity at each point. After that many mathematicians (see, e.g. [1,3,4,5,9,10,11]) were inspired with this result and obtained fixed point results and they have extended the Kannan [9] fixed point results in metric space for using different types of contractive conditions. And recently, M. Aamri and D. El. Moutawakil [1] have proved some fixed point theorems under strict contractive conditions in metric space and also they used the property (E.A) and weakly compatible mappings in their results in metric space. In this paper we obtained a result for metric space with property (E. A), which is a generalization and extension of the results of M. Aamri and D. El. Moutawakil [1].

The following definitions are due to [1] which are useful in our main result.

Definition 1.1: Two self mappings P and Q of metric space X are said to be weakly compatible if they commute at their coincidence points ,that is, If Qx = Sx for some x∈X, the PQx = QPx.

Definition 1.2: Let f and g be self-maps on a set X. If z = px = qx, for some x∈X, then x is called a coincidence point of p and q. z is called a point of coincidence of p and q.

Definition 1.3: Let P and Q be two self mappings of a metric space (X, d), we say that P and Q satisfy the (E.A) property if there exists a sequence (x_n) such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$ for some z∈X.

2. Main Result

In this section we have generalized and improved the results of [1].

Theorem 2.1: P and Q be two weakly compatible mappings of a metric space (X, d) such that

i) P and Q satisfy the property (E.A)

ii)

$$\begin{aligned} & d(Px, Py) \\ & \leq ad(Qx,Qy)+b\max\{(d(Px,Qx),d(Py,Qy)\} \\ & \quad +c\max\{(d(Py,Qx),d(Px,Qy)\} \end{aligned}$$

for a+ b+ c <1.

iii) $PX \subseteq QX$.

If PX or QX is a complete subspace of X, then P and Q have a unique common fixed point.

Proof: Since P and Q satisfy the property (E. A) there exists in X a sequence $\{x_n\}$ satisfying $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Q x_n = t$, for some t∈ X. Suppose QX is complete, then $\lim_{n\to\infty} Qx_n = Qa$ for some a ∈ X and also $\lim_{n\to\infty} Px_n = Qa$. Now to show that Pa = Qa. For suppose Pa ≠ Qa, by (ii) implies

$$\begin{aligned} & d(Px_n, Pa) \\ & \leq a\, d(Qx_n,Qa)+b\max\{(d(Px_n,Qx_n),d(Pa,Qa)\} \\ & \quad +c\max\{(d(Pa,Qx_n),d(Px_n,Qa)\}, \\ & \Rightarrow d(Qa,Pa) \\ & \quad \leq a\, d(Qa,Qa)+b\max\{(d(Qa,Qa),d(Pa,Qa)\} \\ & \quad +c\max\{(d(Pa,Qa),d(Qa,Qa)\}, \\ & \leq b\, d(Pa,Qa)\}+c\, d(Pa,Qa), \\ & =(b+c)d(Pa,Qa)<d(Pa,Qa), \end{aligned}$$

since, b+c <1, which is a contradiction.

Therefore, Pa = Qa.

Since, P and Q are weakly compatible, PQa = QPa and, therefore, PPa = PQa = QQa.

To show Pa is a common fixed point of P and Q. Suppose that Pa ≠ PPa.

$$\begin{aligned}
& d(Pa,\ PPa) \\
& \leq a\ d(Qa, QPa) + b\ max\{(d(Pa, Qa), d(PPa,\ QPa)\} \\
& +c\ max\{(d(PPa, Qa), d(Pa, QPa)\}, \\
& \leq ad(Pa,\ PPa) + cd(PPa, Pa), \\
& = (a\ +\ c)d(PPa,\ Pa) < d(PPa,\ Pa),
\end{aligned}$$

since (a + c) < 1, which is a contradiction.

Hence, PPa = Pa and QPa = PPa = Pa, Pa is a common fixed point of P and Q.

Finally to show that Pa is a unique common fixed point of P and Q.

Suppose Pa is a common fixed point of P and Q.

$$\begin{aligned}
& d(Pa,\ Pa) \\
& \leq a\ d(Qa, Qa) + b\ max\{(d(Pa, Qa), d(Pa, Qa)\} \\
& +c\ max\{(d(Pa,\ Qa), d(Pa, Qa)\}, \\
& \leq a\ d(Pa, Pa) + b\ max\{(d(Pa,\ Pa), d(Pa,\ Pa)\} \\
& +c\ max\{(d(Pa, Pa), d(Pa, Pa)\}, \\
& = (a+b+c)d(Pa, Pa) < d(Pa, Pa),
\end{aligned}$$

since (a + b + c) < 1, which is a contradiction.

Therefore, Pa is a unique common fixed point of P and Q. This completes the proof of the theorem.

3. Conclusion

In this paper we have generalized and improved the results of [1], which are more general than the results of [1].

Conflict of Interest

Author declared there is no fonflict of interest

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