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Some Fixed Point Results in Extended Cone S_b - Metric Space

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Abstract In this paper, we introduce a notion of extended Cone S_b-metric space and prove some fixed point results with various types of contractive conditions. Our results enlarge many results in the literature.

Keywords: *cone metric space, extended S_b - metric space, fixed point*

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1. Introduction and Preliminaries

In 2007, Huang and Zhang [1] introduced the idea of cone metric space, which is a generalization of metric space by replacing the real numbers by ordering Banach space. Consequently, several originators consider the development of cone metric space for mappings that satisfying different contractive conditions [2-14].

The concept of S – metric space was initiated by Sedghi et. al. [15] in 2012, which is distinct from other spaces and established some fixed point results in S - metric space. Many authors enlarged the idea of S - metric space and obtained some fixed point theorems in various contractive conditions [16-22].

A notion of S_b-metric space was initiated by Souayah and Mlaiki [23] in 2016. Dhamodharan and krishnakumar [24] expanded the idea of S - metric space to cone S-metric space in 2017 and established various fixed point results. Several authors developed the idea of cone S-metric space in fixed point theory. [1,23-33].

The concept of cone S_b-metric space was initiated by Singh and Singh [34] in 2018 and obtained some fixed point results. Nabil Mlaiki [31] introduced the concept of extended S_b-metric space and proved some fixed point theorems for mappings satisfying the different contractive conditions [34,35,36,37].

In this paper, we introduce the notion of extended cone S_b-metric space which is a generalization of cone S_b - metric space and prove some fixed point theorems in extended cone S_b - metric space.

Definition 1.1. [15] Let X be a nonempty set and a function $\Gamma: X^3 \to [0, \infty)$ satisfies the following conditions.

1. $\Gamma(v_1, v_2, v_3) \geq 0$.
2. $\Gamma(v_1, v_2, v_3) = 0$ if and only if $v_1 = v = v_3$,
3. $\Gamma(v_1,v_2,v_3) \leq \Gamma(v_1,v_1,t)+\Gamma(v_2,v_2,t)+\Gamma(v_3,v_3,t)$ for all $v_1,v_2,v_3,t \in X$.

Then Γ is called S- metric on X and the pair (X, Γ) is called an S-metric space.

Example 1.1. [15] Let X be a non-empty set and the metric d on X. Then

$$\Gamma(v_1, v_2, v_3) = d(v_1, v_3) + d(v_2, v_3)$$

is an S-metric on X.

Definition 1.2. [23] Let X be a nonempty set and let $b \geq 1$ be real number. Define a function $\Gamma_b : X_3 \to [0,\infty)$ is called an S_b-metric if it is satisfies the following conditions.

1. $\Gamma_b(v_1, v_2, v_3) = 0$ iff $v_1 = v_2 = v_3$,
2. $\Gamma_b(v_1,v_1,v_2) = \Gamma_b(v_2, v_2, v_1)$ for all $v_1, v_2 \in X$.
3. $\Gamma_b(v_1, v_2, v_3) \leq b\,(\Gamma_b(v_1, v_1, t) + \Gamma_b(v_2, v_2, t) + \Gamma_b(v_3, v_3, t))$

Then the pair (X, Γ_b) is called S_b-metric space.

Definition 1.3. [31] Let X be a nonempty set and $\zeta: X^3 \to [1,\infty)$. A function $\Gamma_\zeta : X^3 \to [0,\infty)$ satisfies the following conditions.

(i) $\Gamma_\zeta(v_1,v_2,v_3) = 0$ if and only if $v_1=v_2=v_3$,
(ii) $\Gamma_\zeta(v_1,v_2,v_3) \leq \zeta(v_1,v_2,v_3)(\Gamma_\zeta(v_1,v_1,t) + \Gamma_\zeta(v_2,v_2,t) + \Gamma_\zeta(v_3,v_3,t))$

Then the pair (X, Γ_ζ) is called extended S_b– metric space.

Definition 1.4. [1] Let E be the real Banach space and M be a subset of E is called a cone if it is satisfies the following conditions.

1. M is closed and non-empty $M \neq 0$,
2. $pv_1 + qv_2 \in M$ for all $v_1, v_2 \in M$ and non-negative real numbers p, q.
3. $M \cap (-M) = 0$.

For a given cone $M \subset E$, define a partial ordering $\leq$ on E with respect to M by $v_1 \leq v_2$ if and only if

v_2 - v_1 ∈ M, while $v_1 \le v_2$ will stand for v_2 - v_1 ∈ int M (interior of M).

The cone M is called normal if there is a constant K > 0 such that for all $v_1, v_2 \in E$, $0 \le v_1 \le v_2$ implies $\|v_1\| \le K \|v_2\|$.

Then K is called the normal constant of M.

The cone M is called regular if every increasing sequence which is bounded from above is convergent.

Example 1.2. [1] Let E be the real vector space and K > 1 then,

$$E = \{pv_1 + q : p, q \in R;\ v_1 \in \left[1-\frac{1}{K}, 1\right]\}$$

with supermom norm and the cone M = {pv_1 + q ∈ E: p ≥ 0, q ≥ 0 } in E. The cone M is regular and normal.

Definition 1.5. [1] Let X be a non-empty set and Γ: X x X → E satisfies the following conditions.

1. $0 \le \Gamma(v_1, v_2)$ for all $v_1, v_2 \in X$ and $\Gamma(v_1, v_2) = 0$ if and only if $v_1 = v_2$.
2. $\Gamma(v_1, v_2) = \Gamma(v_2, v_1)$ for all $v_1, v_2 \in X$.
3. $\Gamma(v_1, v_2) \le \Gamma(v_1, v_3) + \Gamma(v_3, v_2)$

for all $v_1, v_2, v_3 \in X$. Then Γ is called a cone metric on X and (X, Γ) is called a cone metric space.

Definition 1.6. [24] Let M be a cone in E (real Banach space) with int M ≠ 0 and ≤ is a partial ordering with respect to M. Let X be a non-empty set and define a function Γ: X^3 → E, if Γ satisfies all the conditions,

1. $\Gamma(v_1, v_2, v_3) \ge 0$
2. $\Gamma(v_1, v_2, v_3) = 0$ if and only if $v_1 = v_2 = v_3$
3. $\Gamma(v_1,v_2,v_3) \le \Gamma(v_1,v_1,t)+\Gamma(v_2,v_2,t)+\Gamma(v_3,v_3,t)$ for all $v_1,v_2,v_3,t \in X$.

Then Γ is called a cone S-metric on X and (X, Γ) is called a cone S-metric space.

Example 1.3. [24] Let E=R^2, M = {(v_1, v_2) ∈ R^2: $v_1 \ge 0$, $v_2 \ge 0$}⊂ R^2, X=R and ȡ: X x X x X → E be the metric on X then Γ: X^3 → E defined by

$$\Gamma(v_1,v_2,v_3) = (ȡ(v_1,v_3)+ȡ(v_2,v_3), \alpha(ȡ(v_1,v_3)+ȡ(v_2,v_3)))$$

is a cone S-metric on X where α > 0 is a constant.

Definition 1.7. [34] Let X be a nonempty set and M be a cone in E(real Banach space) and define Γ_b : X^3 →E is satisfies the following conditions

1. $\Gamma_b(v_1,v_2,v_3) \ge 0$.
2. $\Gamma_b(v_1,v_2,v_3) = 0$ if and only if $v_1 = v_2 = v_3$.
3. $\Gamma_b(v_1, v_2,v_3) \le r[\Gamma_b(v_1,v_1,t)+\Gamma_b(v_2,v_2,t)+\Gamma_b(v_3,v_3,t)]$

for all $v_1, v_2, v_3, t \in X$, where r ≥1 is a constant then Γ_b is called a cone S_b- metric on X and (X, Γ_b) is called an cone S_b-metric space.

2. Main Result

In this section, we introduce an extended cone S_b- metric space and prove some fixed point results in extended cone S_b-metric space.

Definition 2.1. Let X be a non-empty set and ζ : X^3 → [1,∞) be a function. If Γ_ζ : X^3 → E (Real Banach Space) satisfies the following conditions.

1. $\Gamma_\zeta(v_1,v_2,v_3) \ge 0$.
2. $\Gamma_\zeta(v_1,v_2,v_3) = 0$ if and only if $v_1 = v_2 = v_3$.
3. $\Gamma_\zeta(v_1,v_2,v_3) \le \zeta(v_1,v_2,v_3)\big(\Gamma_\zeta(v_1,v_1,t) + \Gamma_\zeta(v_2,v_2,t) + \Gamma_\zeta(v_3,v_3,t)\big)$

for all $v_1,v_2,v_3,t \in X$.

Then (X, Γ_ζ) is called an extended cone S_b- metric space.

Remark 2.1. If $\zeta(v_1,v_2,v_3) = 1$, then the extended cone S_b-metric space reduces to a cone S- metric space.

Remark 2.2. If $\zeta(v_1,v_2,v_3) = b \ge 1$ then the extended cone S_b-metric space is said to be cone S_b -metric space.

Lemma 2.1. Let (X, Γ_ζ) be an extended cone S_b-metric space. Then we have $\Gamma_\zeta(v_1, v_1, v_2) = \Gamma_\zeta(v_2, v_2, v_1)$.

Definition 2.2. Let (X, Γ_ζ) be an extended cone S_b- metric space and M be a normal cone.

1) A sequence $\{v_n\} \in X$ converges to w if and only if w ∈ X such that $\Gamma_\zeta(v_n, v_n, w) \to 0$ as n →∞. we can write this $\lim_{n\to\infty} v_n = w$.
2) A sequence $\{v_n\}$ is said to be Cauchy sequence if and only if $\Gamma_\zeta(v_n,v_n,v_m) \to 0$ as n, m →∞.
3) If every Cauchy sequence $\{v_n\}$ converges to w ∈ X, then (X, Γ) is said to be a complete extended cone S_b- metric space.

Example 2.1. Let E = R^2 and M be a cone in E. Let X = [0,∞) define a function Γ_ζ : X^3 →E such that

$$\Gamma_\zeta(v_1,v_2,v_3) = \{[|v_1 - v_3| + |v_2 - v_3|]^2, \alpha[|v_1 - v_3| + |v_2 - v_3|]^2\},$$

where α > 0 is a constant and a function ζ : X^3 → [1,∞) by $\zeta(v_1,v_2,v_3) = \max\{v_1,v_2\} + v_3 + 1$ then (X, Γ_ζ) is a complete extended cone S_b - metric space

Theorem 2.1. Let (X, Γ_ζ) be a complete extended cone S_b- metric space and T be a self-mapping on X satisfying the following condition

$$\Gamma_\zeta(Tv_1,Tv_2,Tv_3) \le \left\{\begin{matrix} c_1\Gamma_\zeta(v_1,v_2,v_3)+c_2\Gamma_\zeta(v_1,Tv_1,Tv_1) \\ +c_3\Gamma_\zeta(v_2,Tv_2,Tv_2)+c_4\Gamma_\zeta(v_3,Tv_3,Tv_3) \end{matrix}\right\} \quad (1)$$

for all $v_1, v_2, v_3 \in X$ where $0 \le c_1 + c_2 + c_3 + c_4 < 1$ and $\lim_{n\to\infty} \zeta(T^n x, T^n x, T^m x) < \frac{1}{2b}$ for $0 \le b < \frac{1}{2}$, then T has a unique fixed point.

Proof. Let $v_0 \in X$, define a sequence $\{v_n\}$ by $T^n v_0 = v_n$ from (1)

$$\begin{aligned}
&\Gamma_\zeta(v_n,v_{n+1},v_{n+1}) \\
&= \Gamma_\zeta(Tv_{n-1},Tv_n,Tv_n) \\
&\le \left\{\begin{matrix} c_1\Gamma_\zeta(v_{n-1},v_n,v_n)+c_2\Gamma_\zeta(v_{n-1},Tv_{n-1},Tv_{n-1}) \\ +c_3\Gamma_\zeta(v_n,Tv_n,Tv_n)+c_4\Gamma_\zeta(v_n,Tv_n,Tv_n) \end{matrix}\right\} \\
&\le \left\{\begin{matrix} c_1\Gamma_\zeta(v_{n-1},v_n,v_n)+c_2\Gamma_\zeta(v_{n-1},v_n,v_n) \\ +c_3\Gamma_\zeta(v_n,v_{n+1},v_{n+1})+c_4\Gamma_\zeta(v_n,v_{n+1},v_{n+1}) \end{matrix}\right\} \\
&\le (c_1+c_2)\Gamma_\zeta(v_{n-1},v_n,v_n)+(c_3+c_4)\Gamma_\zeta(v_n,v_{n+1},v_{n+1})
\end{aligned}$$

$$\Gamma_\zeta(v_n,v_{n+1},v_{n+1})(1-c_3-c_4) \le (c_1+c_2)\Gamma_\zeta(v_{n-1},v_n,v_n)$$

$$\Gamma_\zeta(v_n,v_{n+1},v_{n+1}) \le \left(\frac{c_1+c_2}{1-c_3-c_4}\right)\Gamma_\zeta(v_{n-1},v_n,v_n)$$

$$\Gamma_\zeta\left(v_n, v_{n+1}, v_{n+1}\right) \le b\Gamma_\zeta\left(v_{n-1}, v_n, v_n\right)$$

where $b = \frac{(c_1 + c_2)}{(1 - c_3 - c_4)}$, $0 \le b < 1/2$ continue this process to obtain

$$\Gamma_\zeta\left(v_n, v_{n+1}, v_{n+1}\right) \le b^n \Gamma_\zeta\left(v_0, v_1, v_1\right)$$

for all m, n ∈ N and n < m. Hence by triangle inequality

$$\begin{aligned}
&\Gamma_\zeta\left(v_n, v_n, v_m\right)\\
&\le \zeta\left(v_n, v_n, v_m\right)(2b)^n \Gamma_\zeta\left(v_0, v_0, v_1\right)\\
&+\zeta\left(v_n, v_n, v_m\right)\zeta\left(v_{n+1}, v_{n+1}, v_m\right)(2b)^{n+1}\Gamma_\zeta\left(v_0, v_0, v_1\right)\\
&+\cdots\\
&+\zeta\left(v_n, v_n, v_m\right)\cdots\zeta\left(v_{m-1}, v_{m-1}, v_m\right)(2k)^{m-1}\Gamma_\zeta\left(v_0, v_0, v_1\right)\\
&\le \Gamma_\zeta\left(v_0, v_0, v_1\right)\\
&[\zeta\left(v_1, v_1, v_m\right)\zeta\left(v_2, v_2, v_m\right)\cdots\\
&\quad \zeta\left(v_{n-1}, v_{n-1}, v_m\right)\zeta\left(v_n, v_n, v_m\right)(2b)^n\\
&+\zeta\left(v_1, v_1, v_m\right)\zeta\left(v_2, v_2, v_m\right)\cdots\\
&\quad \zeta\left(v_n, v_n, v_m\right)\zeta\left(v_{n+1}, v_{n+1}, v_m\right)(2b)^{n+1}\\
&+\cdots+\zeta\left(v_1, v_1, v_m\right)\zeta\left(v_2, v_2, v_m\right)\cdots\\
&\qquad \zeta\left(v_{m-2}, v_{m-2}, v_m\right)\zeta\left(v_{m-1}, v_{m-1}, v_m\right)(2b)^{m-1}]
\end{aligned}$$

by the hypothesis of the theorem

$$\lim_{n\to\infty} \zeta(v_n, v_n, v_m)(2b) < 1$$

by Ratio test series

$$\sum_{n=1}^{\infty}(2b)^n \prod_{i=1}^{n}\zeta(v_i, v_i, v_m)$$

converges.

Let $A = \sum_{n=1}^{\infty}(2b)^n \prod_{i=1}^{n}\zeta(v_i, v_i, v_m)$ and $A_n = \sum_{j=1}^{n}(2b)^j \prod_{i=1}^{j}\zeta(v_i, v_i, v_m)$, for m > n, we have

$$\Gamma_\zeta(v_n, v_n, v_m) \le \Gamma_\zeta(v_0, v_0, v_1)\left[A_{m-1} - A\right]$$

Taking limit as n, m → ∞, the sequence $\{v_n\}$ is a Cauchy sequence. Since X is complete. $\{v_n\}$ converges to $\upsilon \in$ X.

By (1) and the triangle inequality,

$$\begin{aligned}
&\Gamma_\zeta(\upsilon, \upsilon, T\upsilon)\\
&\le \zeta(\upsilon, \upsilon, T\upsilon)[2\Gamma_\zeta(\upsilon, \upsilon, v_n) + \Gamma_\zeta(T\upsilon, T\upsilon, v_n)]\\
&\le \zeta(\upsilon, \upsilon, T\upsilon)[2\Gamma_\zeta(\upsilon, \upsilon, v_n) + k\Gamma_\zeta(\upsilon, \upsilon, v_{n-1})]
\end{aligned}$$

Taking limit as n → ∞,

$$\Gamma_\zeta(\upsilon, \upsilon, T\upsilon) = 0$$

that implies $T\upsilon = \upsilon$. Hence υ is a fixed point of T. To prove that uniqueness, assume that there exists $\upsilon \ne w \in$ X such that T υ = υ and T $w = w$.

Thus,

$$\begin{aligned}
&\Gamma_\zeta(w, \upsilon, \upsilon) = \Gamma_\zeta\left(Tw, T\upsilon, T\upsilon\right)\\
&\le c_1\Gamma_\zeta(w, \upsilon, \upsilon) + c_2\Gamma_\zeta(w, Tw, Tw)\\
&\quad +\left(c_3 + c_4\right)\Gamma_\zeta\left(\upsilon, T\upsilon, T\upsilon\right)\\
&\le b\Gamma_\zeta(w, \upsilon, \upsilon) < \Gamma_\zeta(w, \upsilon, \upsilon)
\end{aligned}$$

which is a contradiction. Therefore, T has a unique fixed point.

If $c_1 = c$ and $c_2 = c_3 = c_4 = 0$ in Theorem 2.1, then the following corollary is obtained.

Corollary 2.1. Let (X, Γ_ζ) be a complete extended cone S_b- metric space and T be a self-mapping on X satisfying the following condition

$$\Gamma_\zeta\left(Tv_1, Tv_2, Tv_3\right) \le c\Gamma_\zeta\left(v_1, v_2, v_3\right) \qquad (2)$$

For all v_1, v_2, v_3 ∈ X where $0 \le c < 1/2$ and $\lim_{n\to\infty} \zeta(T^n x, T^n x, T^m x) < 1/2c$, then T has a unique fixed point.

If $c_1 = 0$ and $c_2 = c_3 = c_4 = c$ in the Theorem 2.1, then the following corollary is obtained.

Corollary 2.2. Let (X, Γ_ζ) be a complete extended cone S_b-metric space and T: X→X satisfy the following conditions

$$\begin{aligned}
&\Gamma_\zeta\left(Tv_1, Tv_2, Tv_3\right)\\
&\le c(\Gamma_\zeta(v_1, Tv_1, Tv_1) + \Gamma_\zeta(v_2 Tv_2, Tv_2) + \Gamma_\zeta(v_3, Tv_3, Tv_3))
\end{aligned}$$

for all v_1, v_2, v_3 ∈ X where $0 \le c < 1/2$ and $\lim_{n\to\infty} \zeta(T^n x, T^n x, T^m x) < 1/2c$, then T has a unique fixed point.

Example 2.2. Let $E = R^2$ and M be a cone in E. Let X = [0, ∞) define a function $\Gamma_\zeta : X^3 \to E$ such that

$$\begin{aligned}
&\Gamma_\zeta\left(Tv_1, Tv_2, Tv_3\right)\\
&= \left\{\left(\left|v_1 - v_3\right| + \left|v_2 - v_3\right|^2,\ \alpha\left|v_1 - v_3\right| + \left|v_2 - v_3\right|^2\right)\right\}
\end{aligned}$$

where α > 0, is a constant and a function $\zeta: X^3 \to [1,\infty)$ defined by

$$\zeta\left(v_1, v_2, v_3\right) = max\left\{v_1, v_2\right\} + v_3 + 1$$

Then (X, Γ_ζ) is a complete extended cone S_b-metric space. Consider the mapping T: X → X defined by

$$Tv_1 = \frac{v_1}{2}$$

Then

$$\begin{aligned}
&\Gamma_\zeta\left(Tv_1, Tv_2, Tv_3\right)\\
&= \left\{\begin{array}{l}\left(\left|\frac{v_1}{2} - \frac{v_3}{2}\right| + \left|\frac{v_2}{2} - \frac{v_3}{2}\right|\right)^2,\\ \alpha\left(\left|\frac{v_1}{2} - \frac{v_3}{2}\right| + \left|\frac{v_2}{2} - \frac{v_3}{2}\right|\right)^2\end{array}\right\}\\
&\le \frac{1}{4}\Gamma_\zeta\left(v_1, v_2, v_3\right)
\end{aligned}$$

where $c \in [0, \frac{1}{2})$, thus T satisfies all the conditions of Corollary 2.1 and hence T has a unique fixed point.

3. Conclusion

Fixed point theory plays an essential role in all branches of Mathematics. In this paper, we introduced an extended cone S_b-metric space and proved some fixed results in various contractive conditions. Our results extends several results in existing literature.

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Conflict of Interest

There is no conflict of interest.

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