American Journal of Applied Mathematics and Statistics, 2022, Vol. 10, No. 1, 1-3
Available online at http://pubs.sciepub.com/ajams/10/1/1
Published by Science and Education Publishing
DOI:10.12691/ajams-10-1-1

Application of Soft Sets to Assessment Processes

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Received November 24, 2021; Revised December 29, 2021; Accepted January 09, 2022

Abstract From the time that Zadeh introduced the concept of fuzzy set in 1965 a lot of research has been carried out for generalizing and extending the corresponding theory on the purpose of tackling more effectively the existing in real life uncertainty. One such generalization is the concept of soft set aiming, among others, to overcome the existing difficulty of defining properly the membership function of a fuzzy set. A new model using soft sets is presented in this paper for assessing human-machine performance in a parametric manner and examples are given to illustrate its applicability in practice. Such kind of models are very useful when the assessment has qualitative rather than quantitative characteristics.

Keywords: *fuzzy sets, soft sets, fuzzy assessment methods*

Cite This Article: Michael Gr. Voskoglou, "Application of Soft Sets to Assessment Processes." *American Journal of Applied Mathematics and Statistics*, vol. 10, no. 1 (2022): 1-3. doi: 10.12691/ajams-10-1-1.

1. Introduction

Probability theory used to be until the middle of the 1960's the unique tool in hands of the experts for dealing with the existing in real life situations of uncertainty. Probability, however, based on the principles of the bivalent logic, has been proved sufficient for tackling problems of uncertainty connected only to randomness.

The *fuzzy set* theory, introduced by Zadeh in 1965 [1], and the connected to it *fuzzy logic* gave to scientists the opportunity to model under conditions of uncertainty that are vague or not precisely defined, thus succeeding to mathematically solve problems whose statements are expressed in our natural language. Since then a lot of research has been carried out for generalizing and extending the fuzzy set theory on the purpose of tackling more effectively the existing uncertainty in problems of science, technology and everyday life [2].

In 1999 Dmtri Molodstov, Professor of the Computing Center of the Russian Academy of Sciences in Moscow, in order to overcome the existing difficulty of defining properly the membership function of a fuzzy set, proposed the *soft sets* as a new mathematical tool for dealing with the uncertainty in a parametric manner [3]. The theory of soft sets has found many and important applications in several sectors of the human activity [4].

The purpose of this article is to present applications of soft sets to assessment problems. The rest of the article is formulated as follows: The concept of the soft set is defined in section 2 and its connection to fuzzy sets is described. The general assessment model using soft sets is presented in section 3 and examples are given in section 4 illustrating its applicability in practice. The article closes with the final conclusion and a hint for future research presented in section 5.

2. Soft Sets

Let U be the universal set of the discourse. It is recalled that a fuzzy set A on U is defined with the help of its *membership function* m: U → [0,1] as the set of the ordered pairs

$$A = \{(x,\ m(x)) : xU\} \qquad (1)$$

The real number m(x) is called the *membership degree* of x in A**.** The greater is m(x), the more x satisfies the characteristic property of *A*. Many authors, for reasons of simplicity, identify the fuzzy set A with its membership function m.

A crisp subset A of U can be considered as a fuzzy set on U with membership function taking the values m(x)=1 if x belongs to A and 0 otherwise. In other words, the concept of fuzzy set is an extension of the concept of the ordinary sets.

The infinite-valued on the interval [0,1] fuzzy logic is defined with the help of the concept of fuzzy set. Through fuzzy logic the fuzzy terminology is translated by algorithmic procedures into numerical values, operations are performed upon those values and the outcomes are returned into natural language statements in a reliable manner. For general facts on fuzzy sets, fuzzy logic and the connected to them uncertainty we refer to the chapters 4-7 of the book [5].

It is of worth noting that there is not any exact rule for defining the membership function of a fuzzy set. The methods used for this purpose are usually empirical or statistical and the definition is not unique depending on the personal goals of the observer. The only restriction about it is to be compatible to the common logic; otherwise the resulting fuzzy set does not give a reliable description of the corresponding real situation. For example, defining the fuzzy set of the young people of a country one could consider as young all those being less than 30 years old and another all those being less than 40 years old. As a result they assign different membership degrees to people with ages below those two upper bounds. The attempt to overcome this difficulty was one of the main reasons that led to the genesis of the concept of soft set.

Let E be a set of parameters, let A be a subset of E and let f be a mapping of A into the set Δ(U) of all subsets of U. Then the soft set F of U connected to A is defined as the set of the ordered pairs

$$F = \{(e, f(e) : e \in A\} \quad (2)$$

In other words, a soft set is a paramametrized family of subsets of U. For example, let U= $\{H_1, H_2, H_3\}$ be a set of houses and let E =$\{e_1, e_2, e_3\}$be the set of the parameters e_1=cheap, e_2=expensive and e_3=beautiful. Let us further assume that H_1, H_2 are the cheap and H_2, H_3 are the beautiful houses. Set A=$\{e_1, e_3\}$, then a mapping f:A → Δ(U) is defined by $f(e_1)=\{H_1, H_2\}$, $f(e_3)=\{H_2, H_3\}$. Therefore, the soft set F of U connected to A and representing the cheap and beautiful houses of U is the set of the ordered pairs

$$F = \{(e_1, \{H_1, H_2\}), (e_3, \{H_2, H_3\})\} \quad (3)$$

A fuzzy set on U with membership function y = m(x) is a soft set on U of the form (f, [0,1]), where f(α)={x ∈ U:m(x) ≥ α} is the corresponding α – cut of the fuzzy set, for each α in [0, 1]. The concept of soft set is, therefore, a generalization of the concept of fuzzy set. For general facts on soft sets we refer to [4].

3. The Assessment Model

Quality is a desirable characteristic of all human actions. This makes assessment one of the most important components of the processes connected to the realization of those actions. The present author has developed in earlier works several methods for assessing human-machine performance under fuzzy conditions, including the measurement of uncertainty in fuzzy systems, the use of the Center of Gravity (COG) defuzzification technique, the use of fuzzy or grey numbers, etc. [6]. Here a new model using soft sets is developed for the assessment of human-machine performance in a parametric manner. Such kind of models are very useful when the assessment has qualitative rather than quantitative characteristics.

The construction of the model is very simple. In this case the set of the discourse U is the set of all objects which are under assessment. Consider the set E=$\{e_1,e_2,e_3,e_4,e_5\}$ of the parameters e_1=excellent, e_2=very good, e_3=good, e_4=mediocre and e_5=failed and the mapping f: E → Δ(U) assigning to each parameter of E the subset of U consisting of all elements of U whose performance is described by this parameter. Then the soft set

$$F = \{(e_i, f(e_i), i = 1, 2, 3, 4, 5\} \quad (4)$$

represents a qualitative assessment of the elements of U in a parametric manner.

The examples that follow in next section illustrate this model and its applicability in practice. Note that, for a more detailed assessment the set E could include more than five parameters, but this is not usually necessary in practice.

4. Examples

The model of the previous section can be applied to all situations involving assessment. Some characteristic examples are the following:

Example 1: Let U = $\{S_1, S_2, \ldots, S_{30}\}$ be the set of the 30 students of a class. Assume that the first four of them are excellent students, the next eight very good, the following 10 good, the next five mediocre and the rest of them weak students. Then, the soft set

$$F = \left\{ \begin{array}{l} (e_1, \{S_1, S_2, S_3, S_4\}), \\ (e_2, \{S_5, S_6, \ldots, S_{12}\}), \\ (e_3, \{S_{13}, S_{14}, \ldots, S_{22}\}), \\ (e_4, \{S_{23}, S_{24}, \ldots, S_{27}\}), \\ (e_5, \{S_{28}, S_{29}, S_{30}\}) \end{array} \right\} \quad (5)$$

represents in a symbolic way the general performance of the class.

Example 2: Consider again the student class of the previous example and let V= $\{C_1, C_2, \ldots, C_{10}\}$ be the set of the different courses taught in the class. Define a mapping f: E → Δ(V) assigning to each parameter of E and for each student of U the subset of V consisting of the courses for which the performance of the student is characterized by this parameter Then the profile of each student can be represented by a soft set of the form

$$F = \{(e, f(e) : e \in E\} \quad (6)$$

For example the soft set

$$F = \left\{ \begin{array}{l} (e_1, \{C_1, C_8\}), (e_2, \{C_2, C_3, C_5, C_9\}), \\ (e_3, \{C_4, C_6, C_{10}\}), (e_4, \{C_7\}), (e_5, \varnothing) \end{array} \right\} \quad (7)$$

represents the profile of a student who demonstrated excellent performance in courses C_1 and C_8, very good performance in courses C_2, C_3, C_5 and C_9, good performance in courses C_4, C_6 and C_{10} and mediocre performance in course C_7.

Example 3: The coach of a football (soccer) club wants to assess the following characteristics of his players:

D=dribbling, P=passing, F= foot kick (shoot), H=head kick, C=creativity and S=speed. Set U={D, P, F, H, C, S} and define a mapping f: E → Δ(U) assigning to each parameter of E and for each player of the club the subset of U consisting of the player's characteristics assessed by this parameter. In this way the coach can represent each player's profile with the help of a soft set. For example, the soft set

$$F = \begin{Bmatrix} (e_1, \{P,\ C\}), (e_2, \{F\}), \\ (e_3, \{D\}), (e_4, \{S\}), (e_5, \{H\}) \end{Bmatrix} \quad (8)$$

corresponds to a player with excellent passing and creativity, very good shoot, good dribbling, mediocre speed, but not good head kick.

In an analogous way one can express the general players' performance with respect to each characteristic of U. Consider for example dribbling (D) and let $V=\{P_1,P_2,\ldots,P_{20}\}$ be the set of all players. Define a map $f{:}E \to \Delta(V)$ assigning to each parameter of E the subset of V consisting of the players whose dribbling was assessed by this parameter. Then, the general players' performance with respect to dribbling is expressed by a soft set of the form (6). We could have, for example, that

$$F = \begin{Bmatrix} (e_1, \{P_1, P_2, P_3\}), (e_2, \{P_4, P_5, \ldots, P_{10},\}), \\ (e_3,\ \{P_{11},\ P_{12}, \ldots,\ P_{15}\}), \\ (e_4, \{P_{16}, P_{17}, P_{18}\}), (e_5, \{P_{19}, P_{20}\}) \end{Bmatrix} \quad (9)$$

This means that the first three players have excellent dribbling, the next seven very good, the next five good, the next three mediocre and the last two players have no good dribbling.

Example 4: *Case-Based Reasoning (CBR)* is the process of solving problems based on the solutions of previously solved analogous problems [7]. The use of computers enables the CBR systems to preserve a continuously increasing "library" of previously solved problems, referred as past cases, and to retrieve each time the suitable one for solving a given new problem. The CBR process involves the following steps:

- Retrieve (R_1) the most similar to the new problem past case, or cases.
- Reuse (R_2) the information and knowledge in that case to solve the new problem.
- Revise (R_3) the proposed solution.
- Retain (R_4) the parts of this experience likely to be useful for future problem-solving.

The quality of a CBR system can be assessed with the help of soft sets as follows:

Set $U=\{R_1,\ R_2,\ R_3,\ R_4\}$ and define a mapping $f{:}E \to \Delta(U)$ assigning to each parameter of E the subset of U consisting of the CBR steps whose quality was assessed by this parameter. For example, the soft set

$$F = \begin{Bmatrix} (e_1, \{R_1,\ R_4\}), (e_2,\ \{R_2\}), \\ (e_3,\ \{R_3\}), (e_4, \emptyset), (e_5, \emptyset) \end{Bmatrix} \quad (10)$$

corresponds to a CBR system which demonstrated excellent performance at the steps of retrieval and retaining of the past cases, very good performance at the step of reusing them and good performance in revising the selected past case for obtaining the solution of the new problem.

Also, given a set of CBR systems, one can compare their performance, similarly to the previous examples, with respect to each of the steps of the CBR process.

5. Conclusion

The discussion performed in this study leads to the conclusion that soft sets offer a potential tool for a qualitative assessment of human-machine performance in a parametric manner.

An interesting subject for future research could be the development of alternative assessment models under fuzzy conditions by using other types of generalizations of fuzzy sets or related theories [2].

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